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1.0 Mining Methods

1.1 Underground Coal Mining

1.1.1 India's Rank in World Coal Production

According to the Statistical Review of World Energy report published in 2013 by British Petroleum, world's coal production was 3845.3 million tonnes in the year 2012. The world's largest coal producers were, in order, China, the USA, Australia, Indonesia India and Russia. India slipped to 5th position in the share of world's coal production in 2012. Indian coal generally has a high ash content and low calorific value, so most coking coal must be imported. Major Indian coal fields are found in Bihar, West Bengal and Madhya Pradesh.

1.1.2 Characteristics of Coal

The characteristics of different types of coal is given below in Table 1.1

Table 1.1 : Characteristics of Coal

Type of Coal	Rank	Depth of Burial	Maximum temperature during burial	Moisture Content	Fixed Carbon Content
Lignite	1	0.2-1.5 km	25-45°C	30-50%	20-35%
Sub bituminous	2	1.5-2.5 Km	45-75°C	10-30%	35-45%
Bituminous	3	2.5-6 km	75-180°C	5-10%	45-80%
Anthracite	4	> 6 km	>180°C	< 5%	80-96%

Lignite Low Grade **[Low calorific value, High V.M.]**
Sub bituminous ↓
Bituminous
Anthracite High Grade **[High calorific value, low V.M.]**

1.1.3 Banded Constituents of Coal

- Vitrain
- Durain
- Clarain
- Fusain

1.1.4 Size Gradation of Coal

- Steam coal : 50 mm to 200 mm
- Rubble coal : 25 mm to 50 mm
- Slack coal : 0 mm to 25 mm

1.1.5 Analysis of Coal

1.1.5.1 Ultimate Analysis : Ultimate analysis of coal and coke is defined as the determination of the C, H, S, and N in the material and expressed on a moisture free basis, as found in the gaseous products of its complete combustion. Ash is determined as in proximate analysis and is calculated on moisture free basis. Then O can be estimated as :

$$\%O = 100 - (\%C + \%H + \%S + \%N + \%\text{Ash})$$

1.1.5.2 Proximate Analysis : Proximate analysis covers the determination of moisture (M), volatile matter (VM), fixed carbon (FC), and ash (A) in coals and cokes, and is used to establish the rank of coals, to show the ratio of combustible to incombustible constituents, or to provide the basis for buying/selling, and otherwise evaluating coal for various purposes. Thus, it is important to both the suppliers and users of coal to have a rapid, accurate, and reliable procedure for obtaining the proximate analysis.

The Fixed carbon (FC) percentage is obtained from the following equation :

$$FC = 100 - (\%M + \%A + \%VM)$$

Moisture percentage can be determined as

$$\%\,M = \frac{\text{Weight loss after drying the sample}}{\text{initial weight of sample}} * 100$$

Ash is the weight of residue obtained after complete combustion of coal sample. This can be determined as

$$\%\,A = \frac{\text{Weight of residue}}{\text{Weight of sample}} * 100$$

Volatile matter is the weight loss obtained on heating coal sample in the absence of air. This can be determined as

$$\%\,VM = \frac{\text{Weight loss due to volatile matter}}{\text{initial weight of sample}} * 100$$

The above equation for determining the percentage of FC considers on wet basis.

If we analyse on dry basis, moisture content should be zero before analysis. Therefore, on dry basis FC percentage is determined as

$$\%\,FC \text{ on dry basis} = 100 - (\%VM + \%A)$$

Ash percentage or VM should be determined as

$$A \text{ or } VM = \frac{\%A \text{ or } \%VM}{(100 - \%M)} * 100$$

If we analyse on dry ash free basis, then both the ash and moisture content should made zero before analysis. Therefore, on dry ash free basis FC percentage is determined as

$$\%\,FC \text{ on dry basis} = 100 - (\%VM)$$

Ash percentage or VM should be determined as

$$VM = \frac{\%VM}{(100 - \%M - \%A)} * 100$$

1.1.6 Types of Coal

1.1.6.1 Non-Coking coal : The categorisation of non-coking coal is done on the basis of heating value of coal and is given below in Table 1.2.

Table 1.2 : Categorisation of non-coking coal

Grades	UHV Range in Kcals/kg
A	> 6200
B	5601 - 6200
C	4941 - 5600
D	4201 - 4940
E	3361 - 4200
F	2401 - 3360
G	1301 - 2400

[Source : D. J. Deshmukh, Vol. 1]

1.1.6.2 Coking Coal : The categorisation of coking coal is done on the basis of ash percentage and is given below in Table 1.3

Table 1.3 : Categorisation of coking coal

Grades	Range of Ash Percentage
Steel 1	<15
Steel II	15-18
Washery - I	18-21
Washery - II	21-24
Washery - III	24-28
Washery - IV	28-35

1.1.7 Caking or Agglutinating Index

The maximum ratio of sand to coal in a mixture which, after carbonization, gives a coherent mass capable of supporting a 500 g weight.

1.1.8 Protodyaknov Index

It is a simple empirical test for determining the strength of coal. The test consists basically of the crushing of a sample of 25 -75 gms of fragments of coal larger than 10 mm contained in a hollow cylinder by five successive drop hammer blows. The index used for cuttability, drillability, amenability to hydraulic mining of coal seams or for design of the pillar. The Protodyaknov index of Indian coals varies from 1-4.

1.1.9 Composition of different Coal Burning Gases

The composition of various coal burning gases and its corresponding heating value is given in Table 1.4.

Table 1.4 : Composition of different Coal Burning Gases

Gas	Main Composition	Heating Value
Coal gas or coke oven gas	CO - 5.8; H_2 - 57.6; CH_4 - 29.6	4000-5000 kcal/m^3
Producer gas	CO - 20-30; N_2 - 50-60; H_2 - 8-20	1250-1600 kcal/m^3
Water gas	CO - 40.2; H_2 - 50	2600 kcal/m^3

1.1.10 Incubation Period of Coal

The time elapses in coal is 1st exposed to air and spontaneous fire of coal called the incubation period of coal.

1.1.11 Mining Methods

1.1.11.1 Bord & Pillar Mining

1.1.11.1.1 Dimension of Pillar and Galleries at different Depths : Dimension of pillar and galleries in u/g Bord & Pillar mining as per Regulation No. 99 of CMR 1957 is given below in Table 1.5.

Table 1.5 : Dimension of Pillar and Galleries

Depth of seam from the surface (m)	Width of galleries			
	< 3	< 3.6	< 4.2	< 4.8
	The distance between centres of adjacent pillars shall not be less than (in m)			
< 60	12.0	15.0	18.0	19.5
60 < D < 90	13.5	16.5	19.5	21.0
90 < D < 150	16.5	19.5	22.5	25.5
150 < D < 240	22.5	25.5	30.5	34.5
240 < D < 360	28.5	34.5	39.5	45.0
D > 360	39.0	42.0	45.0	48.0

1.1.11.1.2 Percentage of Extraction : Percentage of extraction is the ratio of volume coal extraction during development to total in-situ volume of coal. In Figure 1.1, the size of the pillar is $a_1 \times a_2$ and the width of the galleries in dip and rise directions are b_1 and b_2 respectively. The percentage of extraction can be determined by the following formula.

$$R = \frac{(a_1+b_1)*(a_2+b_2)-a_1*a_2}{(a_1+b_1)*(a_2+b_2)}$$

Where, a_1 & a_2 are length and breadth of the pillar, b_1 & b_2 are the size of galleries in dip and rise direction

For a square pillar and same gallery size, $a_1 = a_2 = a$ & $b_1 = b_2 = b$

$$R = \left[1 - \frac{a*a}{(a+b)*(a+b)}\right]$$

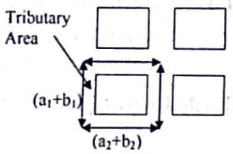


Fig. 1.1 B & P Development Panel

1.1.11.1.3 Tributary Area : It is the area of development gallery shown in Figure 1.1.

Tributary Area (TA) = $(a_1 + b_1) * (a_2 + b_2) - a_1*a_2$

1.1.11.1.4 Load Acting on Pillar : Load acting on the pillar can be determined by

$$P = \gamma * D * \frac{1}{(1-R)} = \gamma * D * \frac{(a+b)^2}{a^2}$$

where, R = percentage of extraction,
D = depth of the pillar from surface
a = length and breadth of the pillar
b = width of the galleries
γ = weight per unit volume of the super-incumbent rock

1.1.11.1.5 Strength of Pillar : The strength of pillar can be determined by the Salamon and munro, 1967 as

$$S = K\frac{W^{\alpha}}{L^{\beta}}$$ **[Salamon and Munro, 1967]**

Where, S = Strength of the pillar in lb/in^2
K = constant (1320 lb/in^2)
W = width of the pillar, in ft.
L = height of the pillar, in ft.
$\alpha = 0.46$ and $\beta = 0.66$

1.1.11.1.6 Factor of Safety of Pillar : Factor of Safety of Pillar is given by

F.O.S. = Strength of pillars (S) / Load actingon pillars (P)

$$= \frac{K * W^{\alpha}}{L^{\beta}} * \frac{(a)^2}{\gamma * D * (a+b)^2}$$

1.1.11.1.7 Line of Extraction :

- Diagonal line of extraction => Generally preferred as it facilitates good roof control
- Step diagonal line of extraction => It is preferred in panels worked in conjunction with hydraulic stowing as it facilitates water drainage
- Steep diagonal line of extraction
- Straight line of extraction

1.1.11.2 Hydraulic Mining : Hydraulic mining of coal comprises two operations viz hydraulic cutting of coal from the seam and the hydraulic transport of the coal and water mixture from the mine. The choice of the mining method depends on the following suitability conditions.

- Can be applicable in any seam thickness but moderate to high desirable for better economy.
- Gradient of 70° required for transport of materials in steel flumes but up to 40° material transportation possible with ultra high molecular weight plastics.
- Coal seam should be soft or ploughable.
- Water pressure of the monitor jet at the nozzle required for breaking of coal is in the range of 10-20 MN/m^2 (101-203 kg/cm^2).
- The cost of hydraulic transport is 10 times less than conventional transport.
- Air pollution problem is less.
- Productivity is higher.
- For pipe line transportations of coal the ratio of coal to pipe diameter should not be more than 1:3.

1.1.11.3 Auger Mining : A mining technique often used by surface mine operations when the overburden becomes too thick for the coal to be mined economically using traditional surface mining methods. Large-diameter, spaced holes are drilled up to 200 ft (61 m) into the coal-bed by an auger. Like a bit used for boring holes in wood, this consists of a cutting head with screw like extensions. As the auger turns, the head breaks the coal and the screw carries it back into the open and dumps it on an elevating conveyor; this, in turn, carries the coal to an overhead bin or loads it directly into a truck. Auger mining is relatively inexpensive, and it is reported to recover 60% to 65% of the coal in the part of the bed where it is used.

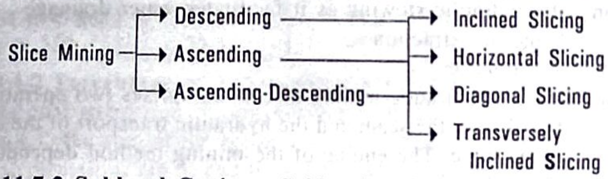
1.1.11.4 Contour Mining : Surface mining that progresses in a narrow zone following the outcrop of a coal seam in mountainous terrain, and the overburden, removed to gain access to the mineral commodity, is immediately placed in the previously mined area, such that reclamation is carried out contemporaneously with extraction.

1.1.11.5 Area Mining : A surface mining method that is carried on in level to gently rolling topography or relatively large tracts of land. Active area mine pits may be several miles long.

1.1.11.6 Strip Mining : Strip mining is usually associated with coal, where a seam or multiple seams are located at a certain elevation or elevations through a mountain or hill.

1.1.11.7 Thick Seam Mining Methods

1.1.11.7.1 Slice Mining : In slice mining, a coal seam is divided into slices of suitable thicknesses and individual slices are worked separately. Slices may be taken in both ascending order or in descending order or in mixed order. The different approaches of mining methods can be categorised as



1.1.11.7.2 Sublevel Caving : Sublevel caving is applicable to thick seams with caveable roof and soft coal. It can also be apply for the hard roof and hard coal, where caving is induced by blasting.

1.1.11.7.3 Integrated sublevel caving or Soutirage Mining : Recovering all the coal of the seam in a single operation from face progressing on the floor.

1.1.12 Explosives, Drilling and Blasting

1.1.12.1 Definition of Explosives : Explosive is a chemical compound or mixture, when exploded by action of heat, impact, gives large volume of gases in a very short time at high temperature & pressure.

1.1.12.2 Classification of Explosives

All commercial explosives are broadly divided in two categories :

- **Low Explosives :** The chemical reaction in low explosives is called deflagration which is a rapid process of combustion without accompanying any shock wave but gives a heaving effect.
  Gun Powder is a type of low explosive and its composition is: Sodium Nitrate - 72% Sulphur - 12% Coal - 16%
- **High Explosives :** Reaction in High Explosives is characterised by an associated shock wave initiated by a detonator. It basically contains. Oxidisers - Such as Ammonium Nitrate Fuel Oil Sensitizers : Methyle Amino Nitrate, Per chlorate Salt, Physical Sensitizer : Entrapped Air Bubbles

1.1.12.3 Permitted Explosive : The explosives incapable of igniting firedamp or coal dust explosion and can be used in u/g.

1.1.12.4 Types of Explosives : Various types of explosives with their brand name is given below in Table 1.6.

Table 1.6 : Types of explosives

Name of the explosives	Type of explosive	Brand Name
P1	Permitted (Unsheathed explosive)	Ajax G, Viking G, Godyne
P2	Non-Permitted in u/g (Sheathed explosive)	Not manufactured in India
P3	Permitted (Equivalent Sheathed explosive)	Unisax G, Permadyne
P4	Explosives approved for special purposes	Not manufactured in India
P5	Solid blasting explosive	Soligex G, Pentadyne

1.1.12.5 Characteristics of Explosives : The characteristics of various types of commercial explosives with respect to velocity of detonation (VOD), composition, form and size of particle is given below in Table 1.7.

Table 1.7 : Characteristics of Explosives

Explosive	Composition	Size	Form	VOD (km/s)
ANFO	Ammonium Nitrate (94 %) + Diesel Oil (6 %)	2 mm	All solid	3.2
Slurry	Oxidiser [$NH_4(NO_3)_2/NaNO_3/Ca(NO3)_2$] + Cross linking agents (Potassium or sodium dichromate/ Antimony/ Boron) + Gel (Starch) + Fuel Sensitisers (TNT/ PETN)	0.2 mm	Solid/ Liquid	3.3
Emulsion	Micro droplets of a ammonium nitrate solution surrounded by fuel phase	.001 mm	Liquid	5-6

1.1.12.5.1 Available Explosive Energy : The energy that an explosive is able to deliver to do useful work is defined as available explosive energy. Actual amount of energy delivered in any blast is unknown as too many variables exist.

1.1.12.5.2 Absolute Weight Strength (AWS) : This is the theoretical absolute energy available, based on the ingredients of the explosive. Thus is usually quoted as MJ/kg of explosive.

1.1.12.5.3 Relative Weight Strength (RWS) : This is the ratio of energies of a unit weight of explosive compared to an equal weight of ANFO. RWS for an explosive is the AWS of the explosive divided by the AWS of ANFO, expressed as a percentage.

$$RWS_{explosive} = \frac{AWS_{explosive} * 100}{AWS_{ANFO}}$$

1.1.12.5.4 Absolute Bulk Strength (ABS) : The energy available in a unit volume of explsive is defined as ABS. ABS for an explosive is its AWS multiplied by its density.

$$ABS_{explosive} = AWS_{explosive} * X\rho_{explosive}$$

Where $\rho_{explosive}$ is the density of the explosive and ABS is in Cal/cc.

1.1.12.5.5 Relative Bulk Strength (RBS) : This is the ratio of the energies available in a given volume of explosive compared to an equal volume of ANFO. RBS for an explosive is the ABS of the explosive divided by the ABS of ANFO, expressed as a percentage.

$$RBS_{explosive} = \frac{ABS_{explosive} \times 100}{ABS_{ANFO}}$$

1.1.12.6 Powder Factor : This is defined as the tonne of coal produced per kg of explosive. For solid blasting it lies in the range of 1.5 to 2.5 te / kg and for cut blasting it is lies in the range of 4 to 6 te/kg.

This can be determined as

$$\text{Power Factor} = \frac{\text{Coal produced (tonne)}}{\text{Explosive required (kg)}}$$

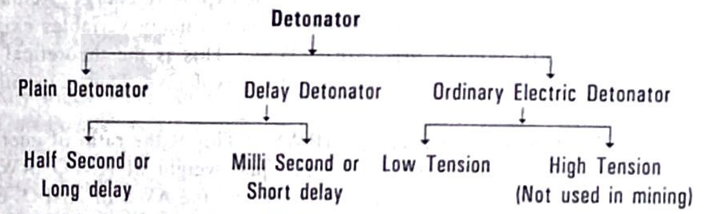
1.1.12.7 Maximum Permissible Charge in a Shot Hole : Maximum permissible charge in a shot hole as per DGMS Stipulations are given in Table 1.8.

Table 1.8 : Maximum Permissible Charge in a Shot Hole

Types of explosives	Degree of Gassiness/type of application	Max. permissible charge per shot hole (gms)
P1	Degree 1 mines, Cut face	800
P3	Degree 1, II & III mines, Cut face	1000
P5	Degree 1 mines, Cut blasting	1000
P5	Degree 1 mines, Solid blasting	565

Note : The coal must be pre-cut to provide a free face and the length of the shot holes must be 15 cm less then the length of the cut.

1.1.12.8 Detonator : Different types of detonators used for blasting in mines are classified as :


Detonator
Plain Detonator
Delay Detonator
Ordinary Electric Detonator
Half Second or Long delay
Milli Second or Short delay
Low Tension
High Tension (Not used in mining)

1.1.12.9 Veolocity of Detonation (VOD) : VOD is the speed that the detonation wave travel through the explosive, usually expressed in metres per second (m/s). It is influenced by rock type, charge diameter, explosive density, particle size, degree of confinement, and primer. VOD is a guide to determining the efficiency of the explosive.

1.1.12.10 Detonation Pressure (P_d) : Pressure in the detionation reaction zone as it progresses along a charge, expressed in MPa. This is what generates the shock pulse in rock. This can be determined as

$$P_d = 0.25 * VOD^2 * \rho$$

Where, P_d is the detonation pressure, VoD is velocity of detonation and ρ is density of explosive.

1.1.12.11 Types of Detonators and their Characteristics : Types of detonator and their characteristics are given below in Table 1.9.

Table 1.9 : Type of Detonators and their Characteristics

Detonator Characteristics	Plain Detonator	Ordinary Electric Detonator	Delay Detonator
Type	Non-delay (Instantaneous)	Non-delay (Instantaneous)	Delay
Base Charge	P.E.T.N*	P.E.T.N	P.E.T.N
Priming Charge	A.S.A**	A.S.A	A.S.A
Fired by	Ignited by Safety Fuse	Passing electric current through fuse head	Passing electric current through fuse head
Delay element	Not used	Not used	Copper or Brass sleeve
–	Ordinary	Low tension and High tension type	Low tension

*P.E.T.N : Penta-Erythritol Tetra-Nitrate

**A.S.A. : Lead Azide, Lead Styphnate and Aluminium Powder

1.1.12.12 DGMS Stipulations on Delay Interval :

- For non-incendive delay detonators in BOS (solid blasting), the maximum delay period between the 1st and last shot is 150 ms in degree I & II gassy coal mine and 100 ms in degree III gassy coal mine.
- The delay period between two consecutive shots with different delay numbers will not exceed 60 ms.
- Distance between two adjacent shots with different delay numbers will not come closer than 0.6 m at the explosive charge ends.

1.1.12.13 Drilling Pattern : The drilling pattern for blasting of coal/ rock depends on the hardness of coal/ rock and the types of blasting.

- Pyramid cut : The shot holes are so inclined towards the centre as to appear to meet at one point and when blasted create a pyramid.
- Wedge cut : Used in BOS in coal in galleries B&P working
- Drag cut :
- Fan cut : Used in BOS in coal in galleries B&P working
- Burn cut : Parallel holes at right angles to the face.
- Coromat cut : Parallel holes at right angles to the face.
- Ring drilling : This is used in blasting galleries in coal mines and sublevel stoping in metal mines.

1.1.12.14 Drilling Method and their Rock Cutting Principles :

Diamond drilling	Rotary abrasive
Percussive drilling	Impact action
Rotary roller drilling	Cutting and ploughing
Jet piercing	Thermal spalling

1.1.12.15 Common Methods of Exploration Drilling : Various types of drilling with their specific applications are given below in Table 1.10.

Table 1.10 : Types of Exploration Drilling

Types of drill	Size of drill hole	Economic depth	Application
Percussive drilling	50–75 mm	300 m	Soft and medium hardness rocks
Churn or Cable drilling	75–500 mm	300 – 600 m	Placer deposits, moderately soft formations
Diamond drilling	30–200 mm	100 -3000 m	Any rock formation except fissured
Calyx or chilled shot drilling	75–1800 mm	< 450 m	Any rock except hard

1.1.12.16 Standards of Drill Hole Diameters and Core Sizes : Standards of drill hole diameters with their core sizes are given below in Table 1.11.

Table 1.11 : Standards of Drill Hole Diameters

Standard	Drill rod outside diameter	Hole diameter	Core diameter
NX/ NW	67	75	54
BX/ BW	54	60	40
AX/ AW	44	47	28
EX/ EW	35	38	21

1.1.12.17 Ground Vibration due to Blasting : Ground vibrations is considered as one of the most undesirable side effects of rock residential buildings in the vicinity of mining operations and thus may lead to conformation and hamper normal production of the mine. Broadly, ground vibrations may be controlled by using the following safety measures.

- Use Delay Detonators
- Optimization of the firing time by trials
- Proper Blast Design
- Lower Charge Per Delay
- Direction of Initiation

This prediction of ground vibration for long cylindrical charges in which any linear dimension should be scaled with square root of the charge weight and is given by the equation

$$V = K\left(\frac{D}{\sqrt{Q}}\right)^{E}$$

Where, V is the peak particle velocity, D is the distance of the measuring transducer and Q is the maximum charge weight per delay. K and B are site constants to be determined by regression analysis.

1.1.12.18 Threshold Value of Ground Vibration : DGMS (India) have specified permissible standards of peak particle velocity (in millimeter per second) depending on type of structure and dominant excitatory frequency. Permissible peak particle velocity at the foundation level of structure in mining areas is given below in Table 1.12.

Table 1.12 : Permissible peak particle velocity

Type of Structure	PPV in mm/sec at a foundation level of Structure at a frequency		
	< 8Hz	8·25 Hz	> 25Hz
Building structure not belonging to owner			
1. Domestic House structure kacha brick & cement	5	10	15
2. Industrial building RCC& Framed Structure	10	25	25
3. Objects of historical importance and sensitive structure	2	5	10
Building belonging to owners with limited span to life			
1. Domestic houses structure Kacha bricks & cement	10	15	25
2. Industrial building RCC & Framed Structure	15	25	50

1.1.13 Subsidence in Mines

1.1.13.1 Introduction : The fractures extend vertically to the surface from the boundaries of the underground excavation.

Geometry of subsidence features over Bord & Pillar workings :

- Sinkholes : these are abrupt depressions in the ground surface that occur as a result of collapse of the overburden into an underlying mine void. the diameter of the sinkhole increases with depth.
- Troughs : these are dish shaped depressions at ground surface. In this case the overburden does not break to form a sinkhole.
- Ground Cracks : These are distinct vertical and horizontal displacements in the overburden.

1.1.13.2 Angle of Draw : This indicates the limit up to which subsidence takes place in the surface. The subsidence profile of three different situation viz. Critical Area, Sub-Critical Area, and Super-Critical Area are shown in Figure 1.2, Figure 1.3, and Figure 1.4 respectively.

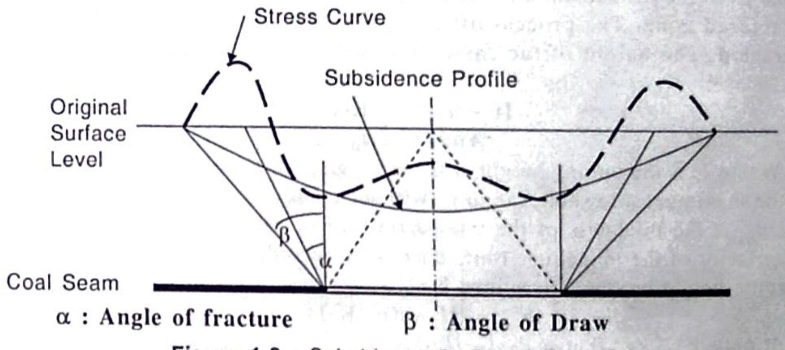


Figure 1.2 : Subsidence Profile of Critical Area

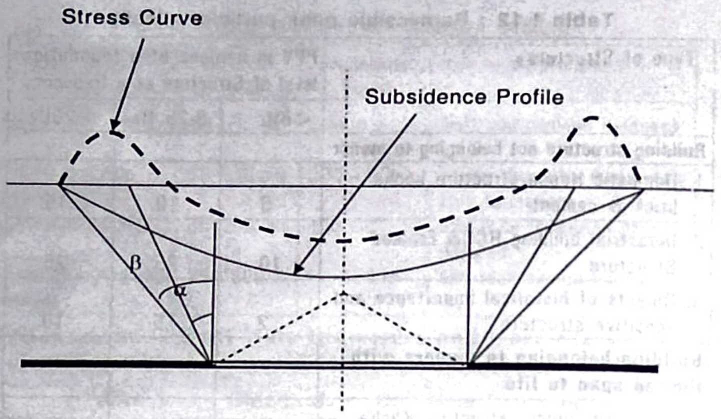


Figure 1.3 : Subsidence Profile of Partial Area or Sub-critical Area

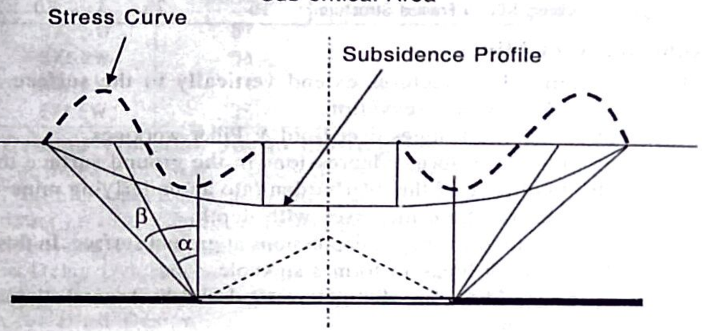


Figure 1.4 : Subsidence Profile of Partial Area or Super-critical Area

1.1.13.3 Caving height or thickness of the immediate roof : Normally, caving initiates from the lowest strata in the immediate roof and propagates upward into the fractured zone. The process of caving starts as soon as the ore body/coal is extracted. The height of the caving holds the following relationship (Peng, 2008).

$$H - d = h_{im}(K-1)$$

$$\text{And, } d \leqslant d_0$$

Where *H* is the mining height, *d* is the sagging of the lowest un-caved strata, d_0 is the maximum allowable sagging (without breaking) of the lowest un-caved strata, h_{im} is the thickness of the immediate roof or caving height, and K is the bulking factor of the immediate roof. Therefore, according to the above equation, the caving height can be determined by:

$$h_{im} = (H - d)/(K-1)$$

It must be noted that if $d=d_0=H$, then $h_{im}=0$, which means no caving: that is, the roof sags gradually until it touches the floor. On the other hand, if $d=d_0=0$:

$$h_{im} = H/(K-1)$$

EXERCISE - 1.1

1. A coal heading 4 m wide and 2.5 m high has an advance of 1 m per cycle. The amount of explosive used in blasting is 6 kg. Taking specific gravity of coal as 1.5, the powder factor is
   a. 1.66 te/ kg b. 2.50 te/ kg
   c. 2.99 te/ kg d. 3.32 te/ kg [GATE 1999]
2. In an underground coal mine, 1800 workers are employed. The average production from the mine is 900 tonnes per day. The O.M.S. of the mine is
   a. 0.24 te b. 0.60 te
   c. 0.65 te d. none of the above [GATE 1999]
3. In a B & P mining, the width of the pillar and galleries are 19.5 m and 3 m respectively. The percentage of extraction during development is
   a. 24.8 b. 25.2
   c. 29.3 d. 30.5
4. In Bord & Pillar panels worked in conjunction with hydraulic stowing, extraction line preferred is
   a. Step diagonal b. Diagonal
   c. Steep diagonal d. Straight line
5. The drilling method which can apply for most wider range of depth is
   a. Percussive drilling b. Cable drilling
   c. Diamond drilling d. Calyx shot drilling
6. Match the following for standard diamond rods

Specification	Outer diameter in mm
P. AW	p. 34.9
Q. BW	q. 44.4
R.EW	r. 54.0
S.NW	s. 66.7

   a. P-r, Q-q, R-r, S-p b. P-r, Q-p, R-s, S-q
   c. P-q, Q-r, R-p, S-s d. P-q, Q-r, R-s, S-p [GATE 2010]
7. A board and pillar panel having square pillars is designed for 30% extraction during development. If the gallery width is 5 m, the side of the pillar in m is
   a. 25.6 b. 28.6
   c. 32.6 d. 24.6 [GATE 2015]
8. A seam is worked at a depth of 70 m from surface and the width of the galleries is 3.2 m, the dimension of pillars should not exceed
   a. 15.0 m b. 16.5 m
   c. 19.5 m d. 13.3 m
9. A seam is worked at the depth of 120 m from surface and has a width of galleries of 3.4 m. The percentage of extraction during development should be restricted to

a. 33.5 % b. 26.2 %
c. 31.8 % d. 42.2 %

10. The average vertical stress in a coal pillar of a B & P panel is found to be 8 MPa. The depth of the pillar and the average density of overburden rock are 200 m and 2500 kg/ m^3 respectively. Assuming pillar stress to be the same in all coal pillars of that panel, the extraction ratio of the panel is
a. 0.387 b. 0.513
c. 0.613 d. 0.751

11. In a coal mine the depth of working is 200 m from surface, width of galleries 3 m, height of galleries 3 m and distance of square pillars between centres of bords is 22.5 m. What is strength of the pillar
a. 128.53 kg/cm^2 b. 138.97 kg/cm^2
c. 142.05 kg/cm^2 d. 139.97 kg/cm^2

12. In any working below ground there should be provided at least two shaft or incline which should be away from each other by a minimum distance
a. 12.5 m b. 13.5 m
c. 14.5 m d. 15.4 m

13. Maximum number of faces available for working in a shift in a 5 heading bord and pillar development district is
a. 10 b. 12 c. 13 d. 15

14. The maximum number of coal faces in an underground bord and pillar development district is 13. The number of headings in the district is
a. 3 b. 5 c. 6 d. 7 [GATE 2011]

15. Coalification has taken place in the order of
a. Peat, lignite, bituminous, anthracite
b. lignite, Peat, bituminous, anthracite
c. Peat, lignite, anthracite, bituminous
d. bituminous, Peat, lignite, anthracite

16. The method of mining a coal seam 8.5 m thick is by
a. B & P method in lifts
b. L/W methods in lifts
c. A combination of B & P and L/W methods
d. L/W sublevel caving

17. In horizon system of mining lateral are driven
a. At right angles to the strike of the seam
b. Parallel to the strike of the seam
c. At 45° to the dip of the seam
d. At any angle to the strike of the seam

18. Which one of the following HEMM is a tyre mounted?
a. LHD b. SDL c. Dozer d. Shovel

19. Wagon drill working at minimum compressed air pressure of
a. 6 kg/ cm^2 b. 7 kg/ cm^2
c. 8 kg/ cm^2 d. 9 kg/ cm^2

20. Square pillars in B & P panel are of size 30 m centre to centre and the galleries are of width 4 m. The tributary area for each pillar in m^2 is
a. 224 b. 256 c. 676 d. 884

21. An underground panel in a coal mine is being developed on B & P system. The pillar size is 30 m centre to centre and the adjacent galleries are 4 m wide. The percentage of extraction during development is
a. 17.6 b. 19.5 c. 24.9 d. 26.2

22. A bord and pillar panel will be developed with square pillars of size 25 m and an extraction ratio of 30 %. The width of the gallery in m is
a. 4.2 b. 4.6 c. 4.8 d. 5.3

23. The mining method used in thick and steep coal seam, where the slices are neither horizontal nor parallel to the inclination of the seam, is called
a. transverse slicing b. inclined slicing
c. horizontal slicing d. integral caving [GATE 2006]

24. In the order of the chronological development, the longwall support systems are arranged as
P Powered support
Q Link bar and friction support
R Frame support
S Hydraulic support
a. P > Q > R > S b. R > S > Q > P
c. S > R > P > Q d. Q > S > R > P [GATE 2008]

25. A longwall face cut by double back shuffle method can be only worked with
a. fixed drum shearer
b. single ended ranging drum shearer
c. double ended ranging drum shearer
d. plough [GATE 2011]

26. 20 plain detonators in series, each of 2d resistance, are fired by a DC exploder supplying a current of 1.25A. If 250 mJ energy is spent to fire the detonators, the time required in millisecond after detonator initiation is
a. 4 b. 8 c. 12 d. 16 [GATE 2011]

27. Select the correct order with respect to the core sizes
a. AX > BX > EX > NX
b. NX > BX > AX > EX
c. AX > BX > NX > EX
d. EX > AX > BX > NX

28. Hydraulic mining of coal is normally applicable to mining seams, which are
a. thick and steeply inclined and have soft coal
b. thick and flat gradient and have soft coal
c. thin and flat gradient and have soft coal
d. thin and steeply inclined and have soft coal

29. For hydraulic transportation of solids in pipelines, the ratio of the size of solids to the pipe diameter should not be more than
a. 1 : 2 b. 1 : 3 c. 1 : 4 d. 1 : 7

30. The layout of the pipelines for hydraulic stowing should be such that the hydraulic profile
    a. is above the geometric profile
    b. is below the geometric profile
    c. is on the geometric profile
    d. there is no relation with the geometric profile

31. Contiguous seams means the parting between two seams is within
    a. 10 m b. 9 m c. 8 m d. 12 m

32. For a delay blasting of 10 circuits, a sequential timer is set on 25 ms between circuit initiations. If the nominal firing time of a detonator is 400 ms, the total add on time in ms for 10 circuits will be
    a. 650 b. 400
    c. 625 d. 500 [GATE 2002]

33. A shock tube initiating system, such as nonel
    a. doesn't need detonators for initiation
    b. It can be used in under-water condition
    c. It is not effected by static electricity or strong currents
    d. It creates lots of noise [GATE 1995]

34. In rock blasting the term "impedance of medium" is the product of
    a. wave pressure and poisson's ratio
    b. wave pressure and density of medium
    c. propagation velocity and poisson's ratio
    d. propagation velocity and density of medium [GATE 1998]

35. A coal heading 4 m wide and 2.5 m high has an advance of 1 m per blasting round. The amount of explosive used in the blasting round is 6 kg. If the sp. gr. of coal is 1.5, the powder factor is
    a. 1.66 te/kg b. 2.50 te/kg
    c. 2.99 te/kg d. 3.32 te/kg [GATE 1999]

36. The delay interval in short delay detonator is
    a. 25 ms b. 30 ms
    c. 15 ms d. 35 ms

37. Emulsion explosive is a mixture of
    a. Ammonium nitrate, TNT, NG
    b. Ammonium nitrate, TNT, guar gum
    c. Ammonium nitrate, water, NG
    d. Ammonium nitrate, water, hollow micro-balloons

38. The weight strength of ANFO of specific gravity 0.8 is 912 kcal/kg. The weight strength of an emulsion explosive of specific gravity 1.2 is 850 kcal/kg. Bulk strength of the emulsion explosive relative to ANFO in percentage is
    a. 101.8 b. 121.6
    c. 139.8 d. 149.6 [GATE 2014]

39. To prevent slope failure the blasting system selected should be
    a. Pre-split b. Muffled
    c. Deck charging d. All of the above

40. The group of holes which are detonated on the first few delays are called
    a. Easers b. Trimmers
    c. Cut holes d. None of the above

41. The ratio of length of explosive column to the hole diameter which will work as a spherical charge, is
    a. $L/D < 6$ b. $L/D > 6$
    c. $L/D < 4$ d. $L/D > 4$

42. For a 25 mm diameter spherical charge, the maximum allowable charge length in cm is
    a. 15.0 b. 25.0
    c. 30.0 d. 150.0 [GATE 2013]

43. Incubation period is NOT related to
    a. crossing point temperature of coal
    b. panel size
    c. seam thickness
    d. explosibility of coal dust [GATE 2013]

44. A developed panel for a coal seam having incubation period of 6 months has 32 square pillars under extraction having size 25m and height 3.0m. Density of coal is 1.4 tonne/m^3. Extraction ratio during depillaring is expected to be 75%. To depillar the panel within the incubation period, assuming 25 working days in a month, the production from the panel in tonne/day is
    a. 420 b. 480 c. 560 d. 680 [GATE 2010]

45. The rotational speed and cutting velocity of a drill are 350 rpm and 71.50 m/min respectively. The diameter of the rotary drill bit in mm is
    a. 65 b. 67 c. 68 d. 70 [GATE 2013]

46. Among the following options, the specific energy for rock-drilling is lowest in
    a. rotary diamond drilling b. rotary roller drilling
    c. percussive drilling d. jet piercing [GATE 2013]

47. An underground coal mine employing 1200 persons experiences 2 fatal injuries, 6 serious injuries and 8 reportable injuries during the year 2013. The total injury rate per 1000 persons employed for the year is
    a. 13.34 b. 12.00 c. 6.34 d. 5.00 [GATE 2015]

48. Given the following,

Machine	**Component**
P. Dint header	1. Cowl
Q. Coal plough	2. Cutting chain
R. Road header	3. Loading apron
S. Shearer	4. Static set of bits

The correct match is
    a. P-4,Q-2,R-3,S-1
    b. P-3,Q-4,R-2,S-1
    c. P-2,Q-3,R-4,S-1
    d. P-2,Q-4,R-3,S-1 [GATE 2013]

49. Given the following,

Mining method	**Technique**
P Thick seam extraction	1. Double unit face
Q Bord and pillar extraction	2. Jet cutting
R Longwall face development	3. Inclined slicing
S Hydraulic mining	4. Half moon method

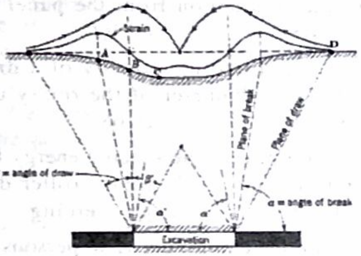
The correct match is
a. P-3, Q-4, R-2, S-1 b. P-1, Q-4, R-2, S-3
c. P-3, Q-2, R-1, S-4 d. P-3, Q-4, R-1, S-2 [GATE 2013]

50. Given the following,

Excavating/loading machine	**Transportation scheme**
P Bucket Wheel Excavator	1. Mine tub
Q Continuous Miner	2. Armoured flexible chain conveyor
R Shearer	3. Shiftable Conveyor
S Load Haul Dumper	4. Shuttle car

The correct match is
a. P-3, Q-2, R-4, S-1 b. P-3, Q-4, R-2, S-1
c. P-3, Q-2, R-1, S-4 d. P-1, Q-4, R-3, S-2 [GATE 2013]

51. A sub-critical subsidence profile is shown in the figure below. The points A, B, C, and D represent respectively the points of



a. zero vertical displacement, maximum tension, inflexion, maximum compression
b. inflexion, maximum tension, maximum compression, zero vertical displacement
c. maximum tension, inflexion, maximum compression, zero vertical displacement
d. maximum compression, maximum tension, inflexion, zero vertical displacement [GATE 2013]

52. In case of an opencast mine blasting, the danger zone comprises of area within the diameter of
a. 400 m b. 600 m
c. 500 m d. 300 m

53. The place in mine, where explosive is stored is called
a. Shelter b. Reserve station
c. Magazine d. None of the above

54. The basic constituents of an emulsion explosive consist of oxidizers, fuels and surfactant with weight percentages of X, Y, Z respectively. Identify the correct relationship from the followings
a. X > Y > Z b. Y > X > Z
c. Y > Z > X d. X > Z > Y

55. In a drift of 2.4 m X 3.0 m size, the pull obtained per blast is 1.8 m. the total explosives consumed per blast is 36 kg. If the tonnage factor is 0.36 m^3/te, the powder factor is
a. 0.5 kg/ te b. 1.0 kg /te
c. 1.5 kg/ te d. 2.0 kg/ te

56. Reaction time of seismic detonators is
a. < 5 ms b. > 10 ms
c. < 1 ms d. < 1 ms & > 5 ms

57. At zero oxygen balance, the ratio of oxidizers to fuel ratio for ANFO is
a. 92 : 8 b. 93 : 7
c. 94 : 6 d. 95 : 5

58. Deflagration is
a. Rapid explosion
b. Explosion and burning
c. Rapid burning but not an explosion
d. None of the above

59. Match the following :

Blast Problem	Cause
P Misfire	1. Poor stemming
Q Vibration	2. Low current
R Blown-out shot	3. Excess charge
S Cut-off shot	4. Improper delays

a. P- 3, Q- 2, R – 4, S-1 b. P- 4, Q- 1, R – 2, S-3
c. P- 2, Q- 3, R – 1, S-4 d. P- 1, Q- 2, R – 4, S-3 [GATE 2008]

60. Match the following :

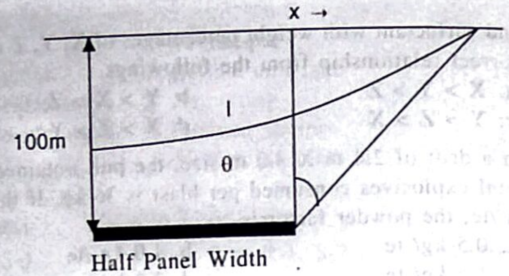
Access	Haulage	Mineralisation
P Shaft	1. Track	a. Moderate depth
Q Decline	2. Trackless	b. Deep seated
R Adit	3. Hoisting	c. Hillock

a. P- 1-a, Q- 3-b, R -2-c b. P- 3-b, Q- 2-a, R–1-c
c. P- 2-a, Q- 1-b, R -3-c d. P- 2-b, Q- 3-c, R – 1-a [GATE 2008]

61. The ground in central portion of the subsidence trough is subjected to
a. lateral tension b. vertical compression
c. lateral compression d. none of the above

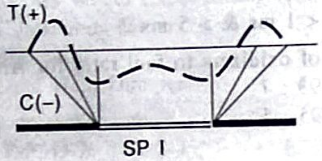
62. The subsidence profile over a longwall panel in a perfectly horizontal seam is given as, s = 0.746 – 0.750 tanh (0.05 x)
Where, s = subsidence at any point on the profile, m
x = horizontal distance measured from point of inflection (I) in the direction of decreasing subsidence, m
If the depth of the panel is 100 m, the angle of draw (θ) in degrees is

a. 15.4
b. 25.4
c. 30.6
d. 59.1

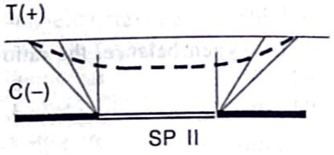


Half Panel Width

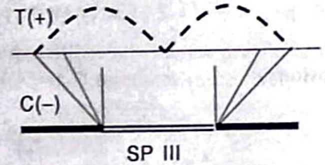
63. Lateral surface strain profile (SP) caused due to extraction of a longwall panel can be approximated by



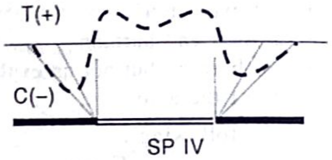
SP I



SP II



SP III



SP IV

a. SP I b. SP II
c. SP III d. SP IV

64. The angle of draw in a trough subsidence helps in determining the
a. Maximum subsidence b.Extent of surface subsidence
c. Plane of fracture d.Critical width of the opening.
[GATE 2010]

65. An underground panel extracted in a coal mine has a strike length of 500 m. After subsidence survey, it was found that the length of a straight line between the extreme limits of the subsided area on surface is 593.2 m along the strike direction. The depth of the extracted panel from the surface is 100 m. The angle of draw for the overlying strata assuming identical subsidence on either side of the centre of the panel is
a. 18° b. 25°
c. 35° d. 28°

66. In an underground coal mine, a freshly exposed roof can be supported by a temporary support in the form of
a. Triangular chocks b. Screw props
c. Safari supports d. Hydraulic props [GATE 2010]

67. The roof bolt that follows the principle of point anchorage is
a. expansion shell bolt b. full column grouted bolt
c. split set bolt d. swellex bolt
[GATE 2012]

68. Equipment used in mining of placer deposits is
a. auger b. wagon drill
c. rope saw d. riffle box
[GATE 2012]

69. Match the following :

Mining Method	Operation
P Bord and Pillar	1. Longhole radial drilling
Q Sublevel caving	2. Splitting and Slicing
R Longwall retreating	3. Loosening under strata pressure
S Integrated Caving	4. Mechanical cutting

a. P- 1, Q- 4, R – 3, S-2 b. P- 2, Q- 3, R – 1, S-4
c. P- 4, Q- 2, R – 3, S-1 d. P- 2, Q- 1, R – 4, S-3
[GATE 2008]

70. The wt% of solids in a sand-water stowing pipe is 60. If the solids density is 3000 kg/m^3, the pulp density of the slurry in kg/m^3 is
a. 1380 b. 1420
c. 1560 d. 1667 [GATE 2008]

71. In a 4.2 m wide and 3.0 m high gallery in a coal seam, twelve shot holes are blasted per round. The holes are charged with 2 explosive cartridges of 435g each. If the powder factor of the blast is 2.2 tonne/ kg and specific gravity of coal is 1.4, the pull per round of blast in m is
a. 1.45 b. 1.70
c. 1.30 a. 4.06 [GATE 2015]

72. In longwall mining subsidence phenomenon, the “angle of break” is the angle between
a. The vertical line at the panel edge and line connecting the panel edge and zero subsidence on the surface
b. The vertical line at the panel edge and line connecting the panel edge and point of critical deformation on the surface
c. The vertical line at the panel edge and line connecting the panel edge and point of the maximum tensile strain on the surface
d. The horizontal line and the line connecting the panel edge and zero subsidence on the surface
[GATE 2009]

73. Pocket and wing technique of pillar extraction is relevant to
a. Room and pillar method b. Bord and pillar method
c. Wongawilli method d. Shortwall method [GATE 2009]

74. Apart from mining of coal, the longwall mining method has been practiced for mining the deposits of
a. Copper b. lead and zinc
c. manganese d. pyrite and phosphate
[GATE 2012]

75. In longwall caving, the thickness of immediate roof is calculated from
a. Bulking factor and width of longwall face
b. Seam thickness and width of longwall face

c. Seam thickness and Bulking factor
d. Bulking factor and length of the panel [GATE 2008]

76. When a double ended ranging drum shearer cuts coal in a longwall face,
a. both the drums rotate in the same direction keeping the front drum up and the rear drum down
b. both the drums rotate in the opposite direction keeping the front drum up and the rear drum down
c. both the drums rotate in the opposite direction keeping the front drum down and the rear drum up
d. both the drums rotate in the same direction keeping the front drum down and the rear drum up
[GATE 2012]

77. Continuous miner and shuttle car combination is NOT applicable in mining with
a. rib pillar extraction technique
b. Wangawilli system
c. room and pillar method
d. longwall method [GATE 2012]

78. A shearer is deployed in a mine where the specific energy consumption for cutting coal is 800 kJ/m^3. The specific gravity of coal is 1.2. If the machine produces 700 te/h, the electrical power consumption in kW of the shearer at 65% motor efficiency is
a. 149.4 b. 199.4
c. 219.4 d. 239.4 [GATE 2012]

79. A longwall panel with a face height of 3.0 m and face length of 150.0 m is worked in 3 shifts per day employing 40 men per shift. The depth of the web of the shearer cutting coal is 0.5m. The unit weight of the coal is 1.4 tonne/m^3. Two full face cuts are executed per shift. The daily production from the panel in tonne is
a. 945 b. 1240
c. 1890 d. 2530 [GATE 2010]

80. In the above question, the panel OMS in tones is
a. 12.75 b. 15.75
c. 8.75 d. 5.25 [GATE 2010]

81. Match the following :

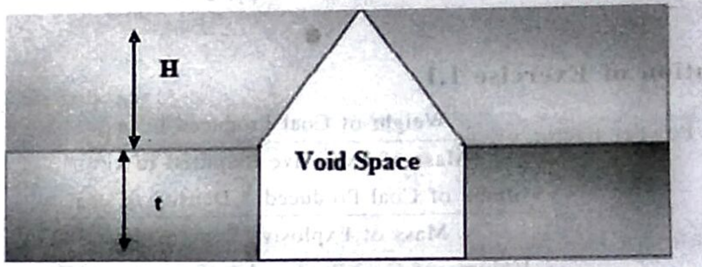
Mining system	Face supports
P Mechanized longwall in flat seam	1. Cable bolting
Q Blasting gallery method	2. Shield support
R Mechanized longwall in steep seam	3. Alpine breaker line support
S Wangawilli method for 3 m thick coalseam	4. Troika shield support

a. P-2, Q-1, R- 4, S-3 b. P-4, Q-1, R-3, S-2
c. P-4, Q-2, R-3, S-1 d. P-2, Q-3, R- 4, S-1 [GATE 2012]

82. A 2.5 m thick coal seam lying at an average depth of 100 m has been developed by bord and pillar method. The width of the square pillars is 30 m (centre to centre) and the gallery width is 4 m. The average density of the overlying strata is 26 kN/m^3 and the pillar strength is 4500 kN/m^2. Extraction ratio during the development of the pillar is
a. 0.129 b. 0.148
c. 0.218 d. 0.249 [GATE 2012]

83. In the above question, the safety factor of the pillar is
a. 1.1 b. 1.3 c. 1.5 d. 1.7 [GATE 2012]

84. A flat coal seam of thickness (t) 3 m is excavated and broken roof rock has completely filled the space created due to extraction as shown in the figure. If the bulking factor of roof rock is 1.2, the caving height (H) in m is



a. 15 b. 16 c. 14 d. 12 [GATE 2014]

85. Six detonators each having resistance of 1.5 ohm are connected in parallel. A 15 V exploder is connected to the detonators by two single-core cables of resistance 3 ohm each. The current in the circuit in Ampere is
a. 1.8 b. 2.4 c. 1.6 a. 3.2 [GATE 2014]

86. As per the Indian Electricity Rules 1956, the maximum permissible length of a flexible cable used with an electric rope shovel in m is
a. 100 b. 200 c. 300 d. 500 [GATE 2014]

87. As per the Indian Electricity Rules 1956, the maximum permissible length of a flexible cable used coal cutting machine for longwall operation in m is
a. 100 b. 200
c. 300 d. 500

88. As per the Indian Electricity Rules 1956, the maximum permissible length of a flexible cable used with any portable or transportable apparatus in mine is
a. 100 m b. 200 m
c. 300 m d. 500 m

89. The maximum voltage for handheld portable apparatus used in underground mines is
a. 30 V b. 125 V
c. 220 V d. 330 V

Answer

1. b 2. d 3. a 4. a 5. c 6. a 7. a
8. d 9. c 10. a 11. d 12. b 13. b 14. c

15. a 16. a 17. a 18. a 19. a 20. a 21. c
22. c 23. a 24. b 25. c 26. a 27. d 28. a
29. b 30. a 31. b 32. c 33. c 34. d 35. b
36. a 37. d 38. c 39. a 40. c 41. a 42. d
43. d 44. c 45. a 46. c 47. a 48. d 49. d
50. b 51. c 52. c 53. c 54. d 55. b 56. c
57. c 58. c 59. c 60. b 61. a 62. c 63. a
64. b 65. b 66. c 67. a 68. d 69. d 70. d
71. c 72. a 73. a 74. d 75. c 76. b 77. b
78. b 79. c 80. b 81. a 82. d 83. b 84. a
85. b 86. c 87. b 88. a 89. b

Solution of Exercise 1.1

1. Powder Factor $= \dfrac{\text{Weight of Coal Produced in te}}{\text{Mass of Explosive Required in kg}}$

$$= \frac{\text{Volume of Coal Produced * Density of coal}}{\text{Mass of Explosive Required}}$$

$$= \frac{\text{Volume of Coal Produced * (S.G. of coal*Density of water}}{\text{Mass of Explosive Required}}$$

$$= \frac{\{(4*2.5*1)*(1.5*1000)\}/1000}{6} = 2.5\ \text{te/kg}$$

2. Output per man shift (O.M.S.)

$$= \frac{\text{output per shift}}{\text{Total manpower per shift}} = \frac{(900/3)}{(1800/3)} = 0.5$$

[Assuming, 3 shifts per day]

3. Percentage of extraction (R) $= \left[1 - \dfrac{a*a}{(a+b)*(a+b)}\right]*100$

Where, a = length and width of pillar [Assuming, a square pillar]
b = width of galleries

Therefore, $R = \left[1 - \dfrac{19.5*19.5}{(19.5+3)*(19.5+3)}\right]*100 = 24.8\%$

7. Given data :
Width of the galleries (b) = 5 m
Percentage of extraction (R) = 30 %
Let the length and breadth of the square pillar is 'a' m
We have, percentage of extraction (R)

$$= \left[1 - \frac{a*a}{(a+b)*(a+b)}\right]*100$$

$$=> 30 = \left[1 - \frac{a^2}{(a+5)^2}\right]*100$$

$$=> 0.30 = \left[1 - \frac{a^2}{(a+5)^2}\right]$$

$$=> \frac{a^2}{(a+5)^2} = 0.7$$

$$=> \frac{a^2}{a+5} = 0.836$$

$$=> a = 25.6\ \text{m}$$

8. Refer Table 1.5,
Minimum distance between adjacent pillar centre to centre corresponding to working depth 70 m and gallery width 3.2 m = 16.5 m
Therefore, pillar width = 16.5-3.2 = 13.3 m

9. Refer to Table 1.5,
Minimum distance between adjacent pillar centre to centre corresponding to working depth 120 m and gallery width 3.4 m = 19.5 m
Therefore, pillar width = 19.5-3.4 = 16.1 m

Percentage of extraction (R) $= \left[1 - \dfrac{a*a}{(a+b)*(a+b)}\right]*100$

Where, a = length and breadth of pillar
[Assuming, a square pillar]
b = width of galleries

Therefore, $R = \left[1 - \dfrac{16.1*16.1}{(16.1+3.4)*(16.1+3.4)}\right]*100 = 31.8\%$

10. Average Load acting on pillar in $N = \gamma * D*(a+b)^2 * g$
= average stress acting on pillary * area of cross section of pillar

$$=> \gamma * D*(a+b)^2 * g = 8*10^6 * a^2$$

$$=> \frac{a^2}{(a+b)^2} = \frac{2500*200*9.81}{8*10^6} = 0.613$$

=> Extraction Ratio = R = $1 - \dfrac{a^2}{(a+b)^2} = 0.387$

Where, a = size (length and breadth) of the pillars, and b = width of galleries

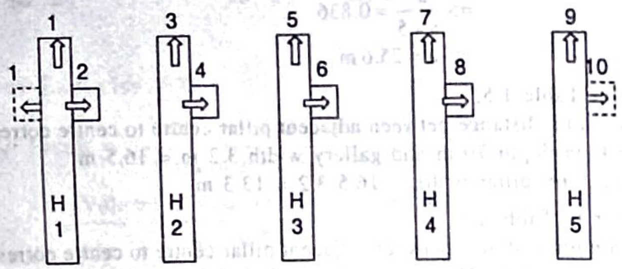
11. Strength of Pillar (s) $= K\dfrac{W^{\alpha}}{L^{\beta}} = 1320*\dfrac{64.07^{0.46}}{9.84^{0.66}}\ \ell b/in^2$

$$= 1902.05\ \ell b/in^2 = 139.97\ kg/cm^2$$

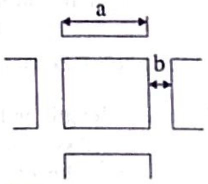
(Refer Salamon and Munro's formula)
Where, K = constant = 1320 ***lb/in***^{2}, w = width of pillar,
L = height of pillar
W = 22.5- 3= 19.5 m = 64.07 ft.
L = 3 m = 9.84 ft.

12. As per CMR 66, clause 3, the minimum distance between two shafts or inclines is 13.5 m.

13. Maximum number of faces available for 5 heading bord and pillar working = 2n+1
= 2*5+1 = 11



14. Maximum number of faces available for working = 2n+1
=> 13= 2*n+1
=> n = 6



20. We have, a = 30 –4 = 26 m
b = 4 m
Tributary Area = $(a+b)^2 - a^2$
= 30^2- 26^2 = 224 m^2

21. Percentage of extraction (R) = $[1- a^2/ (a+b)^2]*100$
= $[1- (26)^2/(30)^2]*100$ = 24.9 %

22. Percentage of extraction (R) $= \left[1-\frac{a*a}{(a+b)*(a+b)}\right]*100$

$$=> 30=\left[1-\frac{a*a}{(a+b)*(a+b)}\right]*100$$

$$=> 0.7=\left[1-\frac{a*a}{(a+b)*(a+b)}\right]$$

$$=> \frac{a}{a+b}=\sqrt{0.7}=0.836$$

$$=> \frac{25}{25+b}=0.836$$

$$=> b=4.88m$$

26. Given data
Number of detonator in series = 20
Resistance of each detonator (R) = 2Ω
Current flow = 1.25A
Energy spent to fire the detonators (E) = 250 mj
We have,
Energy = Power * time
=> E = (V*I) *th
=> E = (Re*I*I)*t
(Re = Equivalent resistance = 20*2 = 40Ω)
=> $t = 250*10^{-3}/40*1.25*1.25 = 4*10^{-3}$ = 4ms

32. First circuit will fire in 400 ms
Second will initiate after 25ms and hence fire take place in (400 + 25)ms
And so on.
Thus, total add on time in ms for 10 circuit
= 400 + 9*25 = 625 ms.

35. Given data
Width of coal heating(W) = 4m
Height of coal face (H) = 2.5m
Pull/Advancement per round of blasting (d) = 1m
Explosive used per roundof blasting (Q) = 6 kg
Specific gravity of coal (λ) = 1.5
Volume of coal extracted per round of blasting (V)
= W*H*d = 4*2.5*1 =10m^3
Weight of coal produced per round of blasting (W) = $V*\gamma*\rho_w$
[where, ρ_w = density of water = 1000 kg/m^3]
⇒ W = 10*1.5*1000 = 15000 kg = 15te
Therefore, Power factor = W/Q = 15/6 = 2.5 te/kg

38. Absolute bulk strength of explosive = absolute weight strength of explosive * density of explosive

$$RBS_{emulsion} = \frac{ABS_{emulsion}*100}{ABS_{ANFO}} = \frac{AWS_{emulsion}*\rho_{emulsion}}{AWS_{ANFO}*\rho_{ANFO}}*100$$

$$= \frac{ABS_{emulsion}*SG_{emulsion}}{ABS_{ANFO}*SG_{ANFO}}*100 = \frac{850*1.2}{912*0.8}*100=139.8\%$$

42. We have, L/D<6
[L = length of charge and D = diameter of spherical charge]
=> L/25 < 6
=> L < 150mm

44. Production from the panel in tonnes per day

$$= \frac{\text{Weight of coal present in the panel for depillaring}}{\text{Number of working days during incubation period}}$$

$$= \frac{\text{Number of coal pillar} * \text{volume of coal in 1 pillar} * \text{density of coal}}{6*25}$$

$$= \frac{32*(25*25*3)*1.4}{150} = 560 \text{ tonne/day}$$

45. We have, $v = \pi * D * N$

Where, V = Cutting Velocity

D = Diameter

N = Rotational Speed

$$\Rightarrow D = \frac{71.5}{\left(\frac{22}{7}\right)*350} m = \frac{71.5*7*1000}{22*350} mm = 65mm$$

47. Given data :

Total number of injuries (N) = 2 + 6 + 8 = 16

Total Number of coal mine workers = 1200

Therefore, injury rate = (16/1200)*1000

= 13.34 per 1000 workers

55. We have, Powder Factor $= \frac{\text{Weight of Coal Produced in te}}{\text{Mass of Explosive Reqired in kg}}$

$$= \frac{\text{Volume of Coal Produced/ tonnage factor}}{\text{Mass of Explosive Required}}$$

$$= \frac{(\text{Area of cross section of drift} * \text{Pull per round of blasting}) / \text{tonnage factor}}{\text{Mass of Explosive Required}}$$

$$= \frac{2.4*3*1.8}{36*0.36} = 1 \text{ kg/te}$$

62. Assume that at a distance of x from point of inflection (I)] the ground shows zero subsidence.

We have,

$s = 0.746 - 0.750 \tanh 0.05x$

For s = 0,

$0 = 0.746 - 0.750 \tanh 0.05x$

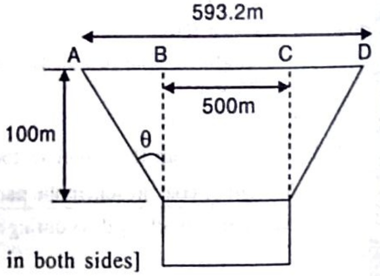
$\Rightarrow \tan h\, 0.746x = \frac{0.746}{0.750} = 0.995$

$\Rightarrow 0.05x = 2.99$

$\Rightarrow x = 59.8$

Again, $\tan\theta = \frac{x}{100} = \frac{59.8}{100} = 0.598$

$\theta = \tan^{-1} 0.598 = 30.6^0$



65. We have,

$$\tan\theta = \frac{AB}{BE} = \frac{(AD-BC)/2}{BE}$$

[Assuming a symmetrical subsidence in both sides]

$$\Rightarrow \tan\theta = \frac{(593.2-500)/2}{100} = \frac{93.2}{200} = 0.466$$

$$\Rightarrow \theta = \tan^{-1} 0.466 = 25^0$$

70. Given that,

Weight% of solids in a sand-water stowing pipe = 60

Therefore, weight % of water in a sand-water stowing pipe = 40

Assuming the weight of pulp = x

Weight of sand = Ws = 0.6x

Weight of water = Ww = 0.4x

$$\text{Pulp density} = \frac{\text{Mass of Pulp}}{\text{Volume of Pulp}}$$

$$= \frac{x}{\text{Volume of Water} + \text{Volume of Sand}}$$

$$= \frac{x}{\frac{0.4x}{1000} + \frac{0.6x}{3000}}$$

$= 3000/1.8 = 1666.7 \cong 1667 \text{ kg/m}^3$

71. Give data :

Width of the gallery (W) = 4.2m

Height of gallery (H) = 3m

Number of holes (n) = 12

Explosive consumption per hole (q) = 2*435g

= 870g = 0.870kg

Therefore, total explosive consumption in 1 round of blasting

= 0.87*n = 10.44 kg

Specific gravity of coal (γ) = 1.4

Let pull per round of blasting is *x*

We have,

$$\text{Powder Factor} = \frac{\text{Weight of Coal Produced in te}}{\text{Mass of Explosive Required in kg}}$$

$$\Rightarrow \text{Powder Factor} = \frac{[\text{Volume of Coal Produced} * \gamma * \rho_w]/1000}{\text{total explosive consumption}}$$

$$\rho_w = \text{density of water} = 1000 \text{ kg/m}^3$$

$\Rightarrow$ Powder Factor =

$$\frac{(\text{Area of cross section of drift} * \text{Pull per round of blasting}) * \gamma * \rho_w \;/1000}{\text{Mass of Explosive Required}}$$

$$\Rightarrow 2.2 = \frac{(4.2*3*x)*1.4}{10.44}$$

$$\Rightarrow x = \frac{2.2*10.44}{4.2*3*1.4} = 1.3$$

78. Given data,
Specific energy consumption for cutting coal = 800 kJ/m3
Specific gravity (SG) of coal = 1.2
Machine production = 700 te/h
Efficiency of the machine = 65%
We have, Volume of coal produced per hour = Machine Production in kg/density of coal
= (700*1000)/1200 m^3
Energy consumption per hour = 800 * (700/1.2) = 560000/1.2 kJ
Energy consumption per second = 560000/ (1.2*3600) kJ = 129.63 kJ
Therefore, the electrical power consumption in kW with 65 % efficiency =
Energy consumption per second/efficiency
= 129.63/0.65 = 199.4 kW

79. Given that, the number of full face cuts executed per shift = 2
Therefore, number of full face cuts executed per day = 3*2 = 6
Depth of cutting coal per day = depth of web in a single cut * number of cut per day
= 0.5 * 6 = 3 m
Daily production = Volume of coal extracted per day * unit weight of coal
= Length of face* height of face * depth of cut per day * unit weight of coal
= 150 * 3 * 3* 1.4 = 1890 te

80. We have, Output per man shift (OMS) =
(Output per day/number of shift)/number of worker deployed per shift
= 1890/3*40 = 15.75 te

82. Given data, distance between centre to centre square pillar = 30m
Gallery width (b) = 4m
Therefore, width of square pillar (a) = 30 – 4 = 26m
Average depth of working (D) = 100m
Average density of the overlying strata (γ) = 26 kN/m^3
Pillar strength = 4500 kN/m^2
We have,

$$\text{Extraction Ratio} = R = 1-\frac{a^2}{(a+b)^2} = 1-\frac{26^2}{30^2} = 1-\frac{676}{900} = 0.249$$

Where, a = size (length and breadth) of the pillars, and b = width of galleries

83. We have,

$$\text{Factor of safety} = \frac{\text{Pillar Strength in kN/m}^2}{\text{Average Load acting on pillar in kN/m}^2}$$

$$= \frac{4500}{\gamma * D * [(a+b)/a]^2} = \frac{4500}{26*100*(30/26)^2} = 1.3$$

[Load of overlying strata of area of cross section $(a + b)^2$ acting on the area of cross section a^2]

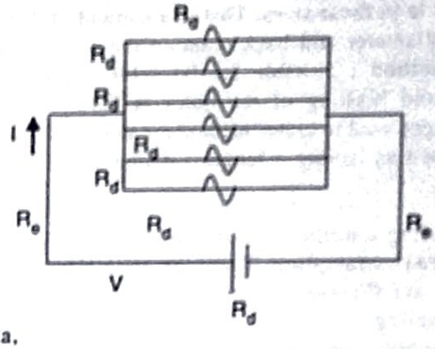
84. We have,

$$H = \frac{t}{k-1}$$

Where, t is the mining height, H is the thickness of the immediate roof or aving height, and K is the bulking factor of the immediate roof.

$$\Rightarrow H = \frac{3}{1.2-1} = 15m$$

85.



Given data,
Resistances of each detonator = Rd = 1.5Ω
Voltage of exploder = 15V
We have,
Voltage (V) = Equivalent Resistant (Re) * Current Flow (I)
15 = (3 + 3 + 1.5/6) * I
I = 15/6.26 = 2.4A

●

1.2 Underground Metalliferous Mining

1.2.1. Gestation Period

The time interval between the mining start and production of ore starts. This is very high in case of metal mines.

1.2.2 Cut-off Grade

The grade at which the mineral resource can no longer be processed at a profit.

1.2.3 Break-even cut-off grade

The breakeven cut-off grade is defined as that grade for which the net value is zero.

1.2.4 Raise or Winze

These are the drivages from one level to another level to connect the level during development stage of metal mining.

1.2.5 Raising Methods

1.2.5.1 Open Raising by drilling, blasting, mucking : Suitable for moderate length (upto 8 m), inclination of 40° - 60° and hard strata & country rock.

1.2.5.2 Two or three compartment raising : Suitable for vertical and very steep raises only and limited to 15-20 m.

1.2.5.3 Raising with Jora raise lift : This is mechanised and suitable for 100 m long but not used now.

1.2.5.4 Raising by Long Hole Drilling : The practical limit of this raising is 15-50 m and the inclination is 35° from vertical. ***This method secures the lowest cost per meter of drivages and highest speed.***

1.2.5.5 Raising with Alimak raise climber : Suitable for greater than 40^0 inclinations from horizontal, length of the raise not more than 150 m.

1.2.5.6 Raise borers : Suitable for limited diameter of the raise (1-3.7 m) as the raise drill bits are available in these sizes. This is a method of drilling a pilot hole of 230 mm to 300 mm diameter and back reaming the hole of suitable sizes.

1.2.5.7 Drop raising method : Suitable for 50 – 60 m long raise and nearly vertical. The charging and blasting of the holes is based on **Vertical Crater Retreat.** Explosives charges used in crater method are normally spherical. Spherical charges have been defined as having a length : diameter (L: D) ratio of 6 : 1.

1.2.6 Sampling

The process of inferring something about a large group of elements by studying only a part of it is referred as sampling. There are mainly four types of sampling method in mines. These are Channel or Groove Sampling, Chip Sampling, Grab Sampling and Bulk Sampling.

Channel sampling in rock can be a difficult chore depending upon the hardness of the rock. This technique is generally used when sampling underground openings (drifts, tunnels, adits, and cross cuts) and trenches dug in rock. The best approach is to chip a sample of rock that is one half to one inch deep and six inches wide for the full length of the wall being sampled. The rock that is chipped off the wall should be collected as it is removed and stored in sample bags which represent a regular sample interval if the rock type is homogeneous. If there are changes in the rock type then the sample interval should correspond to the changing rock types.

1.2.7 Average grade of an ore block

The average grade of the block can be determined by

$$\text{Average grade of the block} = \frac{\sum L_i * w_i * g_i}{L * W}$$

Where, L = total length of the block

W = average width of the block

The average grade determination of the block shown in Figure 1.5 is given below.

$$\Rightarrow \text{Average grade of the block}$$

$$= \frac{L_1 * w_1 * g_1 + L_2 * w_2 * g_2 + L_3 * w_3 * g_3}{L * W}$$

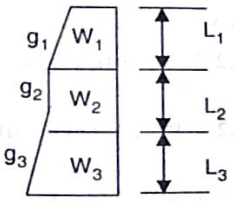


Figure 1.5 : Ore Block

1.2.8 Metal Mining Methods

1.2.8.1 Overhand stoping

- The stoping operation proceeds from lower main level towards the upper main level.
- Applicable in steeply dipping narrow veins or in bedded deposits of 2-3 m thickness

- Strong ore and stable wall rocks
- The ore drilled by compressed air drills

1.2.8.2 Underhand stoping

- The stoping operation proceeds from upper main level towards the lower main level commencing from a winze.
- Applicable in thin steeply dipping (> 40°) veins.
- Strong wall rocks
- Transport of ore is difficult and the blasting efficiency is poor.

1.2.8.3 Breast stoping

- Applicable to deposits of horizontal or mild dip and of thickness up to 3 m (upto 3m thickness mining can be possible with single lift but for more than that two lifts are required).
- Ore and hanging wall should be strong
- This method is of low cost, and preferred for low grade of ore.
- Method lends itself to large scale mechanization in drilling, blasting, loading and transport.
- The worked out space supported by irregular pillars, timber cribs and props.

1.2.8.4 Sublevel stoping

- Sublevel means a level intermediate between the two main levels.
- The stoping operation proceeds from lower main level towards the upper main level.
- Steeply dipping ore and thick ore body
- Stable hanging wall and foot wall
- Competent ore
- Regular ore boundaries
- Ore is produced by drilling and blasting. Long-hole drilling should be of diameter in the range of 50-200 mm and length up to 90 m in **ring-hole pattern or fan pattern.**
- A **draw point system,** consisting of loading cross-cuts with **finger raises** and cones up to the undercut.

1.2.8.5 Long-Hole stoping or Blast-Hole stoping

- Applicable to steeply dipping (> 50°) or vertical deposit.
- Involves large diameter (150 -200 mm) and 60-70 m long drill holes from upper level to lower main level.
- Drill holes are blasted in **VCR** (vertical crater retreat) method.

1.2.8.6 Room and Pillar Mining

- Ore and hanging wall should be strong.
- Applicable to flat or mildly dipping (< 30°) ore bodies.
- The thickness may be in the range of 3-20 m.
- In case of thicker deposit excavation starts with the top section and the remainder of the ore is recovered by benching in one or more steps.

1.2.8.7 Shrinkage stoping

- Applicable to steeply dipping deposit.
- Ore should be of free flow characteristics.

- Ore that is not affected by storage in the stope (certain sulfide ores tend to oxidize and decompose when exposed to atmosphere).
- Suitable for thickness of ore body in the range of 3–12 m.
- Comparatively strong hanging wall and footwall.
- 40% of the broken ore in the stopes has been drawn.
- Recovery of the ore varies from 75-90 %.

1.2.8.8 Cut and Fill Stoping

- Applicable to steeply dipping as well as mildly dipping ore body with firm ore.
- Applicable to small as well as large deposits.
- Offers selectivity
- Dilution of the ore is low
- Stoping operation is cyclic
- This method provides permanent support to surface structures

1.2.8.9 Square Set Stoping

- Square set stoping is a method of mining in which the walls and back of the stope are supported by regular frame work of timber called square sets.
- Applicable for very weak country rock and ore
- For recovery of fracture remnants
- Ore should be of high grade to compensate the costly and labour intensive method

1.2.8.10 Resuing

- Resuing is method of stoping for working narrow veins of valuable ore and involves blasting of a sizeable quantity of barren wall rock in contact with the vein
- Steeply inclined deposits

1.2.8.11 Block Caving

- Steeply dipping deposits or massive ore body
- Weak ore to cave under its own weight
- Low value and grade of ore
- Surface is allowed to cave

1.2.8.12 Sub-Level Caving

- Sub-level caving is a caving method where the overburden and part of the ore is induced to cave in.
- Overburden and ore should be of weak in nature
- Steeply dipping deposit
- Fan drilling is suitable for extraction
- Dilution of the ore vary from 10 to 35%
- Ore loss is in between 5 and 20%

1.2.9 Qualification and Appointment of Competent Persons in U/G Metal Mines

1.2.9.1 Manager : The appointment of a manager in u/g metal mines done on the basis of no. of persons employed in below ground or in the entire mine. The qualifications required for appointment of manager is given in Table 1.13.

Table 1.13 : Qualification and Appointment of a Manager in u/g Metal Mines

No. of persons employed (X)	Qualification
X > 150 in below ground or X > 400 in all the mine	First Class Manager's Certificate
150 > X > 75 in below ground or 400 > X > 150 in all the mine	First Class/ Second Class
In any other case	First Class/ Second Class/ Manager's Permit

1.2.9.2 Assistant Managers / Under Manager : The number assistant manager required in u/g metal mines decided on the basis of no. of persons employed in below ground or in the entire mine. The number required for appointment of assistant manager is given in Table 1.14.

Table 1.14 : Number of Assistant Manager in u/g Metal Mines

No. of persons employed (X)	Number of A.M./ U.M
450 > X > 600 in below ground or 1600 > X > 1200 in all the mine	1
X > 600 in below ground or X > 1600 in all the mine	1 additional A.M. for every additional 300 persons employed in u/g or 800 in all the mine or part thereof

Provided that in a mine where, X > 1200 in below ground or X > 3200 in all the mine, at least 1 A.M. shall hold First Class Managers Certificate.

1.2.9.3 Winding Engineman : The qualification required of a Winding Engineman in u/g metal mine is given below in Table 1.15.

Table 1.15 : Qualification of Winding Engineman in u/g Metal Mine

Type of Engine	Qualification
In case of electric engine of 75 h.p. or more Or Steam or compressed air winding engine which has cylinder exceeding 30 cm in diameter	First Class Engine Driver's Certificate
In any other case Driver's Certificate	Second Class Engine Driver's Certificate

1.2.10 MMR regarding Representative Factor

This is ratio of actual lease hold area to the area shown in the mine plan.

RF = 200 : 1, in case of mica mines and other mines having small scale working below ground

RF = 2000 : 1, in case of mine having large opencast workings and also in case of surface plans of large leasehold areas

RF = 1000 : 1, in other cases

1.2.11 MMR regarding Ladders in u/g metal mines

A. Project at least 1m above the mouth of the shaft winze, rise or stope
B. Inclination of the ladders should not exceed 80° from the horizontal.

EXERCISE 1.2

1. Exploration means
(a) search of ore (b) estimation of reserve
(c) opening up of deposit (d) shaft sinking to access the deposit

2. Ore grade for which revenue from the recoverable reserve exactly equals the cost of mining, treatment, and marketing is known as
(a) cut-off grade (b) average grade
(c) break-even grade (d) liquidation grade [GATE, 2000]

3. A Zn deposit is formed by a geological process is called
(a) Hydrothermal (b) Magmatic separation
(c) Supergene (d) None of the above

4. An ore block of length 30 m and height 20 m having an average width of 1.5 m with sharp boundary walls is mined with a minimum mining width of 2 m. If the tonnage factors are 3 t/m^3 for ore, 2 t/m^3 for wall rock and average grade of ore is 10%, the diluted grade of ore mined in percent is
(a) 7.50 (b) 7.84
(c) 8.18 (d) 8.52 [GATE, 2006]

5. A narrow vertical or inclined excavation made in the ore block along the width and the height of a stope to continue stoping is termed as
(a) trough (b) slot
(c) undercut (d) ore pass [GATE, 2006]

6. Gestation period is
(a) the time interval between the mining start and production start
(b) the time interval between the mining start and mining close
(c) lag on ignition period
(d) time to reach the breakeven point

7. The suitable method of mining a 2 m thick strong sulphide ore deposit within strong wall rocks dipping at 18° and occurring at 60 m below the surface is
(a) overhand stoping (b) top slicing
(c) underhand longwall stoping (d) breast stoping [GATE, 2005]

8. Depending on the decreasing ability of surrounding rock to store strain energy, the underground metal mining methods can be ordered as
a. Cut-and-fill stoping, Sublevel caving, Sublevel open stoping, Block caving
b. Sublevel open stoping, Cut-and-fill stoping, Sublevel caving, Block caving
c. Sublevel caving, Sublevel open stoping, Cut-and-fill stoping, Block caving
d. Block caving, Sublevel caving, Sublevel open stoping, Cut-and-fill stoping
[GATE 2008]

9. The excess pore pressure in backfill material in a cut and fill stope leads to

a. Reduction in strength of the wall rock
b. Enhancement of bearing strength of fill
c. Loss of shear resistance of fill
d. Prevention of progressive failure of crown pillar [GATE 2015]

10. Match the following :

Stoping method	Ore handling systems	Support System
P Breast Stoping	1. LHD	a. In-situ pillars
Q Cut and fill stoping	2. Scraper	b. Unsupported
R Sublevel stoping	3. Gravity flow	c. Mill tailings

a. P- 2-a, Q- 1-c, R -3-b
b. P- 1-a, Q- 3-c, R – 2-b
c. P- 2-b, Q- 1-a, R -3-c
d. P- 1-c, Q- 3-a, R – 2-b [GATE 2008]

11. Resuing stoping method is adopted when ore body is
a. Flat and thick b. Very steep and thick
c. Flat and thin d. Very steep and thin [GATE 2009]

12. The diameter of holes in mm commonly used for ring drilling by drifters in sublevel stoping is between
(a) 25 and 32 (b) 33 and 44
(c) 45 and 64 (d) 65 and 100 [GATE, 2005]

13. High production rates coupled with large scale and extensive subsidence results from the method of
(a) top slicing (b) block caving
(c) sublevel caving (d) VCR mining [GATE, 2000]

14. Cut and fill stoping can be operated
(a) underhand only (b) overhand only
(c) both the above (d) none of the above

15. In sublevel stoping operation the draw system is generally
(a) draw point (b) cross drift
(c) finger raise and chute (d) none of the above

16. Blasting of stope in VCR method consists of
(a) blasting one row after another
(b) creating initial slot going for mass blast
(c) blasting all the holes in slices
(d) blasting one column after another

17. Match the following

Method of mining	Stope support	Ore loading
P. Shrinkage stoping	1. Insitu pillar	a. Overhead mucker
Q. Blasthole stoping	2. Broken ore	b. Pneumatic autoloader
R. Top slicing	3. Timber mat	c. Load haul dumper

(a) P-2-a, Q-1-c, R-3-b
(b) P-2-a, Q-3-c, R-1-b
(c) P-2-b, Q-3-c, R-1-a
(d) P-3-c, Q-2-a, R-1-b [GATE 2011]

18. The equipment that is NOT used in hard rock metal mining drivage is
(a) road header (b) drill jumbo
(c) jack hammer (d) dint header [GATE 2012]

19. The stoping method, where a large part of blasted ore is allowed to accumulate in the stope to serve the purpose of providing working platform for stoping as well as to support the wall-rock, is known as
(a) shrinkage stoping (b) cut and fill stoping
(c) square-set stoping (d) sublevel stoping [GATE 2012]

20. If the RQD of the ore and wall rock are low, the stoping method suitable is
(a) shrinkage (b) sublevel
(c) block caving (d) cut and fill

21. Broken rock of 3684 m^3 volume is heaped into a square pyramid of height 12m. The surface area in m^2 required is
(a) 921 (b) 307 (c) 614 (d) 1228

22. Order the followings in decreasing stability of host rock :
Block caving (1), Cut & Fill (2), VCR stoping (3), Room & Pillar (4)
(a) 1, 2, 3, 4 (b) 2, 3, 4, 1
(c) 4, 3, 2, 1 (d) 4, 2, 3, 1

23. Dilution in sublevel caving is
(a) 10 – 35 % (b) > 40 % (c) < 10 % (d) < 5 %

24. Every ladder way used in a mine shall project above the mouth of at least
(a) 1.0 m (b) 1.5 m (c) 2.0 m (d) 0.5 m

25. Main difference in VRM and Blast Hole stoping is
(a) Drilling technique (b) Blasting technique
(c) Drilling and Blasting technique
(d) None of the above

26. Best suited mining method for thick vein/ massive deposit with strong ore, strong walls and steep dip is
(a) open stope (b) cut and fill
(c) shrinkage stope (d) block caving

27. A block ore between two levels 30 m apart is 40 m long. The ore body thickness, specific gravity and grade are 2 m, 2.8 and 2 % respectively. The tonnage of copper available in the block of ore is
(a) 134.4 (b) 236.2 (c) 156.8 (d) 290.3

28. In-situ ore of 1.5 % Cu got diluted with waste rock of value 0.2 % Cu. If the resultant extracted ore has 1.2% Cu. What is the percentage of dilution?
(a) 30 (b) 25 (c) 23 (d) 20

29. Long-hole drilling with crater blasting is used for the construction of
a. winze b. shaft c. raise d. decline
[GATE 2013]

30. Rill stoping method is a form of
a. block caving b. artificially supported stoping
c. underhand stoping d. overhand stoping [GATE 2013]

31. Out of the support categories given for an underground coal mine, identify the 'active support'.
a. Wire mesh b. Shotcrete
c. fully grouted roof bolt d. Hydraulic Prop [GATE 2013]

32. For a shrinkage stope the following data values are given

Insitu tonnage	9000 tonne
Insitu grade	5.2 g/tonne
Average grade of waste	1.4 g/tonne
Loss of ore in the stope	10%
Dilution	20%

The grade at the mill-head in g/tonne is
a. 6.47 b. 4.57 c. 3.27 d. 5.47 [GATE 2013]

33. A 4.6 m wide vein dipping at 80° is mined by horizontal cut-and-fill stoping method. The fill is to be placed in the stope along the length of 46 m and to a height of 3.0 m. If the specific weight of the fill material is 15.86 kN/m^3 and the porosity is 35%, under fully saturated conditions the volume of water in the fill in m^3 is
a. 222.18 b. 332.40 c. 336.44 d. 634.80 [GATE 2013]

34. In the above question, the mass of solids in saturated fill in tonnes is
a. 820.00 b. 804.10
c. 799.30 d. 788.80 [GATE 2013]

35. In a cut-and-fill stope, the main purpose of back filling is to
a. reduce ore dilution
b. prevent high stress concentrations in far field domain
c. prevent displacement due to dilation of fractured wall rock
d. improve ore rehandling [GATE 2014]

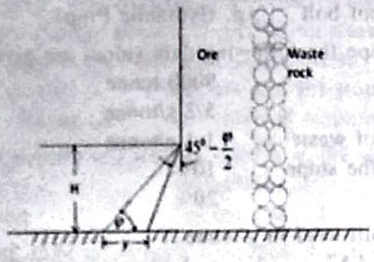
36. In a room-and-pillar stope, bench blasting is conducted using ANFO having density of 800 kg/m^3. The specific gravity of rock is 2.5, hole diameter is 100 mm and spacing to burden ratio is 1.3. The charge length of each blast hole is 80% of the hole length. For a desired powder factor of 0.48 kg/tonne, the spacing and burden of the blast pattern in m respectively are
a. 2.0, 2.6 b. 2.3, 1.8
c. 5.2, 4.0 d. 1.3, 1.0 [GATE 2014]

37. Match the following for ore handling operations in an underground metal mine

Arrangement	Description
(P) Draw point	(I) arrangement that prevents oversized rock to pass
(Q) Ore pass	(II) a system of vertical or near vertical openings for transferring ore from a stope to a single delivery point
(R) Grizzly	(III) a place where ore can be loaded and removed
(S) Finger raise	(IV) a vertical or inclined opening used for transferring ore

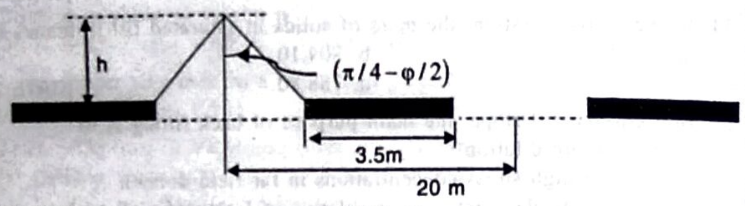
a. P-IV, Q-III, R-II, S-I b. P-III, Q-IV, R-I, S-II
c. P-II, Q-IV, R-I, S-III d. P-III, Q-I, R-II, S-IV
[GATE 2014]

38. The height H of a drawpoint in a sublevel caving stope is 3.0 m. If the angle of repose (??) of broken ore is 35°, the digging depth of the loader as shown in the figure in m is



a. 3.42
b. 1.82
c. 2.72
d. 2.42 [GATE 2014]

39. In block caving operation the draw points are placed at 20 m centre to centre, with the pillar width 3.5 m as shown in figure below. The muck is assumed to have zero cohesion and 35 friction angle. The height of draw cone (h) in m is



a. 12.5
b. 14.6
c. 15.8
d. 16.5 [GATE 2010]

Answer

1. b	2. c	3. a	4. c	5. b	6. a	7. d
8. b	9. c	10. a	11. d	12. d	13. b	14. c
15. c	16. c	17. a	18. d	19. a	20. d	21. a
22. a	23. a	24. a	25. b	26. c	27. a	28. a
29. c	30. d	31. d	32. b	34. b	35. c	36. b
37. b	38. c	39. c				

●

Solution of Exercise 1.2

4. Given data,
Ore block length (L) = 30m
Ore block width (W) = 1.5m
Ore block height (H) = 20m
Tonnage facotor of wall rock (γ_w) = 2 te/m3
Average grade of ore (G_0) = 10%

Volume of ore extraction = L*W*H = 30*1.5*20 = 900 m^3
Therefore, weight of ore production = 900*3 te = 2700 te
For a 2m mining width, the width of wall rock extraction = 2 − 1.5 = 0.5
The volume of wall rock or waster production = 30*0.5*20 = 300m^3
There, weight of waster production = 300*2 te = 600 te
Thus, the diluted grade of ore = (2700*0.1/3300)*100 = 8.18%

21. Given data,
Volume of broken rock = 3684 m^3
Height of pyramid = 12m
We have,

$$\text{Volume} = \frac{1}{3} * \text{Base Area} * \text{Height}$$

$$\Rightarrow 3684 = \frac{1}{3} * A * 12$$

$$\Rightarrow A = 921\,m^2$$

27. Given data,
Height of block (H) = 30m
Length of ore (L) = 40m
Thickness of ore (T) = 2m
Specific gravity of ore (γ) = 2.8 [density of ore = 2800 kg/m^3]
Ore grade (g) = 2%
We have, tonnage of ore available in the block
= Volume of block * density of ore
= (30*40*2)*2800 kg = 6720000 kg = 6720 te
Therefore, tonnage of copper = 6720 * 2/100 = 134.4 te

28. Given data,

Cu grade of in-situ (g_{ci}) = 1.5%

Cu grade of waster rock (g_{cw}) = 0.2%

Cu grade of diluted ore (g_{cd}) = 1.2%

Let V_i, volume of in-situ ore mixed with V_w, volume of waste rock.

Therefore, we have,

$$(V_i + V_w) * g_{cd} = V_i * g_{ci} + V_w * g_{cw}$$

$$\Rightarrow \quad (V_i + V_w) * 1.2 = V_i * 1.5 + V_w * 0.2$$

$$\Rightarrow \quad V_i * 0.3 = V_w * 1$$

We have, Percentage of dilution $= \frac{V_w}{V_i} * 100 = 30\%$

32. Given data,
In-situ tonnage (W_i) = 9000 te
In-situe grade (g_i) = 5.2 g/te
Average grade of waster (g_w) = 1.4 g/te
Loss of ore in the stope = 10%

Dilution = 20%

Therefore,

Actual volume of ore extraction (W_a) = W_i * (Loss of ore) = 9000*0.90
= 8100 te

Weight of ore mix with the actual ore = Wa*dilution = 8100*0.20 = 1620 te

$$\text{Mill head grade (g)} = \frac{W_a * g_i + W_w * g_w}{W_a + W_w} = \frac{8100*5.2+1620*1.4}{8100+1620} = 4.57\ \text{g / te}$$

33. Given data,

Wide of vein (W) = 4.6m

Length of fill (L) = 46m

Height of fill (H) = 3m

Specific weight of fill material (γ_f) = 15.86 kN/m^3 = (15.86/9.81) te/m^3
= 1.6 te/m^3

Porosity (P) = 35%

Therefore, volume of fill = W*L*H = 4.6*46*3 = 634.8 m^3

Volume of water in the fill under fully saturated condition =
Volume of fill * Porosity

= 634.8*0.35 = 222.18 m^3

34. Mass of solids in saturated fill = Mass of filling material – Mass of water
= Volume of fill * Specific weight of fill material
– Volume of water in the fill * density of water

⇒ Mass of solids in saturated fill = 634.8 * 1.61 – 222.18*1
[Density of water = 1000 kg/m^3 = 1 te/m^3]

⇒ Mass of solids in saturated fill = 1026.29 – 222.18 = 804.11 te

36. We have,

$$\text{Powder Factor} = \frac{\text{Mass of Explosive Required in kg}}{\text{Weight of ore Produced in te}}$$

$$= \frac{\text{Volume of explosive} * \text{Density of explosive}}{\text{Volume of ore Produced} * \text{Density of ore}}$$

$$= \frac{\left(\pi * \frac{d^2}{4} * 0.8L\right) * \text{Density of explosive}}{(S*B*H*)*(\text{S.G. of ore} * \text{Density of water})}$$

Where, d = diameter of hole
B = Burden
S = Spacing = 1.3 B (given)
L = Length of the hole

Length of explosive column = 0.8L

$$\Rightarrow 0.48 = \frac{3.14*(0.1*0.1)*0.8L*800}{4*(S*B*L)*(2.5*1000)/1000}\ \text{kg/te}$$

$$\Rightarrow 0.48 = \frac{2}{1.3B*B}$$

⇒ B = 1.80m

Therefore, S = 1.3*B = 2.3m

38. We have,

$$\tan\varphi = \frac{H}{x+y}$$

$$\Rightarrow \tan 35 = \frac{3}{x+y}$$

$$\text{Again, } \tan\left(45+\frac{\varphi}{2}\right) = \frac{H}{x}$$

$$\Rightarrow \tan 62.5 = \frac{3}{x}$$

⇒ x = 1.56

Putting the value of x in the above equation, we get

$$\tan 35 = \frac{3}{1.56+y}$$

⇒ 1.56 + y = 4.28

⇒ y = 2.72m

39. We have,

$$\tan\left(45-\frac{\varphi}{2}\right) = \frac{x}{h} = \frac{(20-3.5)/2}{h}$$

$$\Rightarrow \tan\left(45-\frac{35}{2}\right) = \frac{8.25}{h}$$

⇒ h = 15.84

●

1.3. Opencast Mining

1.3.1 Definition : In this type of mining, all the layers cover the mineral block/ coal extracted to make the access to mineral/coal deposit. The overburden material extracted dump in the area not bearing the mineral/coal block. This type of mining method also called open-pit mining, surface mining and strip mining. Selection of the opencast mining method depends on the stripping ratio.

1.3.2 Stripping Ratio : It can be defined as the ratio of volume of overburden removed to weight of recoverable mineral reserve. Economy of the mining depends on the depth of the overburden because the depth increases, cost of extraction of the overburden material increases.

$$\text{Stripping Ratio} = \frac{\text{Weight of recoverable mineral reserve in tonne}}{\text{Volume of overburden in cubic meter}}$$

1.3.3 Break-even Stripping Ratio : The point beyond which the mineral/ coal cannot be economically extracted out is called the break-even stripping ratio. It can be also defined as the ratio of excess cost required to produce per tonne mineral/coal by underground mining method in comparison to the opencast mining method to the stripping cost per cubic meter of overburden by the opencast method.

$$\text{Break-even Stripping Ratio} = \frac{\text{(Recoverable price per te of ore)} - \text{(O.C. production cost per te of ore} + \text{Profit margin per te of ore)}}{\text{Cost of extraction per cubic meter of overburden material}}$$

1.3.4 Determination of Stripping Ratio : The stripping ratio of the block shown in Figure 1.6 and Figure 1.7 can be determined as given below.

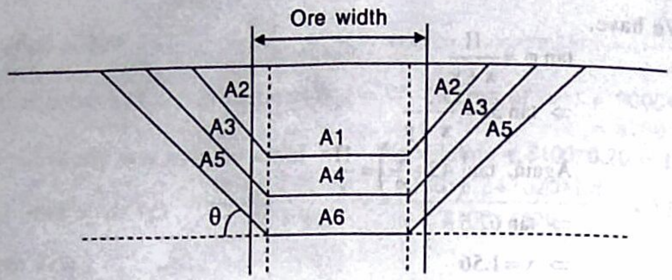


Figure 1.6 : Excavation of Ore Block

$$\textbf{Overall Stripping Ratio (SR)} = \frac{Vw*\rho w(waste)}{Vo*\rho(ore)} = \frac{A_1+2A_2+2A_3+2A_5}{A_4+A_6}$$

Where, A_1, A_2, A_3, A_4, A_5, A_6 : area of cross section of zone shown in the above figure.

V_w = volume of waste material
ρ_w = density of waste material
V_o = volume of overburden material
ρ_0 = density of overburden material

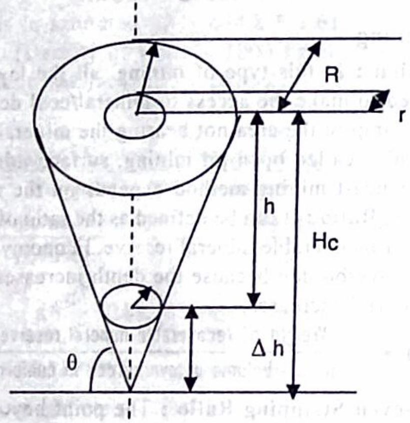
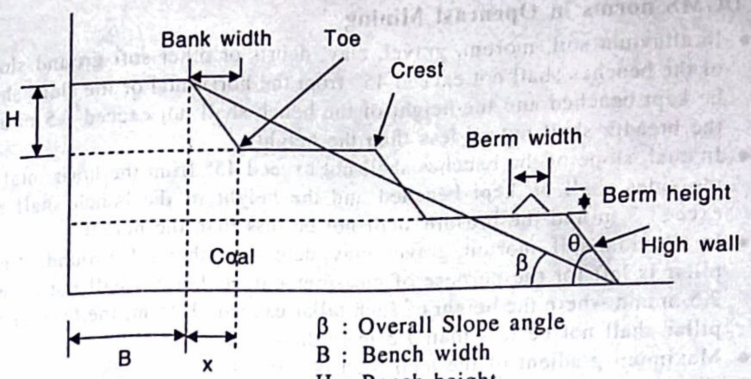


Figure 1.7 : Excavation of Ore Block

Overall Stripping Ratio (SR)

$$= \frac{V_w * \rho_w(waste)}{V_0 * \rho_0(Ore)} = \frac{\left(\frac{1}{3}\pi R^2 H_c - \frac{2}{3}\pi r^2 \Delta h\right) * \rho_w}{\frac{1}{3}\pi r^2 \Delta * \rho_0}$$

1.3.5 Parameters of the Working Bench : In a working bench of opencast mine, the basic parameters are shown in Figure 1.8 and Figure 1.9



β : Overall Slope angle
B : Bench width
H : Bench height
x : Bank width
θ : Bench Slope

Figure 1.8 : Working Bench of Opencast Mine

$$\textbf{Overall Slope Anlge } (\beta) \;= tan^{-1}\frac{3H}{2B+3x} = \frac{3H}{2B+3\frac{H}{tan\theta}}$$

Definition of the terms

- **Spacing (S) :** Distance between the two holes in the same row called spacing
- **Burden (B) :** Distance between the crest of the bench and 1st row of drill holes or the distance between the two rows of holes called burden.
- **Height of the bench (H) :** Vertical distance between the benches called the bench height.
- **Width of the bench :** This is the horizontal gap between the toe of the upper bench to the crest of the lower bench. This should be always greater than the height of the bench.
- **Safety Berms :** This can be referred to the piles of rock material near the crest of the bench to improve the safety the mine safety.
- **Bank width :** This is the horizontal projection of the slope face.
- **Ramps :** This is the passage to make the access from one level to another level. Maximum permissible gradient of this passage is 1 in 10.

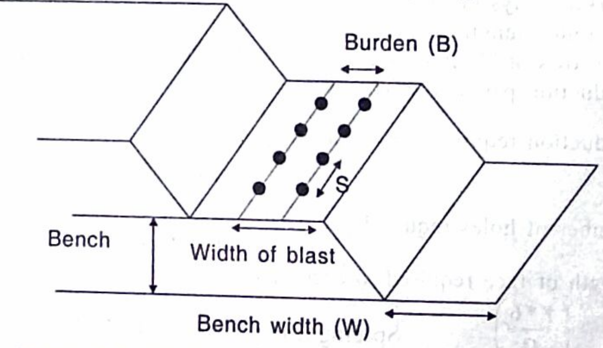


Figure 1.9 : Working Bench of Opencast Mine

1.3.6 DGMS norms in Opencast Mining

- In alluvium soil, morum, gravel, clay, debris or other soft ground slope of the benches shall not exceed 45° from the horizontal or the sides shall be kept benched and the height of the bench shall not exceed 1.5 m and the breadth shall not be less than the height.
- In coal, slope of the benches shall not exceed 45° from the horizontal or the sides shall be kept benched and the height of the bench shall not exceed 3 m and the breadth shall not be less than the height.
- In alluvium soil, morum, gravel, clay, debris or other soft ground, if any pillar is left for the purpose of measurement, its height shall not exceed 2.5 m and where the height of such pillar exceeds 1.25 m, the base of the pillar shall not be less than 1.5 m in diameter.
- Maximum gradient of the haul road is 1 in 14.
- The permissible length of the trailing cable in opencast mines is 300 m as per IER.

1.3.7 Types of Ramps

- **Spiral :** It is not suitable where there is a problem of slope stability. But this layout is preferable when the pit wall is very steep.
- **Switch-back :** This is not spiral in nature but the road turns 180° at the end of the extent.
- **Zigzag :** There is not any uniformity in this type of layout. This type of layout changes the path to make the access to the lower level as per the availability of the suitable/ stable area.

1.3.8 Determination of number of faces required for a target production of 'y'

Assume :
Height of bench = H
Width of the bench = W
Spacing = S
Burden = B
Density of ore = γ
Working days per year = D
Working days per week = 6
Maximum length of face = L
Two rows of blasting in a round [Total width of blast = 2*B]
Production per hole = H * B * S * γ

Production required per week $= \dfrac{Y*6}{D}$

Number of holes required per week $= \dfrac{\left(\dfrac{Y*6}{D}\right)}{H*B*S*\gamma}$

Length of face required to accommodate above number of holes

$$= \frac{\left(\dfrac{Y*6}{D}\right)}{H*B*S*\gamma} * \frac{\text{Spacing (s)}}{\text{Number of rows (2)}} = \frac{Y*6*S}{2*H*B*S*\gamma*D} = \frac{Y*3}{H*B*\gamma*D}$$

For production at shovel face another face of the same length is required for drilling at the same time.

Length of the face $= \dfrac{Y*3*2}{H*B*\gamma*D}$

Again for grade control another face length of the same size required

Total face length required $= \dfrac{Y*3*2*2}{H*B*\gamma*D}$

Number of bench required $= \dfrac{Y*12}{H*B*\gamma*D} * \dfrac{1}{L} = \dfrac{Y*12}{H*B*\gamma*D*L}$

1.3.9 Determination of Powder Factor

$$\text{Powder Factor (PF)} = \frac{\text{Production per hole (te)}}{\text{Explosive consumption per hole (kg)}}$$

$$= \frac{H*b*S*\gamma}{\left(\dfrac{\pi*d^2}{4}\right)*(H-h)*\gamma_e}$$

Where, γ_e = specific gravity of explosive
h = length of stemming
d = diameter of the blast hole

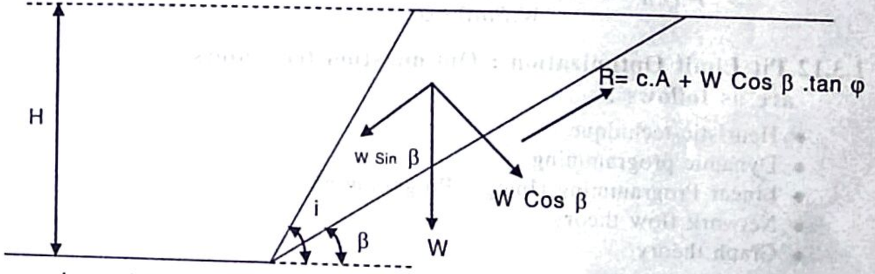
1.3.10 Types of slope failure

There are four types of slope failure occurs in the mining areas. These are as follows :

- Circular failure
- Plane failure
- Wedge failure
- Toppling failure

1.3.11 Determination of Factor of Safety of the Slope

CASE I : When there is no tension crack and water pressure



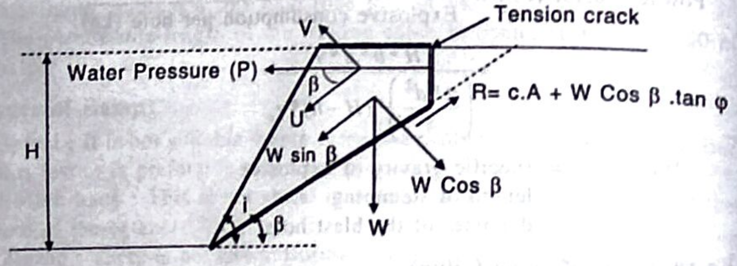
Assuming
Height of the bench = H
Overall slope angle from horizontal = i
Angle of discontinuity from horizontal = β
Weight of the block = W

Friction angle = φ
Cohesion = c
Area of the failure plane = A
Total force tending to induce sliding of the block = W. Sin β
Total force to resist sliding of the block = W. Cos β. tan φ + c. A
[Neglecting the water pressure]

$$\text{Factor of Safety (F.O.S.)} = \frac{\text{Total force to resist sliding}}{\text{Total force tending to induce sliding}}$$

$$\Rightarrow \quad F.O.S. = \frac{c.A + W.Cos\beta.tan\varphi}{W.Sin\beta}$$

CASE II : When there is tension crack and water pressure present



Water pressure along the sliding plan = U = P. Cos β
Water pressure normal to the sliding plane = V = P. Sin β
Total force tending to induce sliding of the block = W. Sin β + U
Total force to resist sliding of the block = (W. Cos β – V). tan φ + c. A
[Neglecting the water pressure]

$$\text{Factor of Safety (F.O.S.)} = \frac{\text{Total force to resist sliding}}{\text{Total force tending to induce sliding}}$$

$$\Rightarrow \quad F.O.S. = \frac{c.A + (W.Cos\beta - V).tan\varphi}{W.Sin\beta + U}$$

1.3.12 Pit Limit Optimization : Optimization techniques are as follows :

- Heuristic technique
- Dynamic programming
- Linear Programming (Integer Programming)
- Network flow theory
- Graph theory

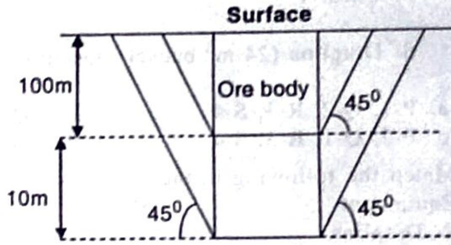
EXERCISE 1.3

1. Which one of the following term is not related to opencast mining?
a. Boxcut b. Highwall c. Crest d. Winze

2. As per MMR 1961, mine plan of a mine having large opencast working shall be on a scale having a representative factor of
a. 200:1 b. 2000:1 c. 1000:1 d. 4000:1

3. The sale value of chromite ore from an open pit mine is Rs. 6500 per tonne. Cost of mining, excluding stripping cost, is Rs. 2450 per tonne. If the cost of stripping is Rs. 1150 per m^3, the breakeven stripping ratio in m^3/tonne is
a. 2.18 b. 3.52 c. 3.65 d. 4.25 [GATE 2011]

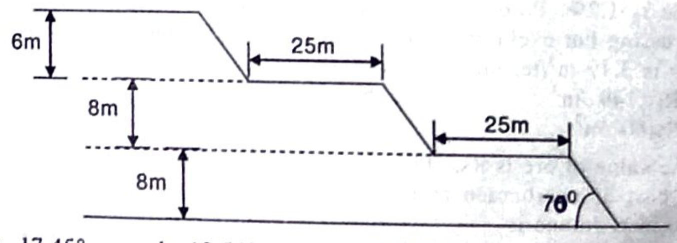
4. A vertical ore body having a consistent width of 100 m has been excavated by the open pit method up to a depth of 100 m at a pit slope angle of 45°. The figure shows the cross section of the open pit. If the ore body is further excavated by a depth of 10 m, maintaining the same pit slope angle, the ratio of waste rock to ore is
a. 1.2 b. 2.1
c. 3.0 d. 3.9



5. An opencast mine has four benches. The height, width and face angle for each bench are 13 m, 30 m and 70° respectively. The overall slope angle of the benches in degrees is
a. 25.5° b. 20.7°
c. 33.4° d. 19.7°
[GATE 2001]

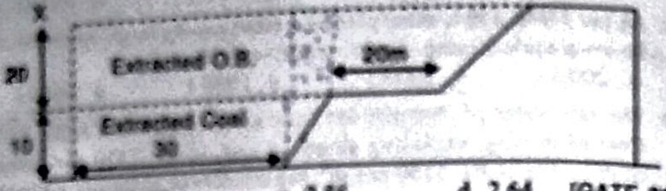
6. Four benches are being worked by the opencast mining system. Height, width and face angle for each bench are 15 m, 50 m and 70° respectively. The overall slope angle of the benches in degrees is
a. 15.45° b. 19.25° c. 32.65° d. 36.25° [GATE 2007]

7. What will be the overall slope of the benches shown in the figure?



a. 17.45° b. 18.56° c. 27.56° d. 43.55°

8. In an opencast mine shown in the figure below, the coal has a density of 1.4 t/m³. Assuming mining operation started from plane XY, the operating stripping ratio under the given conditions in m^3/t is



a. 2.32
b. 2.47
c. 2.56
d. 2.64 [GATE 2010]

9. Match the following in the context of Indian mining practice :

Equipment	Power source
P. Rocker shovel	1. Battery
Q. Locomotive	2. Compressed air
R. Shearer	3. Electricity (Maximum voltage 6.6 kV AC)
S. Dragline (24 m^3 bucket capacity)	4. Electricity (Maximum voltage 1.1 kV AC)

a. P-1, Q-2, R-3, S-4
b. P-2, Q-1, R-4, S-3
c. P-2, Q-1, R-3, S-4
d. P-1, Q-3, R-2, S-4

10. Match the following in the context of Indian mining practice :

Equipment	Slice Thickness (m)	Action
P. Dragline	1. 6-12	a. Crowding
Q. Shovel	2. 30-40	b. Hoisting
R. Surface Miner	3. 0.2-0.4	c. Cutting

a. P-1-b, Q-2-a, R-3-c
b. P-2-b, Q-1-a, R-3-c
c. P-2-a, Q-1-b, R-3-c
d. P-2-b, Q-3-c, R-1-a [GATE 2008]

11. Given bench height: 12 m, burden: 4 m, Spacing: 5 m; explosive per hole : 120 kg; density of rock: 2600 kg/m^3, the powder factor in tonne/kg is
a. 2.0
b. 4.6
c. 5.2
d. 7.3 [GATE 2008]

12. In a mine for one shovel six trucks are assigned. The shovel loading time per truck is 5 min, and truck cycle time 20 min. Calculate the match factor.
a. 0.66
b. 1.00
c. 1.20
d. 1.50

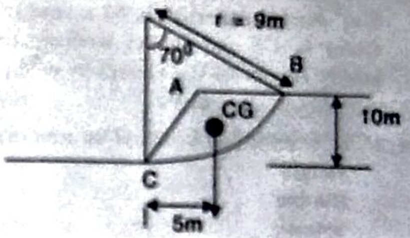
13. For an open pit mining the value of metal is Rs. 210 /kg, and recoverable grade is 1.2%. Production cost per tonne of ore inclusive of mining and processing but excluding stripping is Rs. 2000. If the break even stripping ratio is 3.49 m^3/t, then the stripping cost is
a. Rs. 149 /m^3
b. Rs.72 /m^3
c. Rs.60 /m^3
d. Rs.49 /m^3

14. If the value of ore is Rs. 600 per tonne, production cost Rs. 400 per tonne, and cost of overburden removal Rs. 50 per m^3, the break-even stripping ratio in m^3/tonne is
a. 4:1
b. 3:1
c. 1:3
d. 1:4 [GATE 2008]

15. Total number of injuries in an opencast coal mine employing 800 persons is 16 in a year. As per DGMS norms, the injury rate per 1000 persons employed is
a. 13
b. 15
c. 20
d. 25 [GATE 2008]

16. In a mine deploying shovel and dumper, the distance from the crest of the bench to the centre of the dumper including safety berm and clearance is 7.5 m, the dumping radius and the cutting radii of the shovel are 15 m and 12 m respectively. The width of the working bench of the mine in m is
a. 32.5
b. 30.5
c. 36.5
d. 34.5 [GATE 2005]

17. For a topsoil bench in an opencast mine as shown in the figure, the weight of the sliding block of unit width, ABCA, is 500 kN and the cohesive strength of the material is 20 kN/m^2. Assuming the surface to be totally frictionless, the factor of safety (FOS) for the stability of the slope is
a. FOS < 0.5
b. 0.5 < FOS < 1.0
c. FOS = 1.0
d. FOS > 1.0



18. A haul ramp of 6 % grade is to be constructed connecting a bench of 15 m height to the surface. If the cost of ramp construction is 1 million rupees per km, the cost of this ramp construction in million rupees will be
a. 0.25
b. 0.10
c. 2.50
d. 0.50

19. Maximum permissible gradient for the haul road in opencast mines is
a. 12°
b. 13°
c. 14°
d. 15°

20. Dumper arrives at a shovel at a constant rate of 10 per hour. The service rate is constant at 15 per hour. At a time, the expected number of dumpers in the queue is
a. 2
b. 3
c. 5
d. 4

21. A 15 m^3 dragline is deployed in an O.B. bench of an O.C. mine. It works for 40 days at the rate of 6 hrs per shift and 3 shifts a day. The cycle time, bucket fill factor, and operating efficiency of the dragline are respectively 50 s, 0.8, and 75 %. The total volume of O.B. in m^3 handled by the dragline is
a. 356918
b. 365918
c. 532158
d. 466560

22. In case of an opencast blasting the danger zone comprises of area within the radius of
a. 300 meters from the place of firing
b. 400 meters from the place of firing
c. 500 meters from the place of firing
d. 600 meters from the place of firing

23. In case of an O.C. mine, if the pit wall are too steep to allow suitable bends the layout of haul road geometry selected should be
a. switch back
b. spiral
c. zigzag
d. none of the above

24. To open a mine for deep seated, the cut used is
a. external box cut b. internal box cut
c. trench d. deep cut

25. In ultimate pit analysis the most widely used method is
a. dynamic programming b. linear programming
c. simulation d. heuristic technique

26. For O.B. removal with shovel – dumper combination, the optimum size of the dumper depends on
a. distance of the haul b. size of the shovel
c. distance of the haul and size of the shovel
d. tonnage to be handled

27. In shovel loading of broken iron ore, if loading cycle time of the shovel of 5 m^3 bucket capacity, is 60 seconds, bucket fill ratio is 0.8, rock looseness factor is 1.5 and shovel working time utilization factor is 0.6, then the hourly output of the shovel in m^3/hr will be
a. 92 b. 94 c. 96 d. 97

28. In an overburden bench of an iron ore mine the following data were found :
Height = 10 m
Burden = 4 m
Spacing = 3 m
Sub-grade drilling length = 1m
Collar stemming length = 3 m
If the diameter of the explosive column is 150 mm and the specific gravity of explosive and that of rock are 0.9 and 2.4 respectively, the powder factor in te/kg will be
a. 3.29 b. 4.98 c. 2.26 d. 4.53

29. From a hard rock open pit mine, the following data available
Burden : 3 m
Spacing : 4 m
Bench height : 12 m
Sub-drilling : 1 m
Collar stemming : 4 m
Diameter of hole : 150 mm
Density of rock : 2500 kg/m^3
Density of explosive : 800 kg/m^3
The powder factor in kg/t is
a. 0.314 b. 0.353 c. 2.833 d. 3.183 [GATE 2006]

30. The feasibility study of an opencast mine revealed the following :
Recoverable value per tonne of ore = Rs. 900
Production cost per tonne of ore = Rs. 550
Stripping cost per tonne of waste = Rs. 45
The break even stripping ratio will be
a. 7.77 b. 32.20
c. 1.78 d. 12.22

31. The following data are provided for a surface mine to be excavated by a shovel :
Production target : 10000 te/shift
Available hours per shift : 6 hrs
Shovel loading cycles per hour : 10^6
Bank density of the material mined : 2400 kg/m^3
Swing factor at 120° swing : 0.91
Bucket fill factor : 0.64
Utilization of available time : 83%
No of working days in a year : 300
No of shifts per day : 3
The annual production target in Mte is
a. 5.76 b. 7.00 c. 8.19 d. 9.00 [GATE 2012]

32. In the above question, the size of bucket of the shovel in m^3 is
a. 5.55 b. 9.33 c. 11.22 d. 13.55 [GATE 2012]

33. A tension crack of 5 m depth behind a rock slope crest is completely filled with water. What is the horizontal force exerted on a unit slope length by the water in the tension crack.
a. 122.60 kN b. 245.00 kN
c. 12.26 MPa d. 24.50 Nm

34. Block economic values in Lakhs of Rupees for a section of a block economic model are shown below.

-1	-1	1	-1	0	-1
-1	0	0	0	-1	-2
-5	-3	-2	5	-2	-3

At a permissible slope angle of 1:1, the optimum pit value of the section in Lakhs of Rupees is
a. 0 b. 1 c. 2 d. 3 [GATE 2013]

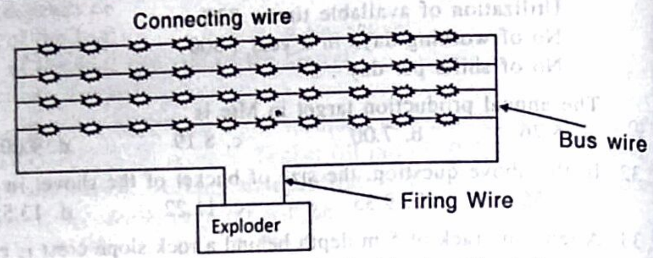
35. A surface miner with 2.0 m cutting drum width excavates coal in windrowing mode from a bench with effective face length 200 m. The cutting speed of the surface miner is 10 m/min and the cutting depth 25 cm. The density of coal is 1.4 tonne/m^3. If the average turning time of the machine at the face end is 5 min, the rate of production in tonne/hour becomes
a. 320 b. 350 c. 360 d. 380 [GATE 2013]

36. A bucket wheel excavator with 20 buckets of capacity 0.5 m^3 each, rotates at 5 rev/min. The bucket fill factor is 80%. The excavator loads on to 1200 mm wide belt conveyor. The cross-section area (m^2) of the material on the belt is $0.1B^2$, where B is the belt width in m. The minimum speed of the belt in m/s to avoid spillage of material is
a. 7.23 b. 5.79 c. 4.63 d. 3.70 [GATE 2013]

37. The blasting technique used for controlled throw of overburden is known as
a. cast blasting b. coyote blasting
c. plaster shooting d. pop shooting [GATE 2012]

38. A surface mine blast pattern shown below has the following details:

Accessory	Resistance (in Ohms)	Number or Length
Detonator	2 per detonator	40 nos.
Connecting wire	0.5/m	100 m
Bus wire	0.5/m	100 m
Firing wire	0.01/m	200 m



If the exploder supplies 440 V, the current in the blasting circuit in ampere is
a. 5.36 b. 7.31 c. 4.83 d. 2.53 [GATE 2008]

39. A surface mine blast design has 9 holes in a row, each of 8 m length and 200mm diameter. The spacing and burden are 6 m and 5 m respectively. The length of subgrade drilling is 1 m and the density of in-situ rock is 2.43 t/m^3. Assuming no back break, the output per blast in t is
a. 4593 b. 5905
c. 6124 d. 6299 [GATE 2009]

40. A surface mine blast design has 9 holes in a row, each of 8 m length and 200mm diameter. The spacing and burden are 6 m and 5 m respectively. The length of subgrade drilling is 1 m and the density of in-situ rock is 2.43 t/m^3. Considering an explosive density of 0.9 t/m^3 and stemming length of 2 m, the powder factor from the blast in t/kg is
a. 4.12 b. 4.00 c. 3.86 d. 3.01 [GATE 2009]

41. A non-electric detonating relay does not contain
a. Delay element b. Fuse head
c. Metal sleeve d. Neoprene connecting tube [GATE 2009]

42. In a surface mine blast, the peak particle velocity (V in mm/s) is estimated from the equation $V = 120(\sqrt{SD})^{-1.0}$, where SD is square root scaled distance. If at a distance of 100 m from the blast site the permissible peak particle velocity is 25 mm/s, the maximum charge per delay in kg is
a. 404 b. 414 c. 434 d. 464 [GATE 2008]

43. Which one of the following is the most likely mode of slope failure for waste dump
a. Circular b. Wedge c. Plane d. Toppling [GATE 2014]

44. A dragline is required to remove 3,00,000 m^3 of rock per month on the bank volume basis. Consider the following data for the dragline operation.
Effective working hours per month = 450
Bucket fill factor = 0.8
Cycle time = 65 s
Swell factor of the rock = 1.25
The minimum bucket capacity of the dragline in m^3 is
a. 7.70 b. 9.63
c. 12.04 d. 18.80 [GATE 2014]

Answer

1. d 2. d 3. b 4. b 5. a 6. b 7. c
8. c 9. b 10. b 11. c 12. d 13. a 14. a
15. c 16. d 17. b 18. a 19. c 20. a 21. d
22. c 23. b 24. b 25. d 26. c 27. c 28. c
29. c 30. a 31. d 32. d 33. a 34. c 35. b
36. c 37. a 38. b 39. a 40. d 41. c 42. c
43. a 44. d

Solution of Exercise 1.3

3. We have,
Break even stripping Ratio (B.E.S.R.)

$$= \frac{\text{Recoverable price per te of ore} - \text{O.C. production cost per te of ore} + \text{Profit margin per te of ore}}{\text{Cost of extraction per cubic meter of overburden material}}$$

$$= \frac{6500-2450+0}{1150} = 4050 = 3.52\ m^3/te$$

[Assuming profit margin is zero]

4. We have,

$$\text{Ratio of waste rock to ore} = \frac{\text{Volume of waste rock}}{\text{Volume of ore}}$$

$$= \frac{2*\left(\frac{1}{2}*110*110*\tan 45\right)*x-2*\left(\frac{1}{2}*100*100*\tan 45\right)*x}{100*10*x}$$

$$= \frac{12100-10000}{1000} = 2.1$$

5. Given data,
Number of benches = 4
Height of each bench = 13m
Width of each bench = 30 m
Face angle = 70^0
Therefore, we have

$$\text{Overall slope angle}(\theta) = \tan^{-1}\left[\frac{4*\text{height of benches}}{3*(\text{width of benches}) + 4*(\text{Bank width})}\right]$$

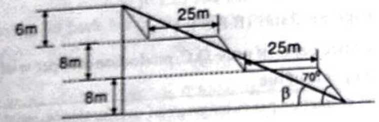
$$= \tan^{-1}\frac{4*13}{3*30+4*\dfrac{13}{\tan 70}} = \tan^{-1} 0.477 = 25.5^0$$

6. Given data.
Number of benches = 4
Height of each bench = 15 m
Width of each bench = 50 m
Face angle = 70°
Therefore, we have

$$\text{Overall slope angle}(\theta) = \tan^{-1}\left[\frac{4*\text{Height of benches}}{3*(\text{Width of benches}) + 4*(\text{Bank width})}\right]$$

$$= \tan^{-1}\frac{4*15}{3*50+4*\dfrac{15}{\tan 70}} = \tan^{-1} 0.35 = 19.25^0$$

7. Given data.
Let the overall slope angle is β



$$\text{Oveall slope angle}(\theta) = \tan^{-1}\left[\frac{\sum \text{Height of benches}}{\sum\left(\text{Width of benches} + \sum \text{Bank width}\right)}\right]$$

$$= \tan^{-1}\frac{6+8+8}{25+25+\dfrac{8}{\tan 70}+\dfrac{8}{\tan 70}+\dfrac{6}{\tan 70}}$$

$$= \tan^{-1}\frac{22}{59} = \tan^{-1}(0.378) = 20.45^0$$

8. $$\text{Operating Stripping Ratio (B.E.S.R.)} = \frac{\text{Volume of overburden removal}}{\text{Weight of ore extraction}}$$

$$= \frac{50*20*x+\frac{1}{2}*20*20\tan 70*x}{\left(30*10*x+\frac{1}{2}*10*10*\tan 70*x\right)*1.4} = \frac{1000+549.5}{(300+137.4)*1.4} = \frac{1549.5}{612.3} = 2.53$$

Where x = width of extraction
Slope angle of O.B. = 70^0
Slope angle of ore banches = 70^0

11. We have,

$$\text{Power Factor} = \frac{\text{weight of rock produced}}{\text{Mass of explosive used}} = \frac{\text{Volume of rock * density of rock}}{\text{Mass of explosive used}}$$

$$= \frac{(12*4*5*2600)/1000}{120} = 5.2$$

12. Number of trucks assigned to one shovel (N) = 6
Truck cycle time (t_c) = 20 min = 1/3 hour
Truck arrival rate = N/t_c = 18/hr.
Service rate of shovel = 1/shovel loading time = 12/hrs.

$$\text{Therefore, Match Factor} = \frac{\text{Truck arival rate}}{\text{Loaderservice rate}} = \frac{18}{12} = 1.5$$

13. We have,
Break even stripping Ratio (B.E.S.R)

$$= \frac{\text{Recoverable price per te of ore} - \text{O.C. production cost per te of ore} + \text{Profit marging per te of ore}}{\text{Stripping cost or cost of extraction per cubic meter of overburden material}}$$

$$\Rightarrow 3.49 = \frac{210000*\frac{1.2}{100} - 2000 + 0}{\text{Stripping Cost}}$$

[Assuming profit margin is zero]

$$\Rightarrow \text{Stripping Cost} = \frac{520}{3.49} = \text{Rs.}150/\text{m}^3$$

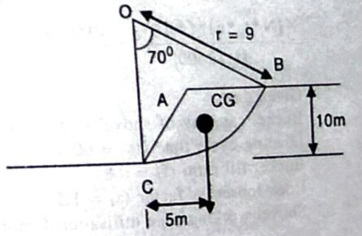
14. We gave,
Break even Stripping Ratio (B.E.S.R.)

$$= \frac{\text{Recoverable price per te of ore} - \text{O.C. production cost per te of ore} + \text{Profit margin per te of ore}}{\text{Stripping cost of extraction per cubic meter of overburden material}}$$

$$= \frac{600-400+0}{50} = 4\ \text{m}^3/\text{te}$$

15. The injury rate per 1000 person employed = (16/800)*1000 = 20

16. Width of the working bench = distance from the crest of the bench to centre of the dumper + dumping radius of shovel + cutting radius of shovel
= 7.5 + 15 + 12 = 34.5m



17. Given data :
We have,

$$\text{Overall slope angle}(\theta) = \tan^{-1}\left[\frac{4*\text{height of benches}}{3*(\text{width of benches}) + 4*(\text{Bank width})}\right]$$

$$= \tan^{-1}\frac{4*13}{3*30+4*\frac{13}{\tan 70}} = \tan^{-1}0.477 = 25.5^0$$

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Number of benches = 4
Height of each bench = 15 m
Width of each bench = 50 m
Face angle = 70°
Therefore, we have

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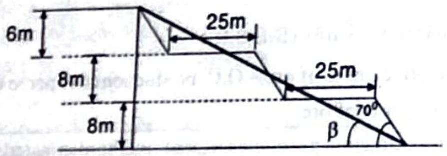
7. Given data,
Let the overall slope angle is β



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$$= \tan^{-1}\frac{6+8+8}{25+25+\frac{8}{\tan 70}+\frac{8}{\tan 70}+\frac{6}{\tan 70}}$$

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8. $\text{Operating Stripping Ratio (B.E.S.R.)} = \frac{\text{Volume of overburden removal}}{\text{Weight of ore extraction}}$

$$= \frac{50*20*x+\frac{1}{2}*20*20\tan 70*x}{\left(30*10*x+\frac{1}{2}*10*10*\tan 70*x\right)*1.4} = \frac{1000+549.5}{(300+137.4)*1.4} = \frac{1549.5}{612.3} = 2.53$$

Where x = width of extraction
Slope angle of O.B. = 70^0
Slope angle of ore banches = 70^0

11. We have,

$$\text{Power Factor} = \frac{\text{weight of rock produced}}{\text{Mass of explosive used}} = \frac{\text{Volume of rock}*\text{density of rock}}{\text{Mass of explosive used}}$$

$$= \frac{(12*4*5*2600)/1000}{120} = 5.2$$

12. Number of trucks assigned to one shovel (N) = 6
Truck cycle time (t_c) = 20 min = 1/3 hour
Truck arrival rate = N/t_c = 18/hr.
Service rate of shovel = 1/shovel loading time = 12/hrs.
Therefore, $\text{Match Factor} = \frac{\text{Truck arival rate}}{\text{Loader service rate}} = \frac{18}{12} = 1.5$

13. We have,
Break even stripping Ratio (B.E.S.R)

$$= \frac{\text{Recoverable price per te of ore} - \text{O.C. production cost per te of ore} + \text{Profit marging per te of ore}}{\text{Stripping cost or cost of extraction per cubic meter of overburden material}}$$

$$\Rightarrow 3.49 = \frac{210000*\frac{1.2}{100} - 2000 + 0}{\text{Stripping Cost}}$$

[Assuming profit margin is zero]

$$\Rightarrow \text{Stripping Cost} = \frac{520}{3.49} = \text{Rs. } 150/\text{m}^3$$

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Break even Stripping Ratio (B.E.S.R.)

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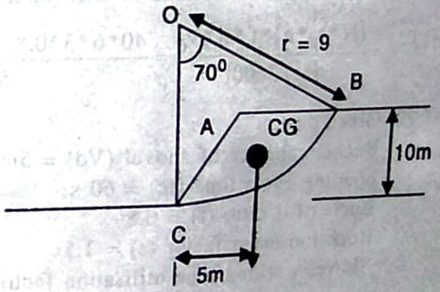
$$= \frac{600-400+0}{50} = 4\ \text{m}^3/\text{te}$$

15. The injury rate per 1000 person employed = (16/800)*1000 = 20

16. Width of the working bench = distance from the crest of the bench to centre of the dumper + dumping radius of shovel + cutting radius of shovel
= 7.5 + 15 + 12 = 34.5m

17. Given data :
We have,



Factor of Safety (FOS) = $= \frac{\text{Restoring Moment about centre O}}{\text{Driving moment abount centre O}} = \frac{C*L*r}{W*x}$

where, L = arc length of
C = cohesion = 20 kN/m2
r = radius of arc = 9m
W = weight of sliding block ABCA of unit thickness = 500 kN
X = perpendicular distance between the vertical line of centre of circle from the centre of gravity = 5m.

Length of arc BC can be determined as

$$L = \frac{70}{360} * 2\pi r = \frac{70}{360} * 2 * 3.14 * 9 = 11\text{m}$$

Therefore, $FOS = \frac{20000*11*9}{500000*5} = 0.792$

18. We have,
The length of haul road = (100/6) *15 = 250 m = 0.25 km
Therefore, cost of construction of haul road = 0.25*1 million rupees
= Rs. 0.25 million

20. Given data,
Arrival rate (λ) = 10 per hour
Service rate (μ) = 15 per hour
Expected number of dumpers in the queue (Lq)

$$= \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{10*10}{15(15-10)} = \frac{4}{3}$$

= 2 nos (nearby integer)
[For details, refer Mine System Engineering Section]

21. Given data,
Bucket capacity of dragline (V_d) = 15m^3
Number of working days (N) = 40
Number of shift (n) = 3
Working hour per shift (t_s) = 6 hrs
Cycle time (tc) = 50s
Bucket fill factor (f) = 0.8
Operating efficiency of dragline (η) = 75%
Therefore, total volume of O.B. handled by dragline

$$= \frac{(N*t_s*n)*f*\eta*V_d}{(t_c/3600)} = \frac{40*6*3*0.8*0.75*15*3600}{50} = 466560\ m^3$$

27. Given data,
Bucket capacity of shovel (Vd) = 5m^3
Loading cycle time (tc) = 60 s
Bucket fill ratio (f) = 0.8
Rock looseness factor (s) = 1.5
Shovel working time utilisation factor (η) = 0.6

Therefore, hourly output of the shovel

$$= \frac{f*\eta*v_d}{s*(t_c/3600)} = \frac{0.8*0.6*3600}{1.5*60} = 96m^3/hr$$

28. Given data,
Height of bench (H) = 10 m
Burden (B) = 4 m
Spacing (S) = 3 m
Sub-grade drilling length (Ls) = 1 m
Collar stemming length (L_c) = 3 m
Diameter of explosive column (d) = 150 mm = 0.15 m
S.G of explosive (λ_e) = 0.9
S.G. of rock (λ_r) = 2.4
We have,

$$\text{Power Factor} = \frac{\text{weight of rock produced}}{\text{Mass of explosive used}}$$

$$= \frac{\text{Volume of rock * density}}{\text{Volume of explosive * density of explosive}}$$

$$= \frac{(H*B*S)*\lambda_r}{\pi*\frac{d^2}{4}*(H-Lc+Ls)*\lambda_e} = \frac{(10*4*3*2.4*10^3)}{3.14*\frac{0.15^2}{4}*(10-3+1)*0.9*10^3}$$

$$= 2260\frac{kg}{kg} = 2.26\ te/kg$$

Note : Height of explosive column = Bench height (H) – Collar stemming length (L_c) + Sub-grade drilling length.
Density of rock or explosive = S.G. of rock or explosive * density of water

29. Given data :
Height of bench (H) = 12 m
Burden (B) = 3 m
Spacing (S) = 4 m
Sub-grade drilling length (Ls) = 1 m
Collar stemming length (Lc) = 4 m
Diameter of hole (d) = 150 mm = 0.15 m
Density of explosive (λ_e) = 800 kg/m^3
Density of rock (λ_r) = 2500 kg/m^3
We have,

$$\text{Power Factor} = \frac{\text{weight of rock produced}}{\text{Mass of explosive used}}$$

$$= \frac{\text{Volume of rock * density}}{\text{Volume of explosive * density of explosive}}$$

$$=\frac{(H*B*S)*\lambda_r}{\pi*\frac{d^2}{4}*(H-Lc+Ls)*\lambda_e}=\frac{(12*3*4*2500)}{3.14*\frac{0.15^2}{4}*(12-4+1)*800}$$

$$=2831\frac{kg}{kg}=2.831te/kg$$

30. We have,

Break even stripping Ratio (B.E.S.R.)

$$=\frac{\text{Recoverable price per te of ore} - \text{O.C. production cost per te of ore} + \text{Profit margin per ti of ore}}{\text{Stripping cost or cost of extracion per cubic meter of overburden material}}$$

$$=\frac{900-550+0}{45}=7.77m^3/te$$

[Assuming profit margin is zero]

31. Given data,

Production target (Q) = 10000 te/shift

Available hours per shift (ts) 6 hrs

Shovel loading cycles per hour (q) = 106

Bank density of the material mined (?) = 2400 kg/m3

Swing factor at 120° swing (s) = 0.91

Bucket fill factor (f) = 0.64

Utilization of available time (η) = 83%

No of working days in a year (N) = 300

No of shifts per day (n) = 3

Therefore, Annual production target = Q*n*N = 10000*3*300
= 9*106 te = 9 Mte

32. We have,

Production target per shift = Vb*ts*q*s*f*η*ρ /1000

$\Rightarrow$ 10000 = Vb*6*106*0.91*0.64*0.83*2400/1000

$$\Rightarrow V_b=\frac{10000}{6*106*0.91*0.64*0.83*2.4}=13.55m^3$$

33. Given data,

Depth of tension crack (z) = 5 m

Since, this is completely filled with water, the height of water column is also be equal to bench.

Therefore, Z_w = 5 m

We have, horizontal force exerted by the water on unit slope length is

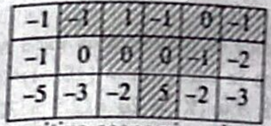
$$V=\frac{1}{2}\rho_w z_w{}^2$$

Where, ρ_w = density of water = 1000 kg/m^3

$$\Rightarrow V=\frac{1}{2}*1000*5*5=12500kg=12500*9.81N=122625N$$

$$=122.6\text{ kN}$$

34. Given economic values for different blocks are :



-1	-1	1	-1	0	-1
-1	0	0	0	-1	-2
-5	-3	-2	5	-2	-3

The maximum positive economic value is +5 and thus this block should be extracted for most economical extraction. Again, the slope is 1:1. Therefore, the highlighted blocks require to extracts before extraction of block having economic value +5.

Optimum pit value = -1 +1 -1+0-1+0+0-1+5 = 2 lakhs

35. Given data,

Width of cutting drum (W) = 2 m

Face length (L) = 200 m

Cutting depth (D) = 25 cm = 0.25 m

Density of coal (γ) = 1.4 te/m^3

Cutting speed (V) = 10 m/min

Average turning time of the machine at face end (tt) = 5 min

Therefore, time required to cut one face length (t) = 200/10 = 20 min

Effective time required to cut one face length = $t_t + t$ = 25 min

Length of face cut in 1 hour (Le) = 200 + 200 + 200/20 = 500 m

The rate of production = $(W*L_e*D)*\gamma$ = 2 * 500 * 0.25 * 1.4 = 350 te/hr

36. Given data,

Number of bucket in excavator (N) = 20

Capacity of each bucket (q) = 0.5m^3

Rotation speed (ω) =5 rev/min = 300 rev/hr

Bucket fill factor (f) = 80% = 0.80

Width of conveyor belt (B) = 1200mm = 1.2m

Cross-sectional area of the material on the belt (A) = 0.1 B^2 = 0.144m^2

To avoid the spillage of the material on the belt,

Material loading rate on the conveyor belt = Material transport rate by the belt

$\Rightarrow$ N*ω*w*f = A*V*3600

[Where, V = minimum speed of belt in m/s]

Length covered in 1 hour = 3600*V

$\Rightarrow$ 20*0.5*300*0.8 = 0.144*V*3600

$$\Rightarrow V=\frac{2400}{518.4}=4.63\,m/s$$

38. Given data,

Resistance of each detonator = 2 Ω

Resistance of connecting wire = 0.5 Ω/m

Resistance of Bus wire = 0.5 Ω/m

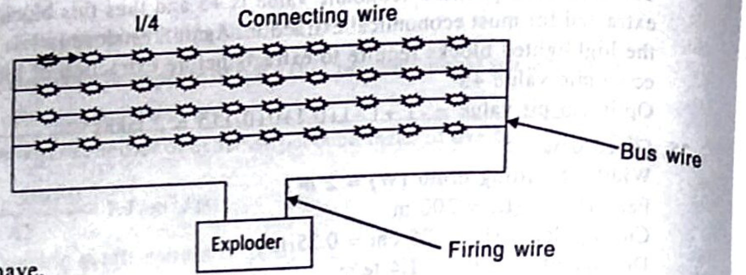
Resistance of Firing wire = 0.01 Ω/m

Number of detonator = 40 (10 in each row)

Length of connecting wire = 100 m

Length of Bus wire = 100 m

Length of Firing wire = 200 m
Exploder supplies voltage = 440 V
Therefore, Equivalent resistance of detonator in each row = 20 Ω
Resistance of connecting wire in each row = (100/4)*0.5 = 12.5 Ω
Equivalent resistance of each row (R) = 20 +12.5 = 32.5 Ω



We have,

$$\frac{1}{R'}=\frac{1}{R}+\frac{1}{R}+\frac{1}{R}+\frac{1}{R}=\frac{4}{R}$$

[Where R' = equivalent resistance of detonation and connecting wires]

$$\Rightarrow R'=\frac{R}{4}=\frac{32.5}{4}=8.125\,\Omega$$

Equivalent resistance of Bus wire (R_b) = 100*0.5 = 50 Ω
Equivalent resistance of Firing wire (R_f) = 200*0.01 = 2 Ω
Equivalent resistance of the circuit (R_e) = R'+ R_b + R_f = 8.125 + 50 + 2
= 60.125 Ω

Therefore, Current flow (I) = V/R = 440/60.125 = 7.32 A

39. Given data,
Number of holes (N) = 9
Spacing (S) = 6 m
Burden (B) = 5 m
Length of drill hole (L) = 8 m
Sub-grade drilling length (x) = 1 m
Diameter of the drill hole = 200 mm = 0.2 m
Density of rock (ρ_r) = 2.43 t/m^3
Output per blast = N*S*B*(L-x)* ρ_r = 9*6*5*(8 – 1)*2.43 = 4592.7 ≅ 4593 te

40. We have,

$$\text{Powder Factor}=\frac{\text{Output per round of blasting in te}}{\text{explosive used per round of blasting in kg}}$$

$$=\frac{4593}{\left(\frac{\pi}{4}*d^2\right)*(L-y)*\rho_e}$$

Where, y = stemming length = 2m
ρ_e = density of explosive = 0.9 t/m^3 = 900 kg/m^3

Therefore,

$$\text{Powder Factor}=\frac{4593}{\left(\frac{\pi}{4}*0.2^2\right)*(8-2)*900}=\frac{4593}{1526.04}=3.01\,\text{te/kg}$$

42. Given equation,
Pack particle velocity (V) in mm/s = 25 = 120* $\left(\sqrt{SD}\right)^{-1}$
Where, SD = square root scaled distance
We have, square root Scaled Distance = $\sqrt{S.D}=D/\sqrt{Q}$
Where, D = distance of the measuring transducer = 100m
Q = Maximum charge weight per delay
Therefore, 25 = 120 * $(100/\sqrt{Q})^{-1}$
$\Rightarrow Q = 434\text{ kg}$

44. Given data,
Target capacity (Q) = 3,00,000 m^3
Effective working hours per month (T) = 450
Bucket fill factor (f) = 0.8
Cycle time (t_c) = 65s
Swell factor (S) = 1.25d
Bucket capacity = V
We have,

$$Q=\frac{V*T*f}{S(t_c/3600)}=\frac{V*450*0.8*3600}{1.25*65}$$

⇒ 300000 = 15950.77*V
⇒ V = 18.8m^3

■

2.0 Rock Mechanics

2.1 Methods of Rock Exploration

2.1.1 By direct penetration

Direct penetration into the rock mass helps in direct observation of the structure and characteristics of the underlying strata. This can be achieved by core boring and recovery.

2.1.2 Geophysical prospercting

By making certain physical measurements from the surface without penetration the characteristics of the underlying strata can be estimated. There are four principle methods of geophysical exploration :

2.1.2.1 Seismic Method : This methods works on the basis of elastic properties and densities of the rocks. Seismic exploration is done by Reflection (for > 600m depth) and Refraction (for shallow depth).

2.1.2.2 Electrical Method : This method works on the principle of variation in electromagnetic field and resistivity of the underlying rocks.

2.1.2.3 Gravity Method : The basic theory involved in this method is the Newton's Law of Gravitation. this method works on the basis of variationin density of rock masses.

2.1.2.4 Magnetic Method : This method works on the basis of variation in magnetic properties of rock masses.

2.2 Rock Quality Designation (RQD)

This is the ratio of the sum of the length of the cores greater than 10 cm to the total length of the core run. This parameter represents the weakness of the rock. The classification of rock sample on the basis of RQD is shown in Table 2.1.

$$\text{RQD (\%)} = \frac{\text{(Sum of the length of the cores greater than 10 cm)}}{\text{(Total length of the core run)}} \text{ X } 100$$

Table 2.1 : Classification of Rock

RQD (%)	Description
0 -25	Very poor
25-50	Poor
50-75	Fair
75-90	Good
90-100	Excellent

- RQD could be obtained from discontinuity spacing measurements made on core or an exposure using the equation

$$\text{RQD} = 100e^{-0.1\lambda}(0.1\lambda + 1)$$

Where λ = the average number of fractures per meter of core length

For values of λ in the range 6 to 16/m, a good approximation to measured RQD values was found to be given by the linear relation

$$\text{RQD} = -3.68\lambda + 110.4$$

- The *RQD* may be estimated from the number of discontinuities per unit volume. The suggested relationship for clay-free rock masses is:

$$RQD = 115 - 3.3\,Jv$$

Where, *Jv* is the sum of the number of joints per unit length for all joint (discontinuity) sets known as the volumetric joint count.

2.3 Physical and Mechanical Properties of Rocks

2.3.1. Unit Weight

The unit weight of any solid can be expressed as

$$\gamma = \frac{W}{V}$$

Where, W = weight of solid
V = volume of solid

Two types of unit weight, namelydry unit weight γ_d and unit weight of water γ_w are commonly used in case of solids.

$$\gamma_d = \frac{W_s}{V}$$

$$\gamma_w = \frac{W_w}{V_w}$$

2.3.2 Specific Gravity of Solids

The specific gravity of any material is the ratio of its density to that of water. In the case of soils, it should be computed for solid phase only. It can be expressed as

$$G_s = \frac{M_s}{V_s\rho_w} = \frac{W_s}{V_s\gamma_w}$$

2.3.3 Determination of Specific Gravity of a Soil Sample

The specific gravity of the soil solids can be determined using the following formula :

$$\text{Specific Gravity } (G_s) = \frac{W_0}{W_0 + (W_A - W_B)}$$

Where,

W_0 = weight of sample of oven-dry soil = W_{PS}- W_P
W_A = weight of pycnometer filled with water
W_B = weight of pycnometer filled with water and soil
W_{PS} = Mass of empty pycnometer + dry soil
W_P = Mass of empty, clean pycnometer

2.3.4 Void Ratio

The relative volume of voids and solids may be expressed using the void ratio, e.

$$e=\frac{V_v}{V_s}$$

2.3.5 Porosity

The porosity of a rock sample is defined as the ratio of the volume of voids to the total volume of the sample.

$$\text{Porosity (P)} = \frac{V_v}{V}*100\% = \frac{e}{1+e}*100\%$$

Where, P = porosity in percentage
V_v = volume of void i.e. volume of air and water present in the pore spaces
V = total volume of the rock sample

It can also be defined as water porosity (Pw) and air porosity (Pa)

$$(P_W) = \frac{V_w}{V}*100\%$$

$$(Pa) = \frac{V_a}{V}*100\%$$

2.3.6 Density

It is defined as mass per unit volume of the rock. There are three types of density viz. dry density, bulk density and saturated density.

2.3.6.1 Dry density : It refers to mass per unit volume when the rock mass is completely dry i.e. void contain only air.

2.3.6.2 Bulk density : It refers to mass per unit volume in normal condition i.e. void contain some liquid and some air.

2.3.6.3 Saturated density : It refers to mass per unit volume when the rock mass is fully saturated.

The relationship between the dry density and bulk densityis

$$Y_d=\frac{Y_b}{1+m}=\frac{Y_b}{1+\frac{W_w}{W_s}}$$

Where, Y_d = dry density
Y_b = bulk density
m = the moisture content of the sample
W_w = the weight of water
W_S = the weight of solids

2.3.7 Moisture Content

It is the ratio of the weight of the water present in the sample to the weight of the sample.

$$\text{Moisture content (w)} = \frac{W_w}{W_s}*100\%$$

Where, W_W = weight of water
W_S = weight of sample

2.3.8 Degree of Saturation

It is the ratio of volume of water in the void to the total volume of voids in the rock sample. The rock mass having higher porosity has higher degree of saturation.

$$S=\frac{V_w}{V_v}$$

Where, V_w = volume of water
V_v = volume of void

2.3.9 Permeability

It is the ability of a porous material to allow a liquid to pass through its pores. Flow of a liquid through a porous mass is given by

Q = K*I*A **[Darcy's Law]**

where, Q = the flow through the area A (m^3/s)
I = the hydraulic gradient (dimensionless)
K = the coefficient of permeability (m/s)
A = Area of cross section (m^2)

2.3.10 Anisotropy and Isotropy

An isotropy material has the same property in three axes whereas an anisotropy material has different property in different axes. Since the rock mass always have some weakness in a particular direction or axis, it always behaves like anisotropy materials.

2.3.11 Stress

It is defined as the load applied on the material per unit area.

$$\text{Stress } (\sigma) = \frac{F}{A}$$

2.3.12 Strain

It is the ratio of change in length of the sample due to applied load to the original length of the sample.

$$\varepsilon=\frac{\Delta l}{l}$$

Where, ε = tensile or compressive strain, m/m
Δl = total elongation in a bar, m
l = original length of the bar, m

2.3.13 Strain Energy

When an elastic body is deformed, the forces that cause the deformation do work on the body as they deform it. In accordance with the principle of conservation of energy, this work is stored in the deform body in the form of elastic strain energy. The strain energy of the rock can be determined as

$$\text{Stored strain energy (U) } \frac{P.\Delta l}{2}$$

$$\text{Again, We have, } E=\frac{(P/A)}{(\Delta l/L)}$$

$$\Rightarrow \Delta l=\frac{PL}{AE}$$

Therefore, $U = \dfrac{P^2L}{2AE}$

Where, U = total energy stored in the bar or strain energy
P = tensile load
Δl = total elongation in the bar
L = original length of the bar
A = cross-sectional area of the bar
E = Young's modulus

2.3.14 Modulus of Elasticity of rock

Rate of change of strain as a function of stress. The property of the material to recover the deformation is known as elasticity. Within the elastic limit the stress is proportional to the strain, called Hook's law.

$$E = \frac{\sigma}{\varepsilon}$$

Where, E = elastic constant, called Modulus of Elasticity
σ = the stress applied on the material
ε = strain in the material due to stress applied

There are two types of modulus of elasticity viz. Tangent modulus of elasticity and Secant modulus of elasticity as shown in Figure 2.1.

Tangent modulus of elasticity is the slope of the stress-strain diagram at any point whereas secant modulus of elasticity is stress divided by strain at any given value of stress or strain. It is also called stress-strain ratio.

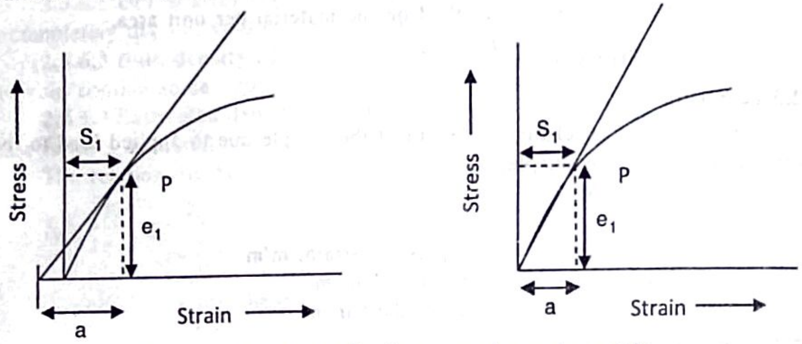


Figure 2.1 : Stress-Strain curve of a rock sample

Tangent modulus of elasticity $= \dfrac{S_1}{a}$

Secant modulus of elasticity $= \dfrac{S_1}{e_1}$

2.3.15 Poisson Ratio

This is the ratio of lateral strain to longitudinal strain. If the material is subject to uni-axial compressive stress (σ_x), then there is a strain in longitudinal (x) direction as well as in the lateral direction (y). If the load applied in a rock sample is P and corresponding change in diameter and length are ΔD and ΔL respectively as shown in Figure 2.2, Poisson ratio can be calculated as

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$$\text{Poisson Ratio} = \frac{\text{Lateral Strain } (\varepsilon_{lat})}{\text{Longitudinal Strain } (\varepsilon_{lon})}$$

$$= \frac{\Delta D / D}{\Delta L / L}$$

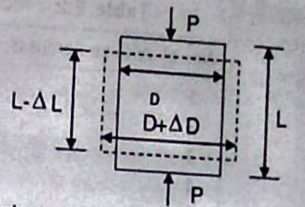


Figure 2.2 : Rock sample with applied load P

2.3.16 Unit Volume Change

The change in unit volume of rock samples is given by

$$\frac{\Delta V}{V} = \varepsilon(1 - 2\nu)$$

where, ΔV = change in volume
V = original volume
ε = strain
ν = Poisson's ratio

2.3.17 Relationship between elastic constants

For homogeneous isotropic simple relatons exist between elastic constants [Young's Modulus (E), Shear Modulus (G), Bulk Modulus(K), and Poisson's Ratio (ν)] that allows calculating them as long as two are known.

$$E = 2G(1 + \nu) = 3K(1 - 2\nu)$$

2.3.18 Deformability

This is defined as the capacity of the rock to strain under applied loads as shown in Figure 2.3.

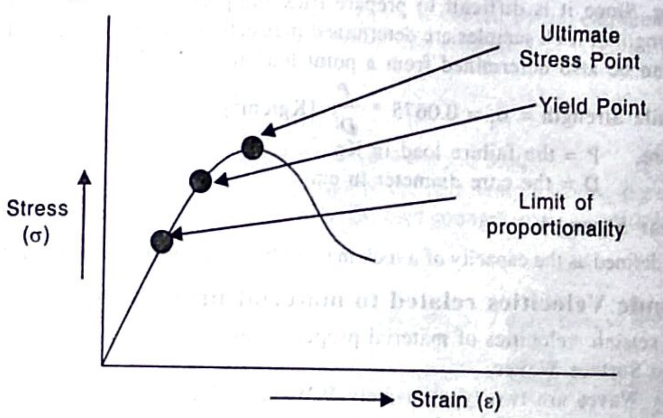


Figure 2.3 : Stress-Strain curve

2.3.19 Hardness

Hardness of rock is defined as its resistance to abrasion. Mohr's scale of hardness of minerals is given below in Table 2.2.

Table 2.2 : Mohr's scale of hardness of minerals

No. of relative hardness scale of rating H	Mineral
1	Talc (**Softest**)
2	Gypsum
3	Calcite
4	Fluorite
5	Apatite
6	Orthoclase Feldspars
7	Quartz
8	Topaz
9	Corundum
10	Diamond (**Hardest**)

2.4 Classification of Rock Strength

2.4.1 Compressive Strength

- Uni-axial Compressive Strength: It is the strength of the rock that can sustain without failure, when the load applied in one direction (parallel to the axis) only.
- Tri-axial Compression Strength: When a rock mass is subjected to an all round pressure and it is further subjected to an additional vertical pressure, then strength exhibited by the rock mass is known as a tri-axial compression strength.

2.4.2 Tensile Strength

It is defined as the maximum tensile stress which a material is capable of developing. Since it is difficult to prepare rock samples for direct tensile tests, tensile strength of rock samples are determined indirectly by Brazilian tests. Tensile strength can be also determined from a point load test (Reichmath, 1963)

$$\text{Tensile Strength} = \sigma_t = 0.0675 * \frac{P}{D^2} \ [\text{Kg/cm}^2]$$

Where, P = the failure load in Kg

D = the core diameter in cm

2.4.3 Shear Strength

It is defined as the capacity of a rock mass to bear the shear load without failure.

2.5 Seismic Velocities related to material properties

The seismic velocities of material properties are of two types namely Body Waves and Surface Waves.

Body Waves are two types namely P-Wave and S-Wave whereas Surface waves are Rayleigh waves and Love waves.

P-waves : P waves (primary waves) are longitudinal or compressional waves. In solids, these waves generally travel almost twice as fast as S waves and can travel through any type of material.

S-waves : S waves (secondary waves) are transverse or shear waves, which mean that the ground is displaced perpendicularly to the direction of propagation.

Rayleigh waves : Rayleigh waves, also called ground roll, are surface waves that travel as ripples with motions that are similar to those of waves on the surface of water.

Love Waves : Love waves are surface waves that cause horizontal shearing of the ground. They usually travel slightly faster than Rayleigh waves, about 90% of the S wave velocity.

For an ideal elastic, isotropic rock the following equations holds good

V_l - longitudinal wave velocity $= \sqrt{\frac{E}{\rho}}$

V_t - transverse wave velocity $= \sqrt{\frac{G}{\rho}}$

Vp - P-wave (cmpressive wave) velocity $= \sqrt{\frac{\frac{4}{3}G+K}{\rho}}$

Vs - S-wave (shear wave) velocity $= \sqrt{\frac{G}{\rho}}$

Where, E = elastic modulus

G = shear modulus

K = bulk modulus

ρ = density of rock

2.6 Rock Mass Strength Indices

2.6.1 Point Load Strength Index

In this test, a rock sample (length is at least 1.4 times the diameter) is loaded between two cones of hardened steel. Failure is caused by development of tensile cracks parallel to the loading axis. Point load strength (I_S) is given by

$$I_{s(d)} = \frac{P}{d^2}$$

where, P = the load at rupture

d = the distance between the point load

$I_{S(d)}$ = point load strength of the core sample having the diameter d.

A common correlation between point load strength and uni-axial compressive strength is given by

$$\sigma_U = 24 * I_{S(50)}$$

where, σ_U = uni-axial compressive strength of a core sample having length to diameter ratio is 2 : 1

$I_{S(50)}$ = point load strength of a core sample of diameter 50 mm.

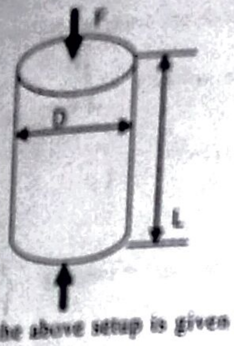
2.6.2 Slake Durability Index

In this test, 500 gm of dry rock is broken into 10 pieces and put in a cylindrical drum of 140 mm diameter and 100 mm length. The cylindrical wall of the drum is made of sieve mesh of 2 mm openings. The drum is turned in a water bath for 10 min at a rate of 20 rpm. The sample of the drum is collected and dried. The percent of rock retained in the drum (on dry weight basis) is known as slake durability index (I_d).

2.7 Testing of Rock Samples

2.7.1 Uni-axial Compressive Strength Test

For cylindrical specimen, Length to diameter ratio should be lying in the range of 2 to 3. For cubical block, L: D is 1.



Compressive strenth of the above setup is given by

$$\sigma = \frac{F}{A} = \frac{F}{\pi D^2/4} = \frac{4F}{\pi D^2}$$

The relationship between the computed ultimate compressive strength of a sample having L : D is 1 and a sample having length is greater than diameter is given by

$$\sigma_{cu} = \frac{\sigma_{u/s}}{0.778 + 0.222\,(D/L)}$$

Where, σ_{cu} = computed ultimate compressive strength of a cubical or cylindrical specimen having L: D is 1

$\sigma_{u/s}$ = ultimate compressive strength of a cylindrical specimen having L: D is greater than 1

D = diameter of a cylindrical sample or the length of a side in case of a cube

L = length or height of a sample

2.7.2 Brazilian Test

For Brazilian test, the specimen should have a length to diameter ratio in the range of 0.5 to 1. The tensile strength is given by a Brazilian test as

$$\sigma_t = \frac{2*F}{\pi*D*L}$$

where, σ_t = tensile stress at failure in Kg/cm^2

F = failure load in Kg

D = diameter of the specimen in cm

L = length of the specimen in cm

2.7.3 Bending Test

In this type of test, a beam is cut out of rock sample with rectangular cross section and the specimen is supported at two ends and load applied at the middle of the sample. The tensile strength is obtained by

$$\sigma_t = \frac{3*P*L*n}{b*d^2}$$

where, σ_t = tensile strength in kg/ cm^2

P = applied load in kg

L = length between the supports in cm

b = width of the specimen in cm

d = depth of the specimen

n = distance from the neutral axis to the farthermost fibre

2.7.4 Punch Shear Test

In this test, the shear strength of the specimen can be determined by punching shear. The specimen should be prepared in disc shape for this type of test. The shear strength can be evaluated with the following relation

$$\text{Shear Strength} = S = \frac{P}{A} = \frac{P}{\pi*d*t}\ [\text{kg/cm}^2]$$

Where, P = load applied to punch the specimen in Kg

A = circumferential shear area, cm^2

t = thickness of the disc, cm

d = diameter of the puncher, cm

2.8 Types of Classification of Rock Mass

The basis of rock mass classification is developed by various scientists and is given in Table 2.3.

Table 2.3 : Classification of Rock Mass

Classification	Basis of classification
Gamble's	Slake durability
Deere's	RQD
Deer's and Miller	Uni-axial compressive strength
Bieniawski's	RMR
Barton's	Q system
Coates	Various types of classification based on uni-axial compressive strength, failure characteristics, deformation, homogeneity, continuity
Hansagi's	C-Factor

2.9 Determination of Shear Strength

In all slope stability analysis, it is necessary to use the shear strength properties of either the discontinuities or of the rock mass and there are different procedures for determining the rock mass strength like

- Discontinuity shear strength can be measured in the field and the laboratory.
- Rock mass shear strength is determined by empirical methods involving either back analysis of slopes cut in similar geological conditions, or by calculation involving strength indices.

2.10 Principle Stresses

If the rock element is rotated to a certain angle, the shear stresses will be zero on all four sides. The planes on each side of this element are represented by points E and F (shown in Figure 2.4), and are known as principal planes. The stresses acting on them are known as principal stresses. The major principal stress σ_1, is the greatest normal stress, whereas the minor principal stress, σ_3, is the smallest normal stress.

The magnitudes of σ_1 and σ_3 are given by

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{zx}^{\ 2}} \text{ and } \sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{zx}^{\ 2}}$$

The angle between the major principal stress (σ_1) and the vertical stress (σ_y) is

$$\theta_z = \frac{1}{2}\cos^{-1}\left(\frac{2\sigma_y - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}\right)$$

Where, σ_1 = major principal stress
σ_3 = minor principal stress
σ_x = horizontal stress
σ_y = vertical stress
τ_{zx} = shear stress acting on a horizontal plane

Stresses on other Planes : Once the principal stresses have been found, we can obtained the normal and shear stress that act on any plane with the following equations.

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 + \sigma_3}{2}\cos 2\theta$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta$$

Where, σ = normal stress acting on a particular plane
σ_1 = major principal stress
σ_3 = minor principal stress
τ = shear stress acting on a particular plane
θ = angle from σ_1 to σ (clockwise is +ve; counter - clockwise is -ve

Cohesion (c) and Internal Friction Angle (φ) : In rock slope design, rock is assumed to be coluomb material in which the shear strength of the sliding surface is expressed in terms of cohesion (c) and the friction angle (φ).

For a test carried out at a constant normal stresses, a typical plot of the shear stress against the shear displacement is shown in the Figure 2.5 (a), 2.5 (b) and 2.5 (c). At small displacement, the specimen behaves elastically and the shear stress inceases linearly with displacement. As the force resisting movement is overcome, the curve becomes non-linear and then reaches a maximum that represents the peak shear strength of the discontinuity. Thereafter, the stress required to cause displacement decreases and eventually reaches a constant value termed the residual shear strength. If the peak shear strength and residual shear strength values from tests are determined at different normal stress levels and plotted, a linear relationship was established. The plot shows the relationship is called the Mohr's diagram.

2.11 Mohr's Circle Analyses

A Mohr's circle describes the two dimensional stresses at a point in a material. It considers the stresses acting on each side of a two dimensional element and plots them on σ vs. τ diagram as shown in Figure 2.4. Each point on the circle represents the normal and shear stresses acting on one side of an element oriented at a certain angle. For example, in this Mohr's circle points A and B represent (σ_y, τ_{yx}) and (σ_y, τ_{xy}), which are the stresses acting on an element aligned with the x and z axes.

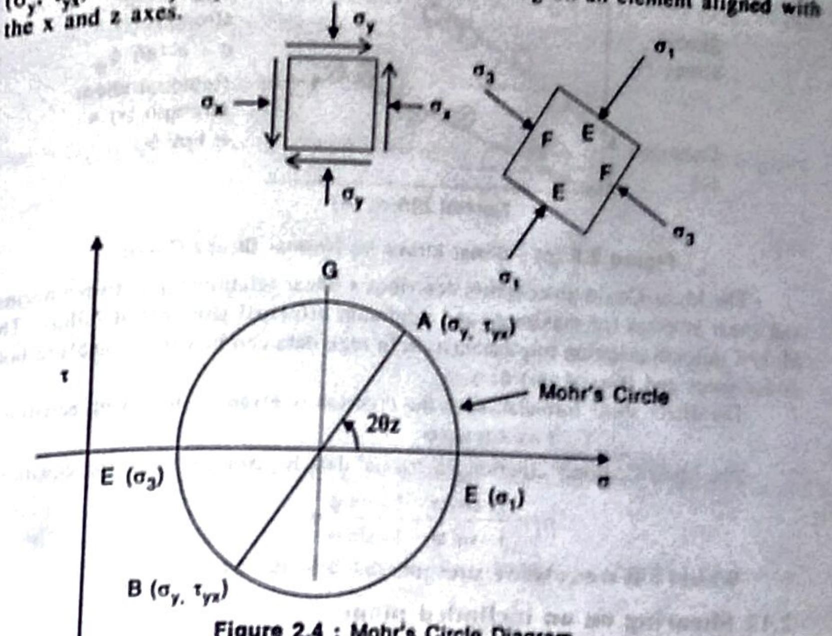


Figure 2.4 : Mohr's Circle Diagram

The relationships between peak shear strength (τ) and normal stress (σ) is given by

Peak shear strength (τ) = c + σ tan φ_p

The relationships between peak shear strength (τ) and normal stress (σ) is given by

Residual shear strength (τ) = σ tan φ_r

Where, φ_p & φ_r are the peak friction angle and residual friction angle respectively.

c is cohesive strength.

For the residual strength condition, the cohesion is lost once displacement has broken the cementing action.

Again, $\varphi_p > \varphi_r$ [Because the shear displacement grinds the minor irregularities on the rock surface and produces a smoother, lower friction surface]

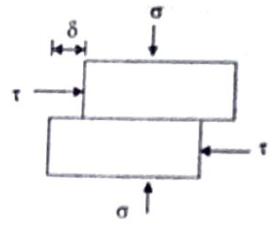


Figure 2.5 (a) : Shearing in plane

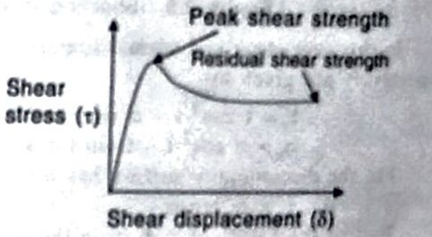


Figure 2.5 (b) : Shear disp. Vs. Shear Stress

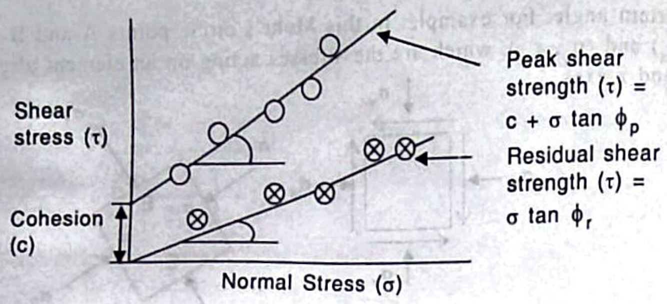


Figure 2.5 (c) : Shear stress Vs Normal Stress Curve

The Mohr-Coulomb criterion describes a linear relationship between normal and shear stresses (or maximum and minimum principal stresses) at failure. The Mohr-Coulomb criterion implementation in rock data can be used to analyze both direct shear and triaxial test data.

The direct shear formulation of the criterion is given by following equation :

$$\tau = c + \sigma \tan \varphi$$

The Mohr-Coulomb criterion for triaxial data is given by following equation :

$$\sigma_1 = \frac{2c \cos \varphi}{1 - \sin \varphi} + \frac{1 + \sin \varphi}{1 - \sin \varphi}\sigma_3$$

Where, c is the cohesive strength, and φ is the friction angle.

2.12 Shearing on an inclinded plane

The shearing of a sample in an inclined plane as shown in Figure 2.6 is explained below.

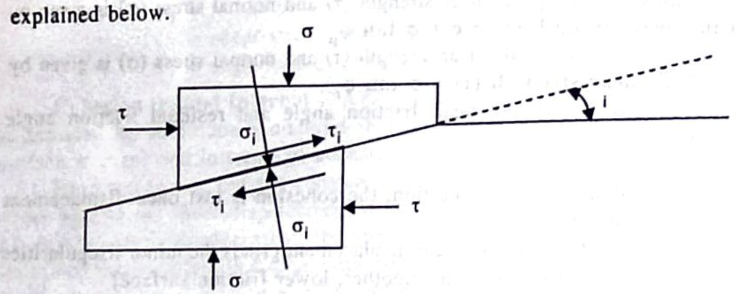


Figure 2.6 : Shearing in an Inclined Plane

In this case shear and normal stresses acting on the sliding surface, τi & σ_i respectively are given by

$$\tau_i = \tau \cos^2 i - \sigma \sin i \cos i$$

$$\sigma_i = \sigma \cos^2 i - \tau \sin i \cos i$$

For the discontinuity surface has zero cohesion, we can get

$$\tau_i = \sigma_i \tan \varphi$$

Putting the values of τ_i & σ_i in the above equation, we get

$$\tau = \sigma \tan (\varphi + i)$$

2.13 Generalised Hoek-Brown Strength Criterion

The generalised Hoek-Brown strength criterion is expressed in terms of the major and minor principal stresses as

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s\right)^a$$

Where, m_b is the reduced value of material constant m_i for intact rock and is given by

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right)$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right)$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right)$$

Where D is a factor that depends upon the degree of disturbance

[D = 0 for undisturbed in-situ rock masses and 1 for very disturbed rock masses]

GSI is geological strength index.

The uni-axial compressive strength of the rock mass is obtained by setting $\sigma'_3 = 0$ in generalized Hoek-Brown equation

$$\sigma_c = \sigma_{ci} * s^a$$

And, the tensile is obtained by setting $\sigma'_1 = \sigma'_3 = \sigma_t$ in generalized Hoek-Brown equation

$$\sigma_t = \frac{s * \sigma_{ci}}{m_b}$$

2.14 Determination of Factor of Safety of the Slope

2.14.1 Plane Failure

CASE I : When there is no tension crack and water pressure : The force diagram of the situation is shown in Figure 2.7.

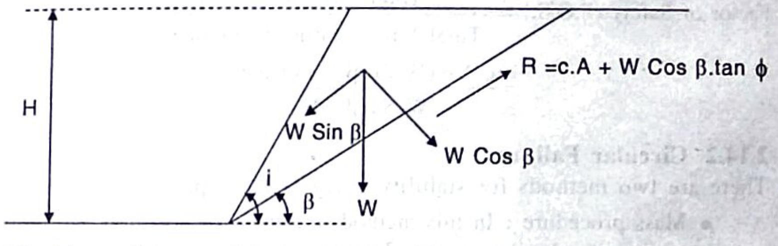


Figure 2.7 : Force diagram of a slope with no tension crack and water pressure

Assuming

Height of the bench = H

Overall slope angle from horizontal = i

Angle of discontinuity from horizontal = β
Weight of the block = W
Friction angle = ϕ
Cohesion = c
Area of the failure plane = A
Total force tending to induce sliding of the block = W.Sin β
Total force to resist sliding of the block = W.Cos β. tan ϕ + c.A

[Neglecting the water pressure]

$$\text{Factor of safety (F.O.S.)} = \frac{\text{Total force to resist sliding}}{\text{Total force tending to induce sliding}}$$

$$= \frac{c.A + W.Cos\,\beta.\tan\phi}{W.Sin\,\beta}$$

CASE II : When there is tension crack and water pressure present : The force diagram of the situation is shown in Figure 2.8.

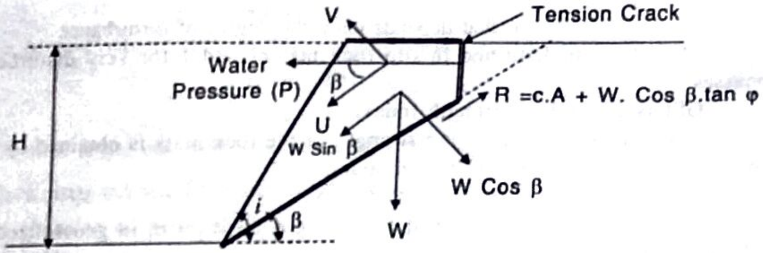


Figure 2.8 : Force diagram of a slope with the tension crack and water pressure

Water pressure along the sliding plane = U = P. Cos β
Water pressure normal to the sliding plane = V = P. Sin β

Total force tending to induce sliding of the block = W. Sin β + U
Total force to resist sliding of the block = (W. Cos β – V). tan φ + c. A

$$\text{Factor of Safety (F.O.S.)} = \frac{\text{Total force to resist sliding}}{\text{Total force tending to induce sliding}}$$

$$= \frac{c.A + (W.Cos\beta - V).\tan\varphi}{W.Sin\,\beta + U}$$

2.14.2 Circular Failure

There are two methods for stability analysis of slope :

- Mass procedure : In this method, the mass of soil above the surface of sliding is taken as a unit. This procedure is useful when the soil that forms the slope is assumed to be homogeneous, although this is not the case in most natural slopes.

- Method of slices : In this procedure, the soil above the surface of sliding is divided into a number of vertical parallel slices. The stability of each slice is calculated separately.

CASE I : When there is no tension crack and water pressure : The force diagram of the situation is shown in Figure 2.9.

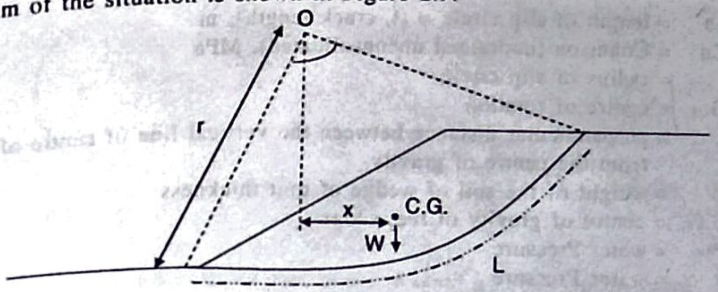


Figure 2.9 : Force diagram of a slope with no tension crack and water pressure

Considering the moment equilibrium about centre O for length of slip arc L, the restoring moment is C_u Lr and driving moment is Wx. Therefore, the factor of safety :

$$FOS = \frac{C_u L_r}{W.X}$$

Where,

L = length of slip arc, m
Cu = Cohesion (undrained unconsolidated), MPa
r = radius of slip circle
O = centre of rotation
W = weight of the soil of wedge of unit thickness
C.G. = centre of gravity of the wedge

CASE II : When there is tension crack filled with water: The force diagram of the situation is shown in Figure 2.10.

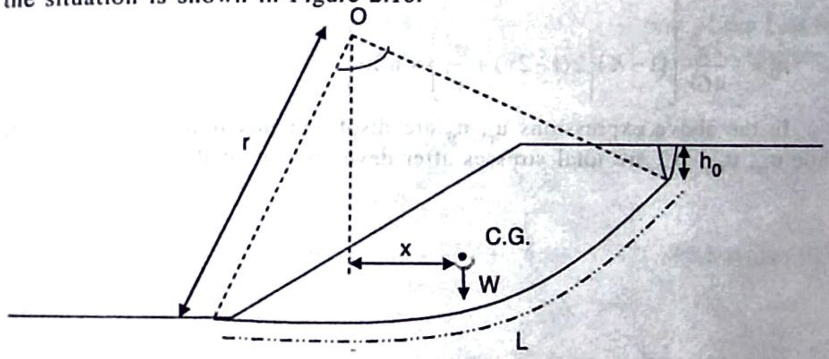


Figure 2.10 : Force diagram of a slope with the tension crack and water pressure

Under this condition, the factor of safety is calculated by the formula

$$\text{FOS} = \frac{C_u L_a r}{W_{w*x} + P_w \gamma}$$

Where,

La = length of slip circle = (L-crack length), m
Cu = Cohesion (undrained unconsolidated), MPa
r = radius of slip circle
O = centre of rotation
x = perpendicular distance between the vertical line of centre of circle from the centre of gravity
W = weight of the soil of wedge of unit thickness
C.G. = centre of gravity of the wedge
Pw = water Pressure
γ = water Pressure

2.15 Stress and Displacement Distribution in Circular Excavation

The stress distribution around a circular excavation as shown in Figure 2.9 can be determined by

$$\sigma_{rr} = \frac{P}{2}\left[(1+K)\left(1-\frac{a^2}{r^2}\right)-(1-K)\left(1-4\frac{a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_{\theta\theta} = \frac{P}{2}\left[(1+K)\left(1+\frac{a^2}{r^2}\right)+(1-K)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_{r\theta} = \frac{P}{2}\left[(1-K)\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta\right]$$

$$u_r = -\frac{Pa^2}{4Gr}\left[(1+K)-(1-K)\left\{2(1-2\nu)+\frac{a^2}{r^2}\right\}\cos 2\theta\right]$$

$$u_\theta = -\frac{Pa^2}{4Gr}\left[(1-K)\left\{2(1-2\nu)+\frac{a^2}{r^2}\right\}\sin 2\theta\right]$$

In the above expressions u_r, u_θ are displacements induced by excavation, while σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ are total stresses after development of the opening.

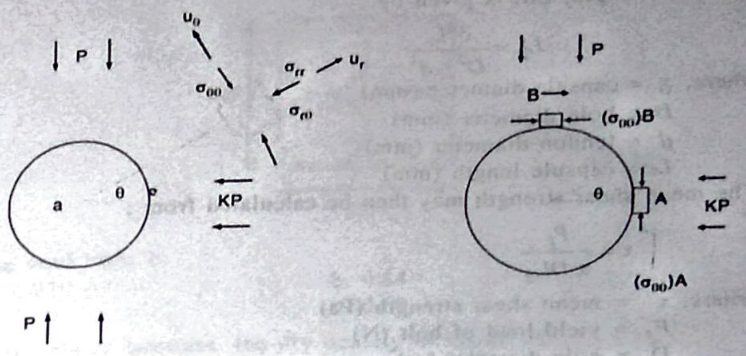


Figure 2.9 : Stress distribution around a circular excavation

By putting r = a in the above equations, the stresses on the excavation boundary are given as

$$\sigma_{rr} = 0$$

$$\sigma_{\theta\theta} = P[(1+K)+(1-K)\,2\cos 2\theta]$$

$$\sigma_{r\theta} = 0$$

Similarly, for $\theta = 0$, and r is very large, the values at point A

$$\sigma_{rr} = KP$$

$$\sigma_{\theta\theta} = P$$

$$\sigma_{r0} = 0$$

2.16 Determination of Length of the Roof Bolt

The optimal roof bolt length depends on the support mechanism. Where bolts are merely acting as skin control, they may be as short as 900 mm (Minney and Munsamy, 1998). In the suspension mode, bolts should obtain at least 300 mm of anchorage in the solid strata (Mark, 2000).

Several investigators have also studied the optimal length of the bolt that should be installed under various conditions. A summary of recommendations is given below.

Lang and Bischoff (1982) : $L = B^{2/3}$
Bieniawski (1987) : $L = B/3$
Unal (1984) : $L = \frac{B}{2}\left(\frac{100 - RMR}{100}\right)$

Where, *RMR* is the rock mass rating (Bieniawski, 1987) which ranges from 20 to 80 depending on roof rock conditions.

B is span (m) and *L* is bolt length (m).

2.17 Determination of mean shear strength using short anchor test

In these tests a short capsule (250 mm) is used, and the bolt is pulled to failure.

The bond length, *LB*, is given by :

$$L_B = \frac{\delta^2 L_c}{D^2 - d^2}$$

where, δ = capsule diameter (mm)
D = hole diameter (mm)
d = tendon diameter (mm)
Lc = capsule length (mm)

The mean shear strength may then be calculated from :

$$\tau = \frac{P_f}{\pi D L_B}$$

where, τ = mean shear strength (Pa)
P_f = yield load of bolt (N)
D = hole diameter (m)
Lc = capsule length (m)

Once τ has been determined by short encapsulated pull tests, the calculation may be reversed in order for the required capsule length to be found through substituting P_f for a design load. A suitable safety factor should also be used.

EXERCISE

1. A sandstone specimen of diameter 54 mm and height 60 mm has moisture content of 20 %, and specific gravity 2.65. The porosity of the sandstone is
a. 0.346 b. 1.53
c. 1.732 d. 2.887

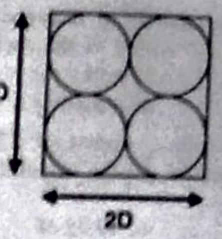
2. Porosity of a coarse grain sandstone sample is 15 %. The sp. gr. of sandstone is 2.8. What is the void ratio in the sandstone sample?
a. 0.150 b. 0.176
c. 0.850 d. 1.176

3. Porosity of a coarse grain sandstone sample is 15 %. The sp. gr. of sandstone is 2.8. If the sandstone is fully saturated in water, the saturated density of the sample in kg/m³ is
a. 1590 b. 2234
c. 2438 d. 2531

4. A dry rock sample of diameter 50 mm and length 100 mm weighs 300 g. After saturating in brine solution of specific gravity 1.05, its weight increased to 330g. The porosity of the rock sample in percentage is
a. 10.45 b. 14.55
c. 12.55 d. 16.55
[GATE 2014]

5. A phreatic surface experiences a pressure
a. Less than atmospheric pressure b. Equal to atmospheric pressure
c. More than barometric pressure d. Less than barometric pressure

6. The granular media in an ore bin assumed to be of regular spherical shape represented by the geometry as shown in the figure. The unit weight of solids is 25 kN/m³.



The void ratio is
a. 0.91 b. 0.84
c. 0.78 d. 0.69
[GATE 2010]

7. In the above question, the dry density in kN/m³ is
a. 13.09 b. 12.50
c. 11.74 d. 10.87
[GATE 2010]

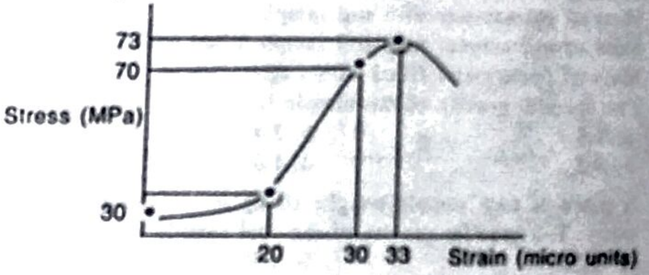
8. The grain size distribution of soil is known as
a. Permeability b. Structure
c. Porosity d. Texture
[GATE 2008]

9. Dimension of hydraulic permeability expressed with L, M and T is
a. L^2 b. LT^{-1}
c. LT^{-2} d. MLT^{-1}

10. Dimension of modulus of elasticity (E) expressed with L, M and T is
a. $M L^{-1} T^{-2}$ b. $M L^{-1} T^{-1}$
c. $M L T^{-1}$ d. $M L T^{-2}$

11. The average Young's modulus and Poisson ratio values of a limestone sample are 60×10^3 MPa and 0.3 respectively. The shear modulus in MPa is
a. 23.07 b. 230.7
c. 2307.0 d. 2307.0
[GATE 2010]

12. From the stress-strain diagram shown below, the tangent and the secant moduli of elasticity in GPa are



a. 4.0, 2.2 b. 3.3, 2.3
c. 3.3, 1.5 d. 4.0, 1.5
[GATE 2008]

13. Increase in moisture content of rock, the effect on bearing capacity of the rock is
a. Increases b. Decreases
c. No relation with the moisture content d. Depends on the rock type

14. Given k is the thermal conductivity, ρ is density and c is specific heat of a rock sample, the thermal diffusivity of the rock sample is
a. $k\rho/c$ b. $\rho c/k$
c. kc/ρ d. $k/\rho c$ [GATE, 2014]

15. A rock specimen had moisture content of 2.5 %. Volume of solids was found to be 75 % and specific gravity of solid is 2.5. Degree of saturation of the rock sample is
a. 1.975 % b. 19.75 %
c. 0.1975 % d. 30 %

16. The grain density and bulk density of a dry coarse gained sandstone rock sample are 3.0 gm/cc and 2.7 gm/cc respectively. The void ratio of the sample in percentage is
a. 8.4 b. 10.0
c. 11.1 d. 30.5

17. During a laboratory permeability experiment on mill tailings 20 ml of water percolated in 30 min through tailings of 5 cm diameter, and 20 cm length. If the tailing column is subjected to a steady hydraulic gradient of 1.2 m during the experiment, what is the hydraulic conductivity of the tailings?
a. 94 Darcys b. 172 Darcys
c. 0.94 μm/ s d. 17.2 m^3/ s [GATE, 2000]

18. A confined aquifer of 75 m thickness has 2 monitoring wells spaced 2500 m apart along the direction of water flow. The hydraulic conductivity of the aquifer is 40 m per day. The water head difference between the wells is 1.5 m. Applying the Darcy's law, the rate of flow per meter of distance perpendicular to the direction of flow in m^3/day is
a. 2.1 b. 1.8
c. 1.45 d. 1.21 [GATE 2008]

19. In an experiment to determine specific gravity of a soil sample, the following data is obtained :

Mass of empty pycnometer	20.4 g
Mass of pycnometer with soil sample	51.6 g
Mass of pycnometer with soil sample filled with water	88.6 g
Mass of pycnometer filled with water	70.4 g

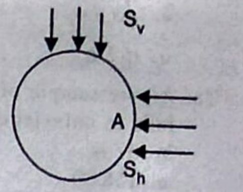
The specific gravity of the sample is
a. 1.8 b. 2.4
c. 3.2 d. 1.6 [GATE 2013]

20. A piece of coal sample weighs 10 kg in air and 2 kg when immersed in water. The specific gravity of the coal sample is
a. 1.25 b. 1.50
c. 2.25 d. 1.00 [GATE 2014]

21. Stress concentration at a point on the wall of a vertical shaft results in a compressive stress of 59.66 MPa. The wall rock mass has an unconfined compressive strength of 89.49 MPa. The safety factor of the shaft wall at the point is

a. 0.67 b. 0.86
c. 1.23 d. 1.50 [GATE 2011]

22. A circular opening in a biaxial stress field is as shown, with S_v and S_h being the vertical and horizontal compressive stresses respectively. The point A is under tensile stress when
a. $S_h = 0$ b. $S_h = S_v/3$
c. $S_h = S_v$ d. $S_h = 3\ S_v$



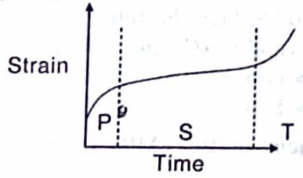
[GATE, 2000]

23. In a triaxial compression test if σ_1 is axial stress and σ_2 and σ_3 are confining stesses, then
a. $\sigma_3 > \sigma_2 = \sigma_1$ b. $\sigma_1 > \sigma_2 = \sigma_3$
c. $\sigma_1 = \sigma_2 > \sigma_3$ d. $\sigma_1 < \sigma_2 = \sigma_3$

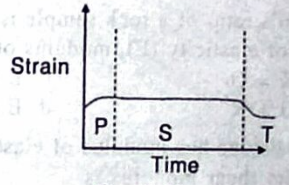
24. The ratio of uniaxial compressive strength to uniaxial tensile strength of a sandstone specimen is 8:1. The theoretical value of angle of internal friction of the specimen in degree is
a. 51 b. 41
c. 32 d. 7

25. A circular tunnel is made underground where far field vertical and horizontal stresses are P_0 and KP_0 respectively. The tangential stress ($\sigma_{\theta\theta}$) at the boundary of the tunnel for $\theta = 45^0$ from the horizontal plane is $3P_0$. The value of K is
a. 0 b. 1
c. 2 d. 3

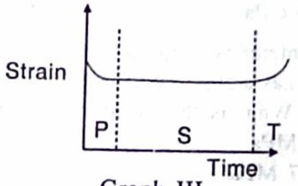
26. Which one of the following graph typically represents the standard strain-time creep behaviour of an isotropic rock material under constant temperature? P, S and T in the figures refer to primary creep, secondary creep and tertiary creep respectively.



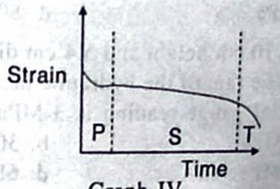
Graph I



Graph II



Graph III



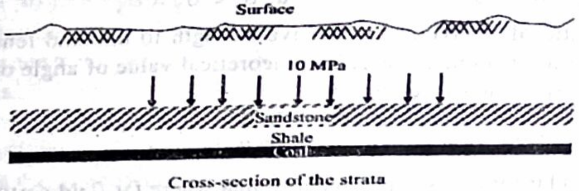
Graph IV

a. Graph I b. Graph II
c. Graph III d. Graph IV [GATE 2009]

27. If σ_s is the induced stress and σ_i is the in-situ stress at a point below ground, the 'stress concentration' at that point is
a. $\sqrt{\sigma_s/\sigma_i}$ b. $\sqrt{\sigma_i/\sigma_s}$
c. σ_i/σ_s d. σ_s/σ_i [GATE, 2014]

28. A core sample of 54 mm diameter having Young's modulus of 68.97 GPa fails in uniaxial compression at 0.1% axial strain. The axial load at failure in kN is
a. 158.00 b. 68.97
c. 58.00 d. 15.80 [GATE, 2011]

29. A typical case of gravity loading under complete lateral restraint in flat strata is shown in the figure below. The physico-mechanical parameters of the strata are given in the table. The in-situe stresses (σ_Z, σ_H) on the top of the coal seam in MPa are



Cross-section of the strata

Strata	Thickness (m)	Specific Gravity	Young's Modulus (GPa)	Sher Modulus (GPa)
Sandstone	50	2.35	26.4	12.5
Shale	25	2.15	20.50	8.25
Coal	20	1.52	2.41	0.95

a. (10.17, 2.54) b. (10.17, 3.69)
c. (11.68, 3.69) d. (11.68, 2.54) [GATE, 2011]

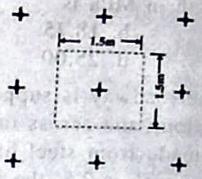
30. If Poisson's ratio of a rock sample is 0.25, then the relationship among the modulus of elasticity (E), modulus of rigidity (G) and bulk modulus (K) is
a. E = K = G b. E > G > K
c. E = G > K d. E > K > G [GATE 2012]

31. Weak sandstone has modulus of elasticity of 1000 MPa and Poisson's ratio is 0.25. Its shear modulus is
a. 4 GPa b. 400 GPa
c. 500 GPa d. 800 GPa [GATE, 2000]

32. A core of 10 cm height and 5.4 cm diameter is subjected to diametricalpoint loading. The ram of the hydraulic jack has a cross sectional area of 12.5 cm^2 and the dial gauge reading is 3 MPa. What is the point load strength?
a. 1.63 MPa b. 30 MPa
c. 37.5 MPa d. 68.7 MPa

33. A resin grouted rock bolt is able to provide yield strength of 200 MPa. If the bolt diameter is 2.5 cm and the grout length 154 cm, what is the adherence (shear strength) between the bolt and the grout?
a. 800 kg/ cm^2 b. 800 kPa
c. 80 MN d. 8000 N/m

34. The bolts are spaced at 1.5m centre-to-centre in a square pattern as shown in the figure below. The tensile stress in 22mm diameter bolt rod is 193.35 MPa. The unit weight of the roof layer is 25 kN/m^3. The axial load in the bolt rod in kN is



a. 294.0 b. 173.5
c. 147.0 d. 73.5 [GATE, 2011]

35. In the above question, at equilibrium, the thickness of the roof layer supported by the bolt in m is
a. 1.31 b. 2.4
c. 2.62 d. 3.08 [GATE, 2011]

36. A tension crack of 5 m depth behind a rock slope crest is completely filled with water. What is the horizontal force exerted on a unit slope length by the water in the tension crack?
a. 122.6 kN b. 245 kN
c. 12.26 MPa d. 24.5 Nm

37. The shear stress on a plane of major principal stress acts is
a. half the major principle stress
b. twice the major principle stress
c. shear stress is absent
d. three times the major principle stress

38. Following is the state of stress in a biaxial stress field :
σ_{XX} = 100 MPa σ_{YY} = 30 Mpa
σ_{XY} = 15 MPa
The major principal stress is equal to
a. 103 MPa b. 130 MPa
c. 145 MPa d. 165 MPa

39. A cylindrical rock specimen is uniaxially loaded under compression and fails at 50 MPa. The fracture plane is inclined at an angle of 45^0 with the axial direction. The normal and shear stresses respecively on the failure plane in MPa are
a. 50, 50 b. 0, 50
c. 50, 0 d. 25, 25 [GATE, 2012]

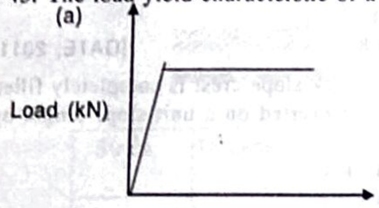
40. The components of state of stress at a point in x-y plane are given as σ_{xx} = 5 MPa, σ_{yy} = 10MPA and τ_{xy} = 2 MPa. The sum of the principal stresses acting on the x-y plane in MPa is
a. 10.7 b. 11.7
c. 12.7 d. 13.7 [GATE 2014]

41. A series of triaxial tests of sandstone samples reveal the cohesion and the angle of internal friction as 21.65 MPa and 30?respectively. Based on the assumption that the sandstone samples follow the Mohr-Coulomb's failure criteria, the tensile strength in MPa is
a. 12.50 b. 18.75
c. 21.65 d. 25.00 [GATE 2012]

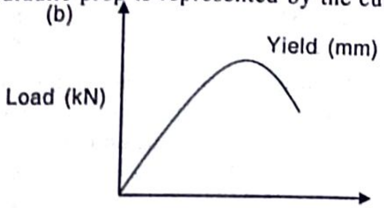
42. Immediate roof of a mine roadway is supported by 1.5 m long roof bolts placed at 1.2 m centers along and across the roadway. The bolts of 4 cm^2 cross sectional area are made from steel having tensile strength 300 MN/m^2. Taking sp. gr. of roof rock is 2.5, the factor of safety of the roof bolt is
a. 1.86 b. 2.26
c. 2.98 d. 4.34

43. The load yield characterstic of a hydraulic prop is represented by the curve

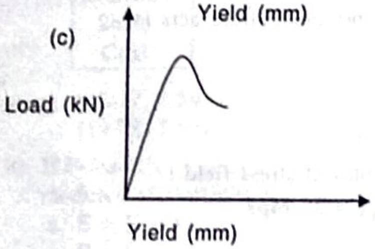
(a)



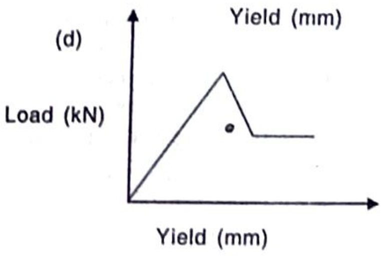
(b)



(c)



(d)



[GATE 2008]

44. The rock specimen obtained from the overlying strata in a coal mine were tested in uni-axial compression and the following results were obtained at 50 % of average compressive strength
Longitudinal strain = 0.06 and Lateral strain = 0.012
The ratio of vertical to horizontal stress in the mine is
a. 3.0 b. 4.0
c. 2.5 d. 0.3

45. A cylinder specimen of sandstone of 50 mm diameter and 125 mm length, tested in uni-axial compression gave the following results :

Load at failure = 100 kN and longitudinal deformation upto failure = 1.5 mm. Assuming load-deformation curve to be straight line up to failure, find the uni-axial compressive strength and the modulus of elasticity of the sandstone.
a. 12.72 MPa; $1.09*10^9$ Pa b.15.72 MPa; $1.8*10^6$ Pa
c. 1.72 MPa; $1.9*10^9$ Pa d.12.72 MPa; $1.09*10^{10}$ Pa

46. A rock specimen was subjected to diametrical point load testing. The pressure gauge reading at failure is 12.5 MPa. If the ram area is 13.36 cm^2, and the diameter of the core tested is 54 mm, the point load strength index is
a. 5.73 MPa b. 5.21 MPa
c. 4.85 MPa d. 5.46 MPa

47. For a circular opening in rock under hydrostatic condition at great depth, the maximum tangential tensile boundary stress will occur at
a. 0° with vertical b. 45° with vertical
c. 90° with vertical d. no where along the boundary

48. For a typical hard rock with E = 10^5 MPa and Poisson's ratio (υ) = 0.25, the modulus of rigidity in GPa is
a. 40 b. 80 c. 66.67 d. 133.33

49. The length of consecutive cores obtained from 2m section of a borehole (NX size) are 89, 32, 18, 172, 80, 255, 280, 176, 53, 302, 102, 153, 40, 147 and 101 mm. Then the RQD value of rock in that section is
a. 66.9 % b. 84.4 % c. 95.5 % d. 100 %

50. In a borehold log of 1.2m in length, recovery of rock cores in cm is given below
20, 8, 15, 8, 8, 4, 3, 9, 10, 1, 5, 10
The RQD in percentage is
a. 29.2 b. 31.8
c. 45.8 d. 50.0 [GATE 2014]

51. The corrected value (C_o) of the uni-axial compressive strength for the specimen size not in 1:1 is given by the relation where C_p is observed uni-axial compressive strength

a. $C_0 = \frac{C_p}{0.778 + 0.222 * (D/L)}$ b. $C_0 = \frac{C_p}{0.222 + 0.778 * (D/L)}$

c. $C_0 = \frac{C_p}{0.778 + 0.222 * (L/D)}$ d. $C_0 = \frac{C_p}{0.222 + 0.778 * (L/LD)}$

52. The rock mass classification system that considers 'active stress' factor is
a. Q-system b. RMR
c. RQD d. GSI

53. Barton classified the rock mass on the basis of
a. RMR b. Q-system
c. Slake durability d. RQD

54. Hansagi classified the rock mass on the basis of
a. C-factor b. RMR
c. Q-system d. RQD

55. The term RQD/ Jn expressed in Q system refers to
a. Sizes of joint block
b. Shear strength of block surfaces
c. Environmental conditions influencing the behaviour of the rock mass
d. Roughness of joint surfaces
[GATE, 2006]

56. The instrument Flat jack is used for measuring the
a. In-situ stress in rock b. Load
c. Bed separation resistance d. Roof convergence

57. The instrument, Tape extensometer is used for measuring the
a. In-situ stress in rock b. Load
c. Bed separation resistance d. Roof convergence

58. Match the following :

Failure criteria	Relation
P. Drucker-Prager	1. $\sigma_1 = \sigma_3 + \sqrt{m\sigma_3 + s^2}$
Q. Hoek-Brown	2. $\tau = c + \sigma_n \tan\varphi$
R. Hoek-Brown	3. $\sqrt{2/3}\left[\left(\frac{\sigma_1-\sigma_2}{2}\right)^2+\left(\frac{\sigma_2-\sigma_3}{2}\right)+\left(\frac{\sigma_3-\sigma_1}{2}\right)^2\right]^{1/2}$

a. P-1, Q-3, R-2 b. P-3, Q-1, R-2
c. P-1, Q-2, R-3 d. P-3, Q-2, R-1

59. In a point load test on 50 mm diameter core specimen, rupture was observed at a load of 5000 kg. The point load strength of the specimen was
a. 300 kg/ cm^2 b. 200 kg/ cm^2
c. 400 kg/ cm^2 d. 500 kg/ cm^2

60. In a point load test on 50 mm diameter core specimen, rupture was observed at a load of 5000 kg. The uni-axial compressive strength of the specimen was
a. 3800 kg/ cm^2 b. 4000 kg/ cm^2
c. 4800 kg/ cm^2 d. 5000 kg/ cm^2

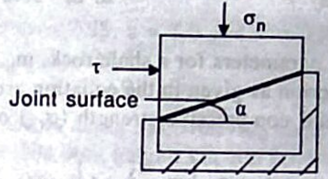
61. During Barzilian test, a rock specimen of diameter 5 cm, and height 5 cm, failed at a compression load of 19.6 kN. The indirect tensile strength of the specimen is
a. 0.5 kPa b. 2.5 MPa
c. 5 MPa d. 25 kPa

62. During Barzilian test of a cylindrical rock core of 54 mm diameter and 22 mm thickness, failure occurred at diametrical loading of 11.4 kN. The tensile strength of rock in MPa will be
a. 1.24 b. 2.49
c. 3.05 d. 6.10
[GATE, 2004]

63. For a bulking factor of 1.5, the height of the immediate roof for an extraction thickness of H is
a. H b. 1.5 H
c. 2 H d. 2.5 H

64. The specific gravity of gypsum mineral is 2.4. If the density of a gypsum core is 2100 kg/m^3, the porosity of that core is
a. 0.062 b. 0.125
c. 0.5 d. 0.875
[GATE, 2006]

65. In the direct shear test, the joint surface is inclined α° with the direction of the applied shear load. The bottom half of the specimen is fixed inside the shear box. The friction angle of the joint surface is ϕ°. The shear strength envelope for the experimental set up can be expressed by
a. $\tau = \sigma_n \tan(\phi + \alpha)$
b. $\tau = \sigma_n \tan(\phi - \alpha)$
c. $\tau = \sigma_n \tan\phi$
d. $\tau = \sigma_n \tan\alpha$



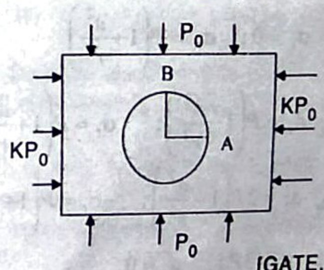
[GATE, 2006]

66. The cohesion and the angle of internal friction of a sandstone rock are 6 MPa and 35° respectively. The magnitudes of theoretical unconfined compressive and tensile strength of sandstone in MPa are
a. 6.24, 0.66 b. 11.53, 3.12
c. 16.14, 4.37 d. 23.05, 6.24
[GATE, 2006]

67. The classification of rock mass on the basis of RQD is given by
a. Gamble b. Deere
c. Bieniawski d. Barton

68. A rock specimen of average thickness 0.8 cm failed at an applied load of 1000 kg during determination of shear strength by the punch test. The diameter of the punch is 2.8 cm. The shear strength of the rock specimen in kg/ cm^2 is
a. 50.75 b. 142.10
c. 284.20 d. 446.43
[GATE, 2006]

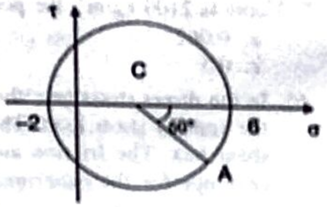
69. A circular tunnel is opened in sandstone rock with far-field stresses of P_O and KP_O as shown in the figure. If the tangential stress at point A is 3 times that of the point B (both points are located on the boundary of the tunnel), the value of K is
a. 0.3
b. 0.6
c. 1.0
d. 3.0



[GATE, 2006]

70. In a bi-axial stress field the vertical stress is 10 MPa and the Poisson ratio for the rock mass is 0.2. The horizontal stress in MPa is,
a. 1.5 b. 2.5
c. 2.0 d. 5.0 [GATE, 2008]

71. The major and minor principal stresses of a rock specimen are found to be 6 MPa and − 2 MPa respectively. They are plotted in a Mohr circle as shown in the figure. The centre of the Mohr circle is denoted by C. The normal and shear stresses in MPa acting on the plane A are



a. 2, −2.0 b. 2, −3.46
c. 2, −6.93 d. 2, −3.46
[GATE, 2006]

72. Rock mass parameters for a shale rock, m_b, s, and a of the Hoek and Brown failure criterion as given in the equation are 2.4, 0.01 and 0.52 respectively. The uni-axial compressive strength (σ_{CI}) of intact shale is 50 MPa.

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b \frac{\sigma_3}{\sigma_{ci}} + s\right)^a$$

The global rock mass strength of shale in MPa is
a. 1.2 b. 10.29
c. 20.83 d. 50.00

73. Rock mass parameters for a shale rock, m_b, s, and a of the Hoek and Brown failure criterion as given in the equation are 2.4, 0.01 and 0.52 respectively. The uni-axial compressive strength (σ_{CI}) of intact shale is 50 MPa.

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b \frac{\sigma_3}{\sigma_{ci}} + s\right)^a$$

The uniaxial compressive strength of rock mass is
a. 1.2 b. 4.59
c. 20.83 d. 50.00

74. For a circular opening of radius a, where the far-field stress value is P. The radial and tangential stresses at a distance r from the center of the opening are

a. $\sigma_r = 0;\ \sigma_t = P\left(1 + \frac{a^2}{r^2}\right)$

b. $\sigma_r = P\left(1 - \frac{a^2}{r^2}\right);\quad \sigma_t = P\left(1 + \frac{a^2}{r^2}\right)$

c. $\sigma_r = 2P\left(1 - \frac{a^2}{r^2}\right);\quad \sigma_t = P\left(1 + \frac{a^2}{r^2}\right)$

d. $\sigma_r = 3P;\quad \sigma_t = 0$ [GATE, 2001]

75. The ratio of horizontal to vertical in-situ stresses, K, at a mine field varies with depth, D (in m) as

$$K = \frac{267}{D} + 1.25$$

If the unit weight of overburden rock is 25 kN/m^3, determine the horizontal stress in MPa at a depth of 400m.
a. 19.175 b. 9.175
c. 11.75 d. 13.75 [GATE, 2015]

76. The uniaxial compressive strength of a limestone sample is 80 MPa. The sample is confined at a pressure of 20 MPa in a triaxial compressive strength test. Based on Hoek-Brown failure criteria the maximum principal stress at failure in MPa is
(Consider rock constants as m = 7.88, s = 1.0 and a = 0.5)
a. 117.9 b. 132.3
c. 137.9 d. 157.9 [GATE 2013]

77. A room and pillar stope at 100 m depth has overlying rock density of 2800 kg/m^3. For a square pillar of side 4 m, height 3 m and for room width of 2.4 m, the mean pillar stress in MPa is
a. 3.2 b. 3.8
c. 19.6 d. 7.0 [GATE, 2001]

78. A bord and pillar operation is planned at a depth of 300 m in a strata of average unit weight 24.5 kN/m^3 and compressive strength 15.5 MPa. If the width of the opening is 6 m considering a factor of safety of 1, the maximum possible extraction ratio in percentage is
a. 28 b. 34
c. 45 d. 53 [GATE 2008]

79. For a cylindrical specimen of cross sectional area A, length L, Young's modulus E, and axial load P, the strain energy stored is
a. $P^2L/2AE$ b. $PL^2/2AE$
c. P^2L^2/AE d. P^2L/AE [GATE, 2001]

80. Correlation between the rock mass classification systems of RMR and Q is
a. RMR = 9 $\log_e$ Q + 44 b. Q = 8 $\log_e$ RMR + 34
c. RMR = 9 $\log_{10}$ Q + 44 d. Q = $\log_e$ RMR + 44
[GATE, 2001]

81. In a compression strength test, the increase in confining pressure leads to a reduction in the rock
a. ductility b. angle of internal friction
c. brittleness d. ultimate bearing load
[GATE, 2001]

82. The ratio of specific weight of an intact rock to that of loose rock is
a. fill factor b. loose density
c. swell factor d. compaction factor
[GATE, 2001]

83. For a wooden prop of modulus of elasticity 60 MPa, critical bearing strength 25 MPa, length 2.5 m and diameter 0.16 m, the slenderness ratio is

a. 15.625	b. 0.064
c. 2.4	d. 0.416

84. During uniaxial compressive testing of a rock specimen of diameter 54 mm and length 120 mm, the axial strain was found to be 6.5 X 10^{-4} mm/mm. If the Poisson's ratio of the rock is 0.3, the volumetric strain of the rock specimen in mm/mm is

a. 1.95 * 10^{-4}	b. 2.60 * 10^{-4}
c. 2.34 * 10^{-2}	d. 3.12 * 10^{-2}

85. Dilatancy of rock is associated with
a. Increase in surface area after fragmentation
b. Decrease in shear strain due to cracking of rock
c. Increase in shear strain due to cracking of rock
d. Increase in volume due to cracking of rock [GATE, 2015]

86. An immediate roof of RMR 45 will be supported by roof bolts. If the width of the gallery is 4.6 m, the minimum length of the bolt in m will be

a. 1.035	b. 1.265
c. 2.070	d. 2.530

[GATE, 2004]

87. The correct order of hardness of rock in their increasing order
a. Gypsum < Flourite < Quartz < Topaz
b. Flourite < Gypsum < Quartz < Topaz
c. Flourite < Quartz < Gypsum< Topaz
d. Topaz < Quartz < Flourite < Gypsum

88. A rock specimen of 120 mm length and 54 mm diameter is subjected to uniaxial compressive loading. Two strain gauges, one axially and other radially, are attached to the specimen. At the breaking load, if these two gauges read 0.0082 m/m and 0.0023 m/m respectively, the Poisson's ratio of the specimen is

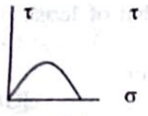
a. 3.56	b. 0.25
c. 0.28	d. 0.32

[GATE, 2002]

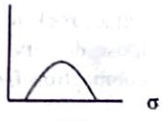
89. A rock specimen was tested in compression. The lateral and longitudinal strains were found 0.6 % and 3 % respectively. The Poisson's ratio of the specimen is
a. 0.2 b. 0.3 c. 0.4 d. 0.18

90. From a core drilling of shale, the average number of fractures per meter of core length is found to be 20. The RQD (%) of the rock is
a. 60 b. 37 c. 49 d. 53

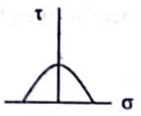
91. Identify the 'Pure shear' state of stress condition from the following four Mohr circle diagrams



(1)



(2)



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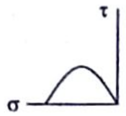


(4)

a. 3	b. 2
c. 1	d. 4

92. From a series of tri-axial tests conducted on some sandstone specimens, the relationship between the major and the minor principal stresses in MPa is found to be $\sigma_1 = 100 + 5\,\sigma_3$. The angle of internal friction in degrees of that sandstone is

a. 38	b. 79
c. 42	d. 32

[GATE, 2002]

93. From a series of tri-axial tests conducted on some sandstone specimens, the relationship between the major and the minor principal stresses in MPa is found to be $\sigma_1 = 100 + 5\,\sigma_3$. The coefficient of cohesion of that sandstone is

a. 5.3	b. 10.5
c. 22.3	d. 18.5

94. A sample of 3 cm thickness and 8 cm diameter is subjecte to punch shear test. The shear failure of the sample occurs at 200 kg load. What is the shear strength of the sample?

a. 2.65 kg/cm^2	b. 3.55 kg/cm^2
c. 8.21 kg/cm^2	d. 12.65 kg/cm^2

95. The values of normal and shear stresses are $\sigma_x = 7.99$ MPa, $\sigma_y = 6.90$ MPa, and $\tau_{xy} = 0.07$ MPa. The values of maximum and minimum principal stresses in MPa are

a. 6.90, 9.00	b. 7.50, 1.09
c. 8.00, 6.90	d. 8.50, 1.55

[GATE, 2005]

96. The measurements of a site investigation are as follows :
Jn = 9, Jr = 1.5, Ja = 2.0, Jw = 1.0, SRF = 2.5. The RQD values taken from five boreholes of the area are 94.2, 95.6, 99.4, 95.3 and 98.0 respectively. The rock mass quality index (Q) is

a. 3.20	b. 5.60
c. 10.14	d. 20.10

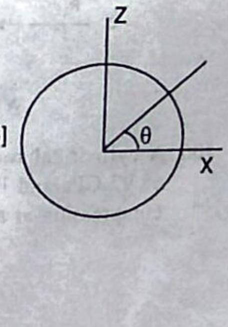
[GATE, 2005]

97. The Poisson's ratio (μ) of a rock specimen is 0.27. If the compressive and shear wave velocities are denoted by V_P and V_S, the ratio V_p^2/ V_S^2 is

a. 0.315	b. 3.174
c. 1.781	d. 10.07

[GATE, 2005]

98. A 3 m diameter circular tunnel at 450 m depth is driven in rock of unit weight (λ_w) is 6 kN/ m^3, uniaxial compressive strength (σ_c) is 25 MPa and uniaxial tensile strength (σ_t) is 3.0 MPa. The state of stress condition for K = 0.3 is
a. $\sigma_\theta = \sigma_c$, failure in compression at θ = 90°
b. $\sigma_\theta = \sigma_t$, failure in tension at θ = 90°
c. $\sigma_\theta = \sigma_t$, failure in tension at θ = 0°
d. $\sigma_\theta = \sigma_c$, failure in compression at θ = 0°
[GATE, 2005]



99. The number of experimental set ups of flat jack to determine 3 dimensional state of stress in a rock mass is

a. 6 b. 3
c. 2 d. 1 [GATE, 2005]

100. The uni-axial compressive strength and modulus of elasticity of a rock sample are 90 MPa and 30 GPa respectively. The stored strain energy per unit volume in kN.m/ m^3 of the sample at the time of failure is
a. 1.5 b. 140
c. 270 d. 135

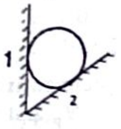
101. What is the seismic S-wave velocity in the near surface earth if the density of the earth surface rock is 2500 $kg/m^3$and the shear modulus is 10^{10} Pa?
a. $2*10^3$ b. $5*10^4$
c. $25*10^4$ d. $5*10^{-4}$

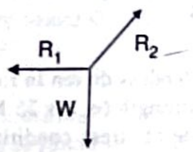
102. What is the seismic P-wave velocity in the near surface earth if the density of the earth surface rock is 2500 kg/m^3, the shear modulus = 10^{10} Pa, and the Poisson's ratio is 0.25?
a. $3.46*10^3$ b. $5.26*10^3$
c. $5.00*10^4$ d. $8.45*10^3$

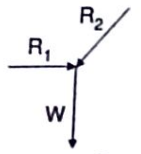
103. The velocity of the wave type that determines the 'rippability' of rockmass is
a. P wave b. S wave
c. Raleigh wave d. Love wave [GATE 2008]

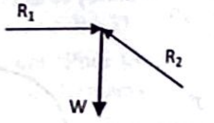
104. In the Mohr's scale of hardness, the minerals in increasing sequence of hardness are
a. calcite, gypsum, topaz, diamond b. topaz, gypsum, calcite, diamond
c. calcite, gypsum, diamond, topaz d. gypsum, calcite, topaz, diamond [GATE 2013]

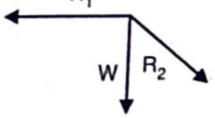
105. A ball of weight W is supported on smooth walls as shown in the following figure. R_1 and R_2 are reactions from the walls 1 and 2. The free body diagram of the ball is represented by



a.


b.


c.


d.


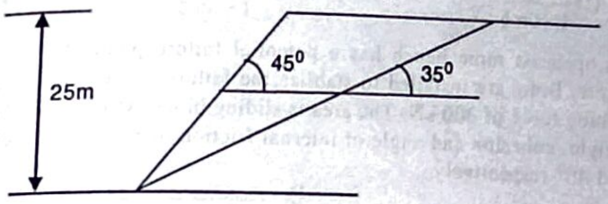
106. A cylindrical rock specimen of diameter 54 mm has Young's modulus of 68.97 GPa and Poisson's ratio of 0.35. The rock specimen fails in uniaxial compression at a lateral strain of 0.01%. The axial load at failure in kN is

a. 45.15 b. 40.23
c. 35.26 d. 55.20 [GATE 2013]

107. A core sample of a rock, having diameter 54 mm and length 108 mm, is subjected to axial loading. If the axial strain and Poisson's ratio are 2000 x 10^{-6} and 0.28 respectively, the value of volumetric strain , represented in micro-strain is
a. 860 b. 870
c. 880 d. 890 [GATE 2013]

108. The pressure on a phreatic surface is
a. less than atmospheric pressure
b. greater than atmospheric pressure
c. equal to atmospheric pressure
d. independent of atmospheric pressure [GATE 2013]

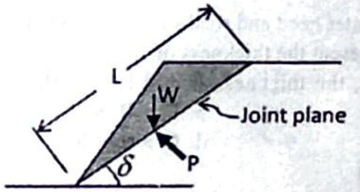
109. An open pit mine bench has a potential failure plane as indicated below. The unit weight, cohesion, and angle of internal friction of the rock mass are 24.5 kN/m^3, 0.02 MPa and 30° respectively.



The force for failure, on the potential failure plane is
a. 187 N b. 1.87 kN
c. 18.7 kN d. 1.87 MN [GATE 2008]

110. In the above problem, the factor of safety of slope is
a. 0.7 b. 0.9
c. 1.1 d. 1.3 [GATE 2008]

111. A joint plane of length L and dip δ intersectsthe toe of a slope as shown in the figure. The weight of the shaded block is W. Uniform water pressure P acts normal to the joint plane. If the cohesion and angle of internal friction of the joint surface are c and ϕ respectively, then the expression for 'safety factor' of the shaded block is



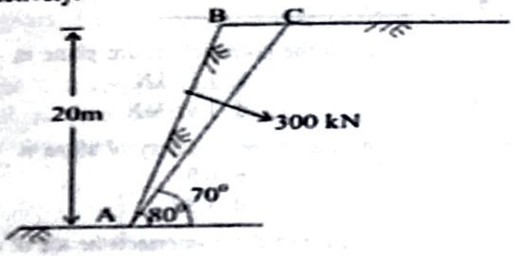
a. $\dfrac{Lc+(W \sin\delta - LP)\tan\varphi}{W\cos\delta}$ b. $\dfrac{Lc+(W\cos\delta + LP)\tan\varphi}{W\sin\delta}$

c. $\frac{Lc+(W\cos\delta-LP)\tan\varphi}{W\sin\delta}$ d. $\frac{Lc+(W\sin\delta+LP)\tan\varphi}{W\cos\delta}$ [GATE 2014]

112. A series of tri-axial compression tests conducted on sandstone samples reveal the following relationship between major and minor principal stresses
$\sigma_1 = 50 + 3\sigma_3$ [stresses are in MPa]
The cohesion in MPa and angle of internal friction in degrees of sandstone respectively are
a. 14.43, 30.0 b. 14.43, 60.0
c. 0.21, 73.9 d. 0.21, 16.1 [GATE 2014]

113. In a Mohr-Columb failure criterion, the ratio of the uniaxial compressive strength to the tensile strength is
a. $\frac{1+\sin\phi}{1-\sin\phi}$ b. $\frac{1-\sin\phi}{1+\sin\phi}$
c. $C^*\frac{1+\sin\phi}{1-\sin\phi}$ d. $2C^*\frac{1+\sin\phi}{1-\sin\phi}$ [GATE 2010]

114. An opencast mine bench has a potential failure plane AC as indicated in figure. Bolts are installed to stablize the failure plane providing a resultant bolting force of 300 kN. The area is sliding block ABC is 37.45m^2. The unit weight, cohesion and angle of internal friction of rock are 25 kN/m^3, 20kPa and 40^0 respectively.



The factor of safety of slope when bolts are installed perpendicular to the failure plane is
a. 0.79 b. 1.08
c. 1.78 d. 3.46 [GATE 2012]

115. Under identical water head and roadway conditions for water dam construction, if P, Q and R represent the thickness of flat dam, cylindrical dam and spherical dam respectively, the thickness would follow the order.
a. R > P > Q b. P > R > Q
c. P > Q > R d. Q > P > R [GATE 2008]

Answer

1. a 2. b 3. d 4. b 5. b 6. a 7. a
8. d 9. b 10. a 11. d 12. a 13. b 14. d
15. b 16. c 17. c 18. b 19. b 20. a 21. d

22. d 23. b 24. a 25. c 26. a 27. d 28. a
29. c 30. d 31. b 32. b 33. a 34. d 35. a
36. a 37. c 38. a 39. d 40. a 41. d 42. b
43. a 44. b 45. a 46. a 47. d 48. a 49. b
50. a 51. a 52. a 53. b 54. a 55. a 56. a
57. d 58. b 59. b 60. d 61. c 62. d 63. c
64. b 65. b 66. d 67. b 68. b 69. b 70. b
71. b 72. b 73. b 74. b 75. a 76. d 77. d
78. d 79. a 80. a 81. d 82. c 83. a 84. b
85. d 86. b 87. a 88. c 89. a 90. b 91. c
92. c 93. c 94. a 95. c 96. a 97. b 98. b
99. b 100. d 101. a 102. a 103. a 104. d 105. c
106. a 107. c 108. c 109. b 110. d 111. c 112. a
113. a 114. b 115. c

■

SOLUTION OF EXERCISE 2

1. Given data,
Moisture content (MC) = 20%
Diameter of sample (D) = 76 mm
Height of sample (H) = 60 mm
Specific gravity of sample (SG) = 2.65

We have,

$$MC = \frac{\text{weight of water present in the sample}}{\text{Weight of solid}} = \frac{W_W}{W_S} = \frac{V_W * \rho_w}{V_s * \rho_S} = \frac{V_W}{V_s(\rho_S/\rho_W)}$$

$$= \frac{V_W}{V_S*(SG)}$$

$$\Rightarrow 0.2 = \frac{V_W}{V_S*(2.65)}$$

$$\Rightarrow \frac{V_S}{V_W} = \frac{1}{0.53}$$

$$\text{Again, Porosity (P)} = \frac{V_V}{V_V+V_S} = \frac{V_W}{V_W+V_S} = \frac{0.53}{0.53+1} = 0.346$$

[for saturated condition, void area is fully filled with water. Thus, $V_V=V_W$]

2. Given data,
Porosity of sample (P) = 15% = 0.15
Specific gravity of sample (SG) = 2.8

We have,

$$P = \frac{V_V}{V_S+V_V} = 0.15$$

$$\Rightarrow \frac{V_V}{V_S} = \frac{0.15}{0.85}$$

$$\text{Therefore, void ratio} = \frac{V_V}{V_S} = 0.176$$

3. Given data,
Porosity of sample (P) = 15% = 0.15
Specific gravity of sample (SG) = 2.8 = ρ_s/ρ_w

We have,

$$\text{Saturated density (S)} = \frac{M_S}{V_S} = \frac{V_s\rho_s + V_w\rho_w}{V_s+V_w}$$

Again, for 100% saturation, volume of void filled with water and this is equal to volume of water

$$\frac{V_V}{V_s+V_w} = \frac{V_w}{V_s+V_w} = 0.15$$

$$V_S = 5.67 * V_w$$

$$\text{Therefore, } S = \frac{5.67*V_w*2.8*\rho_w + V_w*\rho_w}{5.67*V_w+V_w} = 2.531\rho_w = 2531$$

[density of water = 1000 kg/m^3]

4. Given data,
diameter of sample (D) = 50 mm
Length of Sample (L) = 100 mm
Weight of dry sample = 300 g
Specific gravity of brine solution = 1.05

We have,

Weight os saturated sample – weight of dry sample = Volume of void * density of brine solution

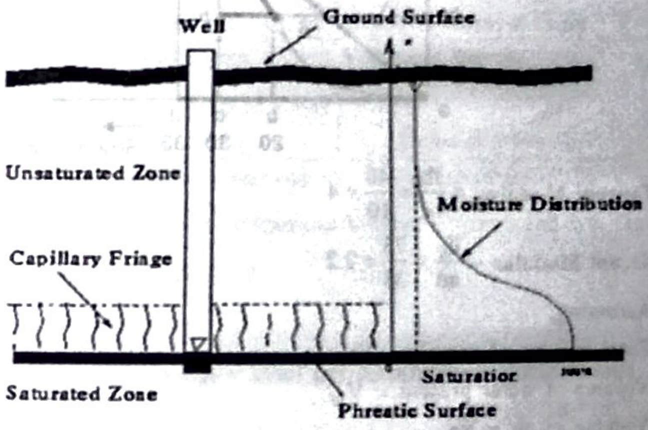
$$\Rightarrow (330-300)*10^{-3} = V_v*1050$$

$$\Rightarrow V_v = 28.571*10^{-6}\,m^3 = 28.571\,cm^3$$

Therefore,

$$\text{Porosity} = \frac{\text{Volume of void}}{\text{Volume of sample}} = \frac{28.571}{\pi\left(\frac{D}{2}\right)^2 L} = \frac{28.571}{3.14*2.5^2*10} = 0.1455 = 14.55\%$$

. The phreatic surface (Water tale), defined as the surface at every point of which the pressure in the water is atmospheric, separates between the saturated and the unsaturted zones. The unsaturated zone is bounded from above by grond surface, and from below by the water table (=phreatic surface).



6. We have,

Void ratio =

$$\frac{\text{Volume of Void}}{\text{Volume of Sample}} = \frac{V_V}{V_S} = \frac{(2D*2D*D) - 4*\left(\frac{4}{3}*\pi*\frac{D^3}{8}\right)}{4*\left(\frac{4}{3}*\pi*\frac{D^3}{8}\right)} = \frac{4-2.09}{2.09} = 0.91$$

7. We have,

$$\text{Dry density}(\gamma_d) = \frac{\text{Mass of Solids + Mass of air present in voids}}{\text{Volume of sample}}$$

$$= \frac{\text{Volume of Solids * Units weight of solids}}{2D*2D*D} \quad \text{[Weight of air is neglected]}$$

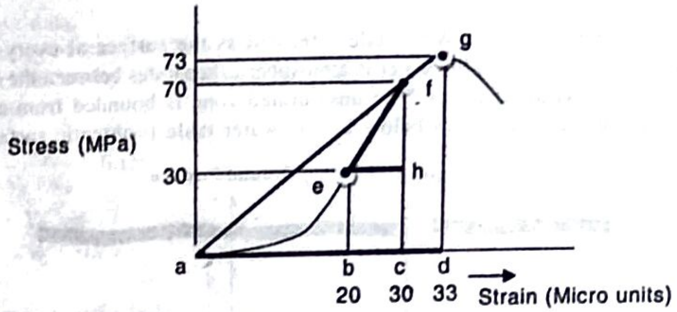
$$= \frac{4*\left(\frac{4}{3}*\pi\frac{D^3}{8}\right)*25}{4D^3} = 13.09$$

11. We have,

'Young's Modulus (E) = 2*Shear Modulus(G)* [1 + Poisson's ratio (v)]

$$\Rightarrow G = \frac{E}{2(1+v)} = \frac{60*10^3}{2*(1+0.3)} = 23070\,\text{MPa}$$

12.



$$\text{Tangent Modulus} = \frac{fh}{bc} = \frac{40}{10} = 4$$

$$\text{Secant Modulus} = \frac{dg}{ad} = \frac{73}{33} = 2.2$$

15. Assuming,

Total volume of sample = Vt

Volume of water present = V_W

Volume of air = Va

Given data,

$V_s = 0.75V_t$

Moisture Content (MC) = 2.5%

We have,

Total volume = Volume of water + Volume of air + Volume of Solids

$\Rightarrow V_t = V_W + V_a + 0.75*V_t$

$\Rightarrow V_W + V_a = 0.25\,V_t$

Again, MC = Weight of water (W_W)/Total weight of sample (W_t) = 0.025

$$\Rightarrow \frac{v_w\rho_w}{v_w\rho_w + v_s\rho_s} = 0.025 \quad \text{[Weight of air neglected]}$$

$$\Rightarrow \frac{1}{1+(v_s/v_w)(\rho_s/\rho_w)} = 0.025$$

$$\Rightarrow 1+(V_s/v_w)\,2.5 = 40$$

$$\Rightarrow V_s/V_w = \frac{39}{2.5} = 15.6$$

$$\Rightarrow V_W = \frac{0.75\,V_t}{15.6} = 0.048V_t$$

Therefore,

$$\text{Degree of saturation} = \frac{V_w}{V_w + V\alpha}*100\% = \frac{0.0480V_t}{0.25V_t} = 19.23\%$$

16. Given data,

Grain density (ρ_g) = 3 gm/cc

Bulk density (ρ_b) = 2.7 gm/cc

We have,

$$\text{Void ratio} = \frac{\text{Volume of Void}}{\text{Volume of Grains}} = \frac{\text{Total Volume} - \text{Volume of Grain}}{\text{Volume of Sample}} = \frac{v - v_g}{v_g}$$

$$= \frac{V}{V_g} - 1 = \frac{\left(\frac{\text{Mass of air + Mass of grains}}{\text{Bulk density}}\right)}{\text{(Mass of grains/ grain density)}} - 1 = \frac{\text{(Mass of grains/2.7)}}{\text{(Mass of grains/3)}} - 1$$

[Mass of air is very small in comparison to mass of grains and thus neglected]

$$= \frac{3}{2.7} - 1 = 0.111 = 11.1\%$$

17. Given data,

Quantity of water flow (Q) = 20ml per 30 minutes = 20/1800 ml/s

$= (20/18)*10^{-8}$ m³/s

Hydraulic gradient (i) = 1.2m per 20cm length = 1.2/0.2=6

Area of cross section = $\pi D^2/4 = (3.14*25*10^{-4})/4 = 19.625*10^{-4}$ m²

We have,

$Q = K * i * A$

Where, K is the hydraulic conductivity

Q = Quantity

$\Rightarrow 20 = K * i * A$

$$\Rightarrow K = \frac{Q}{i*A} = \frac{\left(\frac{20}{18}\right)*10^{-8}}{6*19.625*10^{-4}} m/s$$

$\Rightarrow K = 0.94\ \mu m/s$

18. We have,

$Q = K*i*A$

Where, Q = Quantity of flow
i = hydraulic gradient
A = area of cross section
K = hydraulic conductivity

$$\text{Therefore, } Q = 40*\left(\frac{1.5}{2500}\right)*(75*1) = 1.8 m^3/day$$

[For unit width, A=75*1]

19. Given data,

M_A = Mass of pycnometer filled with water = 70.4g

M_B = Mass of pycnometer filled with water and soil = 88.6

M_{PS} = Mass of empty pycnometer + dry soil = 51.6g

M_P = Mass of empty, clean pycnometer = 20.4g

Therefore, $M_0 = M_{PS} - M_P = 51.6 - 20.4 = 31.2g$

$$\text{Specific Gravity } (G_s) = \frac{M_0}{M_0 + M_A - M_B} = \frac{31.2}{31.2 + 70.4 - 88.6} = 2.4$$

20. Assuming, Density of coal = ρ_c

Density of water = ρ_W

Mass of coal = m_c

Volume of coal = V_c

Given data, Mass of coal sample in air (m_c) = 10 kg

Mass of coal sample when immersed in water = 2kg

$\Rightarrow m_c - V_c \rho_w g = 2$ [Applying, Archimedes' Principle]

$\Rightarrow 10 - V_c \rho_W = 2$

$\Rightarrow V_c = 8/1000 = 8*10^{-3} m^3$

$\Rightarrow \rho_c = 10/\left(8*10^{-3}\right) = 1.25*10^3\ kg/m^3$

Therefore,

Specific Gravity of coal = $\rho_c/\rho_w = 1.25*10^3/1000 = 1.25$

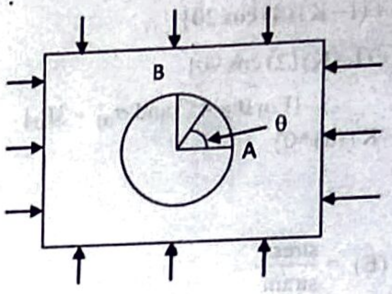
21. We have,

Safety Factor of the Shaft Wall (SF)

$$= \frac{\text{unconfined compressive/ tensile strength of shaft wall rock } (C_0/T_0)}{\text{peak compressive and tensile stresses at the shaft wall } (\sigma_c/\sigma_t)}$$

$$= \frac{89.49}{59.66} = 1.5$$

22. Give condition :



Let the tangential stress at point A is $\sigma_{\theta A}$

We have,

$$\sigma_{\theta A} = \frac{S_V}{2}\left[(1+K)\left(1+\frac{a^2}{r^2}\right) + (1-K)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

At the boundary point, r = a

Therefore,

$$\sigma_{\theta A} = S_V[(1+K) + 2(1-K)\cos 2\theta]$$

At point A, θ = 0^0

Therefore, $\sigma_{\theta A} = S_V[(1+K) + 2(1-K)\cos\theta] = S_V(3-K)$

The point A is under tensile stress, when $\sigma_{\theta A} \geq 0$

$\Rightarrow S_V(3-K) \leq 0$

$\Rightarrow (3-K) \leq 0$

$\Rightarrow K \geq 3$

$\Rightarrow \frac{S_h}{S_v} \geq 3$

$\Rightarrow S_h \geq 3S_v$

24. We have,

$$\frac{\sigma_c}{\sigma_t} = \frac{1+\sin\varphi}{1-\sin\varphi}$$

Where, σ_c = uniaxial compressive strength
σ_t = uniaxial tensile strength
φ = angle of internal friction

Therefore, $\frac{8}{1} = \frac{1+\sin\varphi}{1-\sin\varphi}$

$\Rightarrow \sin\varphi = \frac{7}{9}$

$\Rightarrow \varphi = 51^0$

25. The tangential stress ($\sigma_{\theta\theta}$) at the boundary of the tunnel is given by

$\sigma_{\theta\theta} = \frac{P_0}{2}[(1+K)(2)+(1-K)(4)\cos 2\theta]$

$\Rightarrow 3P_0 = P_0[(1+K)+(1-K)(2)\cos 90]$

[For $\theta = 45^0$ and $\sigma_{\theta\theta} = 3P_0$]

$\Rightarrow 3 = [(1+K)+(1-K)(2)*0]$

$\Rightarrow K = 2$

28. We have,

Young's Modulus (E) $= \frac{\text{stress}}{\text{strain}}$

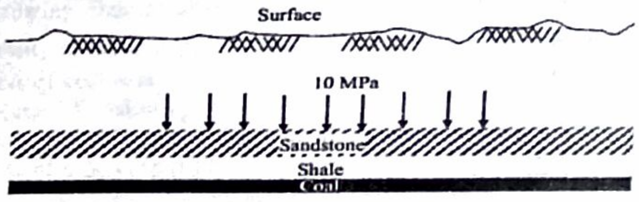
$\Rightarrow E = \frac{\text{Axial Load (F) / Area of cross section (A)}}{\text{Strain}}$

$\Rightarrow 68.97*10^{12} = \frac{F/\pi*0.027^2}{0.001}$

$\Rightarrow F = 68.97*10^9*0.001*\pi*0.027^2 = 15.8*10^4 \text{ N} = 158 \text{ kN}$

29. We have,

Strata	Thickness (m)	Specific Gravity	Young's Modulus (GPa)	Shear Modulus (GPa)
Sandstone	50	2.35	26.4	12.5
Shale	25	2.15	20.50	8.25
Coal	20	1.52	2.41	0.95



Cross-section of the strata

We have,

Vertical in-situ stress above coal seam (σ_Z)

$= 10*10^6$ Pa + Vertical stress due to sandstone and shale layer

Again, vertical stress due to sandstone layer = Load acting/Surface area

$= (50*1*1)*2350*9.81/1*1$ [for unit area of cross section]

$= 117.5 \text{ kN/m}^2$

Similarly, vertical stress due to shale layer = $(25*1*1)*2150*9.81/1*1$

$= 53.5 \text{ kN/m}^2$

Therefore, $\sigma_Z = 10*10^6 \text{ N/m}^2 + 117.5*10^3 \text{ N/m}^2 + 53.5*103 \text{ N/m}^2$

$= 11.68*10^6 \text{ N/m}^2 = 11.68$ MPa

Again we have,

$$\sigma_h = \frac{\nu}{1-\nu}\sigma_v = \frac{\frac{E}{2G}-1}{1-\left(\frac{E}{2G}-1\right)}\sigma_v = \frac{\frac{20.5}{2*8.25}-1}{1-\left(\frac{20.5}{2*8.25}-1\right)}*53.5 = 3.69 \text{ MPa}$$

Where, σ_h = horizontal stress above coal seam

ν = Poisson ratio of shale layer

E = young's modulus of shale layer

G = shear modulus of shale layer

30. The Poisson ratio (ν) of rock sample is 0.25

We have,

$E = 2G(1 + \nu) = 3K\ (1-2\nu)$

$\Rightarrow E = 2G(1+0.25) = 3K(1-2*0.25)$

$\Rightarrow E = 22.5G = 1.5K$

Since each of the modulus are greater than zero, the above relationship indicate that

$E > K > G$

31. Given data,

Modulus of elasticity (E) = 1000 MPa

Poisson's ratio (ν) = 0.25

Therefore, Shear Modulus (G) = $E/2*(1 + \nu) = 1000/(2*1.25) = 400$ MPa

32. Given data,

Height of sample = 10cm = 0.1m

Diameter of sample = 5.4cm = 0.054m

Cross sectional area of ram = $1.25\text{cm}^2 = 12.5*10^{-4} \text{ m}^2$

Dial gauge reading = 3 MPa

We have,

Point Load Strength (P_d) $= \frac{P}{d^2}$

Where, P = Load at rupture = stress*area of cross section

d = distance between the point load

Therefore, $P_d = \frac{3*10^6*12.5*10^{-4}}{0.054^2} = 1.286*10^6$ Pa

Therefore, uniaxial compressive strength = $24*P_d = 30.8$ MPa

33. We have,

Adherence Strength $(\tau) = \dfrac{P_f}{\pi DL}$

Where, P_f = yield load in N
D = diameter of bolt
L = length of bolt

$$\text{Therefore, } (\tau) = \frac{200*10^6*\left(\pi*D^2/4\right)}{\pi DL} = \frac{50*10^6*D}{L} = \frac{50*10^6*2.5*10^{-2}}{154*10^{-2}}$$

$$= 0.811*10^6\ \text{N/m}^2$$

$$= 796\ \text{kg/cm}^2 \approx 800\ \text{kg/cm}^2$$

34. Axial load = tensile stress in bolt* area of cross section of bolt
= $(193.35*10^6)*(\pi*D^2/4)$ = $(193.35*10^6)*(3.14*0.022*0.022/4)$
=$73.5*10^3$ N = 73.5 kN

35. At equilibrium,

Axial Load on bolt = Load acting per bolt due to strata

$\Rightarrow 73.5*10^3 = 25*10^3*(1.5*1.5*x)$

[Area of cross section covered by each bolt = 1.5*1.5, and thickness of roof layer supported by bolt is x]

Therefore,

x = 73.5/(25*1.5*1.5)= 1.31

36. We have,

Horizontal force exerted on a unit slope length (F)
= Pressure due to presence of water (P) * area of cross section (A)

$\Rightarrow F = (h*\rho_w*g)*(L*h)$

$\Rightarrow F = (5*1000*9.81)*(5*1) = 122.6$ kN

Where, h = depth of water = 5m
ρ_w = density of water = 1000 kg/m^3
L = length = 1m

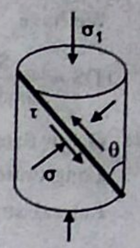
38. Given data

$\sigma_{xx} = 100$ MPa
$\sigma_{yy} = 30$ MPa
$\sigma_{xy} = 15$ MPa

We have, major principal stress

$$\sigma_1 = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy^2}} = \frac{100+30}{2} + \sqrt{\left(\frac{100-30}{2}\right)^2 + 15^2}$$

39. Given data :



Uniaxial load at the point of fail (σ_1) = 50 MPa

Inclination of fracture plane (θ) = 45^0

Let the normal and shear stresses on the failure plane are σ and τ respectively.

The normal stress along the failure plane is given by

$$\sigma = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1+\sigma_3}{2}\cos 2\theta$$

$$\Rightarrow \sigma = \frac{50+0}{2} + \frac{50+0}{2}\cos 90 = 25+0 = 25\ \text{MPa}$$

The shear stress along the failure plane is given by

$$\tau = \frac{\sigma_1-\sigma_3}{2}\sin 2\theta$$

$$\Rightarrow \tau = \frac{50-0}{2}\sin 90 = 25*1 = 25\ \text{MPa}$$

40. Given data :

$\sigma_{xx} = 5$ MPa
$\sigma_{yy} = 10$ MPa
$\sigma_{xy} = -2$ MPa

We have, major principal stress

$$\sigma_1 = \frac{\sigma_{xx}+\sigma_{xy}}{2} + \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy^2}} = \frac{5+10}{2} + \sqrt{\left(\frac{5-10}{2}\right)^2 + (-2)^2}$$

= 10.70 MPa

41. We have,

$$\sigma_t = \frac{2c\cos\varphi}{1+\sin\varphi}$$

Where, c = cohesion
σ_t = uniaxial tensile strength
φ = angle of internal friction

$$\text{Therefore, } \sigma_t = \frac{2*21.65*\cos 30}{1+\sin 30} = 25\ \text{MPa}$$

42. Given data :

Length of roof bolt (D) = 1.5 m
Area covered by one roof bolt (A) = 1.2*1.2 = 1.44 m^2
Area of cross section of bolt (a) = 4 cm^2
Tensile strength of bolt (St) = 300 MN/m^2 = $300*10^6$ N/m^2
Sp. gr. of roof rock (SG) = 2.5
Therefore, density of rock (λ) = 2.5*density of water = 2500 kg/m^3
Let the facor of safety of the roof bolt is FOS

We have,

$$FOS = \frac{\text{Strength of roof bolt}}{\text{Load acting on bolt by strata}} = \frac{S_t * a}{A * D * \gamma * g} = \frac{300*10^6*4*10^{-4}}{1.44*1.5*2500*9.81} = 2.26$$

44. Given data :

Longitudinal strain (ε_{lon}) = 0.06

Lateral strain (ε_{lat}) = 0.012

Therefore, Poisson Ratio (v) = $\frac{\varepsilon_{lat}}{\varepsilon_{lon}} = \frac{0.012}{0.06} = 0.2$

Again, we have,

Horizontal stress $(\sigma_h) = \frac{v}{1-v} *$ Vertical stress (σ_v)

$$\Rightarrow \sigma_h = \frac{0.2}{1-0.2}\sigma_v$$

$$\Rightarrow \frac{\sigma_v}{\sigma_h} = 4$$

45. Given data :

Radius of cylindrical specimen of sandstone (D) = 50 mm = 0.05 m

Length of cylindrical specimen of sandstone (L) = 125 mm

Load at failure (F) = 100 kN

Longitudinal deformation (Δl) = 1.5mm

Let the uniaxial compressive strengthis σ and the modulus of elasticity is E

The compressive strength of the rock can be determined as

$$\sigma = \frac{F}{A} = \frac{100*10^3}{\pi*(0.05)^2} = \frac{7*100*10^3}{22*(0.05)^2} = 12.72\text{ MPa}$$

Again, modulus of elasticity is given by

$$E = \frac{\sigma}{(\Delta l/L)} = \frac{12.72}{(1.5/125)} = 1060\text{ MPa} = 1.06*10^9\text{ Pa}$$

46. Given data

Diameter of the core tested (D) = 54 mm = 0.054 m

Cross sectional area of ram (A) = 13.36 cm^2 = 13.36 $*10^{-4}$ m^2

Pressure gauge reading at failure = 12.5 MPa

We have,

Point Load Strength (Pd) $= \frac{P}{d^2}$

Where, P = load at failure = stress * area of cross section

d = distance between the point load = diameter of the core = 0.054 m

$$\text{Therefore, } P_d = \frac{12.5*10^6*13.36*10^{-4}}{0.0542^2} = 5.73\text{ MPa}$$

47. The tangential stress at the boundary of a circular opening in underground is given by $\sigma_{\theta\theta} = \frac{P_0}{2}[(1+K)(2)+(1-K)(4)\cos 2\theta]$

Where, P_0 is the vertical load

K is the ratio of horizontal to vertical stress.

θ is the angle between horizontal line along the the circular opening and the line joining the point of stress measurement with centre.

In hydrostatic condition, stresses along horizontal and vertical directions are equal (that is, $\sigma_h = \sigma_v$). Therefore, K = 1

Putting the value of K in the above equation, we get

$$\sigma_{\theta\theta} = \frac{P_0}{2}[6+0] = 3P_0$$

Thus, the value is independent of θ and positive. And, for tensile boundary stress, this should be negative. so, tensile stress will not occurred along the boundary.

48. Given data,

Modulus of elasticity (E) = 10^5 MPa

Poisson's ratio (v) = 0.25

Therefore, Modulus of rigidity or shear modulus (G) = E/2*(1+v)

= $10^5/(2*1.25)$ = 40000 MPa = 40 GPa

49. We have,

$$RQD(\%) = \frac{\text{Sum of the length of the cores greater than 10 cm}}{\text{Total length of the core run}} * 100$$

$$= \frac{172+255+280+176+302+102+153+147+101}{2000}*100 = \frac{1688}{2000}*100 = 84.4\%$$

50. We have,

$$RQD(\%) = \frac{\text{Sum of the length of the cores greater than 10 cm}}{\text{Total length of the core run}} * 100$$

$$= \frac{20+15}{120}*100 = \frac{35}{100}*100 = 29.16\% \approx 29.2\%$$

59. Given data,

Diameter of the core tested (D) = 50mm = 0.05m

Load at rupture = 5000 kg

We have,

Point Load Strength $(P_d) = \frac{P}{d^2}$

Where, P = load at failure = stress * area of cross section

d = distance between the point load = diameter of the core = 0.05m

$$\text{Therefore, } P_d = \frac{5000}{0.05^2} = 2*10^6\text{ kg/m}^2 = 200\text{ kg/cm}^2$$

60. From the above solution we get,

Point Load Strength (P_d) = 200 kg/cm^2

Again we have,

Uniaxial compressive strength (σ_{cu}) = $24*P_d = 24*200 = 5000\ kg/cm^2$

61. Given data,

Diameter of the core tested (D) = 5cm

Height of the core tested (L) = 5cm

Compression Load at failure = 19.6 kN

We have,

Indirect tensile strength of the specimen (σ_t)

$$= \frac{2*F}{\pi*D*L} = \frac{2*19.6*10^3}{3.14*5*10^{-2}*5*10^{-2}} N/m^2$$

$$= 5*10^6\ Pa = 5\ MPa$$

62. Given data,

Diameter of the core tested (D) = 54mm = 5.4cm

Height of the core tested (L) = 22mm = 2.2cm

Load at failure = 11.4 kN

We have,

Indirect tensile strength of the specimen

$$(\sigma_t) = \frac{2*F}{\pi*D*L} = \frac{2*11.4*10^3}{3.14*5.4*10^{-2}*2.2*10^{-2}} N/m^2$$

$$= 6.11*10^6\ Pa = 6.11\ MPa$$

63. We have,

$$\text{Height of immediate roof } (h_{im}) = \frac{\text{Extraction thickness}(H)}{\text{Bulking factor}(K) - 1}$$

$$\Rightarrow h_{im} = \frac{H}{1.5-1} = 2H$$

64. Given data,

Specific gravity of gypsum mineral (S_g) = 2.4

Density of gypsum core (ρ_{gm}) = 2100 kg/m^3

We have,

Density of gypsum mineral (ρ_{gc}) = S_g*density of water = 2.4*1000

= 2400 kg/m^3

Therefore,

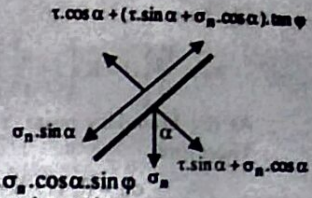
$$\text{Porosity} = \frac{\text{Volume of void}(V_v)}{\text{Volume of sample}(V)} = \frac{\text{Volume of core sample}(V) - \text{Volume of gypsum}(V_g)}{\text{Volume of core sample}(V)}$$

$$= 1 - \frac{V}{V_g} = 1 - \frac{\text{mass of core sample}(m)}{\text{density of gypsum core}(\rho_{gc})} * \frac{\text{density of gypsum core}(\rho_{gm})}{\text{mass of gypsum}(m_g)}$$

$$= 1 - \frac{m_g*\rho_{gm}}{\rho_{gc}*m_g} = 1 - \frac{2100}{2400} = \frac{1}{8} = 0.125$$

[Note : Assuming the sample is dry and thus voids are filled with air. Mass of air is very small in comparison to gypsum and hence neglected. Therefore, mass of core sample = mass of void + mass of gypsum = 0 + m_g = m_g]

65.



In equilibrium conditon,

We have,

$\sigma_n.\sin\alpha = \iota.\cos\alpha\,(\tau.\sin\alpha + \sigma_n.\cos\alpha).\tan\varphi$

$\Rightarrow \sigma_n.\sin\alpha.\cos\varphi = \tau.\cos\alpha.\cos\varphi + \tau.\sin\alpha.\sin\varphi.\sigma_n.\cos\alpha.\sin\varphi$

$\Rightarrow \sigma_n.(\sin\alpha.\cos\varphi - \cos\alpha.\sin\varphi) = \tau.(\cos\alpha.\cos\varphi + \sin\alpha.s)in\varphi$

$\Rightarrow \sigma_n.\sin(\alpha-\varphi) = \tau.\cos(\alpha-\varphi)$

$\Rightarrow \tau = \sigma_n.\tan(\alpha-\varphi)$

66. We

$$\sigma_t = \frac{2c\cos\varphi}{1+\sin\varphi}$$

$$\sigma_c = \frac{2c\cos\varphi}{1-\sin\varphi}$$

Where, c = cohesion

σ_t = uniaxial tensile strength

σ_c = uniaxial compressive strength

φ = angle of internal friction

$$\text{Therefore, } \sigma_t = \frac{2*6*\cos 35}{1+\sin 35} = \frac{9.83}{1.57} = 6.24\ MPa$$

$$\sigma_c = \frac{2*6*\cos 35}{1-\sin 35} = \frac{9.83}{1.57} = 23.05\ MPa$$

68. Shear Strength (S) $= \frac{P}{A} = \frac{P}{\pi*d*t}\ [kg/cm^2]$

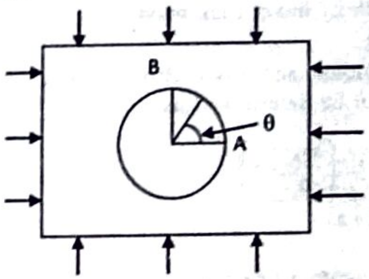
Where, P = load applied to punch the specimen = 1000 kg

A = circumferential shear area, cm^2

t = thickness of the disc = 0.8 cm

$$\text{Therefore, } S = \frac{1000}{3.14*2.8*0.8} = 142.17$$

69. Give data



Let the tangential stress at point A is $\sigma_{\theta A}$ and the same at B is $\sigma_{\theta B}$

Given that, $\sigma_{\theta A} = 3\sigma_{\theta B}$

We have,

$$\sigma_\theta = \frac{P_0}{2}\left[(1+K)\left(1+\frac{a^2}{r^2}\right)+(1-K)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

At the boundary point, $r = a$
Therefore,

$$\sigma_\theta = P_0\ (1+K)+2(1-K)\cos 2\theta$$

At point A = $\theta = 0$

$$\sigma_{\theta A} = P_0\ (1+K)+2(1-K)\cos 0\ = P_0(3-K)$$

At point B, $\theta = 90^0$

$$\sigma_{\theta A} = P_0\ (1+K)+2(1-K)\cos 180\ = P_0(3K-1)$$

Again,

$$\sigma_{\theta A} = 3\,\sigma_{\theta B}$$

$$\Rightarrow P_0(3-K) = 3P_0(3K-1)$$

$$\Rightarrow 10K = 6$$

$$\Rightarrow K = 0.6$$

70. We have,

$$\sigma_h = \frac{\varphi}{1-\varphi}\sigma_v$$

Where, σ_h = horizontal stress
σ_v = vertical stress
v = Poisson's ratio

Therefore,

$$\sigma_h = \frac{0.2}{1-0.2}*10 = 2.5\text{ MPa}$$

71. We have,
Radius of the circle = (6 + 2)/2 = 4
Therefore, normal stress (σ) = 4 cos 60 = 2 MPa
And, shear stress (τ) = –4 sin 60 = – 3.46 MPa (–ve sign because of downward direction]

72. Given data,
Uniaxial compressive strength of intact rock mass (σ_{ci}) = 50 MPa
m_b = 2.4, s = 0.01, a = 0.52
From equivalent Mohar Coulomb and Hoek Brown failure envelops, the global rock mass strength can be determined as

$$\sigma_{cm} = \sigma_{ci}\frac{[m_b+4S-a(m_b-8S)]*\left(\frac{m_b}{4}+S\right)^{a-1}}{2(1+a)(2+a)}$$

$$\Rightarrow \sigma_{cm} = 50*\frac{2.4+4*0.01-0.52(2.4-8*0.01)\ *\left(\frac{2.4}{4}+0.01\right)^{0.52-1}}{2(1+0.52)(2+0.52)}$$

$$\Rightarrow \sigma_{cm} = 10.24\text{ MPa}$$

73. We have,

$$\sigma_1 = \sigma_3+\sigma_{Ci}\left(m_b\frac{\sigma_3}{\sigma_{Ci}}+s\right)^a$$

$\sigma_1 = \sigma_c$ (uniaxial compressive strength)
and, $\sigma_3 = 0$
Therefore from above equation, we get
$\sigma_c = 0 + 50 * (0 + 0.01)^{0.52} = 50*0.091 = 4.56$ MPa

75. Given equation :

$$K = \frac{267}{D}+1.25$$

Where, K is the ratio of horizontal stress to vertical stress

$$\Rightarrow \frac{\sigma_h}{\sigma_v} = \frac{267}{D}+1.25$$

At 400m depth, the vertical load due to overburden rock having unit weight 25 kN/m^3 is
σ_v = depth*unit weight of rock = 400*25 kN/m^2 = 10MN/m^2
Therefore, from above equation, we get

$$\frac{\sigma_h}{10} = \frac{267}{400}+1.25$$

$$\Rightarrow \frac{\sigma_h}{10} = \frac{267}{400}+1.25 = 1.9175$$

$$\Rightarrow \sigma_h = 19.175$$

76. According to Hoek-Brown failure criteria,

$$\text{Maximum principal stress} \Rightarrow \sigma_1 = \sigma_3+\sigma_{Ci}\left(m\frac{\sigma_3}{\sigma_{Ci}}+s\right)^a$$

$$= 20+80\left(7.88*\frac{20}{80}+1\right)^{0.5} = 157.9\text{ MPa}$$

77. We have,

$$\text{Mean Pillar Stress} = \frac{\text{Average Load acting on Pillar}}{\text{Area of cross section of Pillar}} = \frac{\gamma*D*(a+b)^2 g}{a^2}$$

Where, a = size (length and breadth) of the pillars, and b = width of galleries, γ = rock density, D = depth of overlying strata

$$\Rightarrow \text{Mean Pillar Stress} = \frac{2800*100*(4+2.4)^2*9.81}{4^2} = 7*10^6\text{ Pa} = 7\text{MPa}$$

78. We have,

$$\Rightarrow \text{Mean Pillar Stress} = \frac{\text{Average Load acting on Pillar}}{\text{Area of cross section of pillar}} = \frac{\gamma*D*(a+b)^2}{a^2}$$

Where, a = size (length and breadth) of the pillars, and b = width of galleries, γ = unit weight of rock, D = depth of overlying strata

$$\text{Mean Pillar Stress} = \frac{24500*300*(a+6)^2}{a^2} N/m^2$$

Again,

$$\text{Factor of Safety of Pillar (FOS)} = \frac{\text{Strength of Pillar}}{\text{Stress on Pillar}} = \frac{15.5*10^6}{24500*300*(a+6)^2/a^2}$$

$$\Rightarrow 1 = \frac{2.1*a^2}{(a+6)^2}$$

$$\Rightarrow \frac{a^2}{(a+6)^2} = 0.47$$

$$\text{Therefore, Extraction Ratio} = R = 1 - \frac{a^2}{(a+6)^2} = 1 - 0.47 = 0.53 = 53\%$$

83. We have, Slenderness ratio (SR) = L_e/r
Where, Le = equivalent length of prop = L/2 = 2.5/2 = 1.25m
r = radius of prop = D/2 = 0.08 m
Therefore, SR = 1.25/0.08 = 15.625

$$\left[\text{Critical slenderness ratio} = \pi\sqrt{\frac{\text{Modulus of elasticity (E)}}{\text{critical bearing strength } (\sigma_{cr})}}\right]$$

84. We have,

$$\text{Volumetric Strain} = \frac{\Delta V}{V} = \varepsilon*(1-2v)$$

Where, ε = axial strain, and v = Poisson's ratio
Therefore, Volumetric strain = $6.5*10^{-4}*(1-2*0.3) = 2.6*10^{-4}$ mm/mm

86. We have,

$$\text{Minimum length of bolt (L)} = \frac{B}{2}\left(\frac{100-RMR}{100}\right)$$

Where, B = width of galleries

$$\text{Therefore, } L = \frac{4.6}{2}\left(\frac{100-45}{100}\right) = 1.265m$$

88. We have, Posson's ratio = Lateralstrain/Longitudinal strain
= 0.0023/0.0082 = 0.28

89. We have, Posson's ratio = Lateral strain/Longitudinal strain = 0.006/0.03 = 0.2

90. We have, RQD (%) = 110.4 – 3.68*λ
Where, λ = number of fractures per meter of core length
Therefore, RQD = 110.4 – 3.68*20 = 37%

92. Given relationship between the major and minor principal stresses is
$\sigma_1 = 100 + 5\sigma_3$
Again, we have,

$$\sigma_1 = \frac{2c\cos\varphi}{1-\sin\varphi} + \frac{1+\sin\varphi}{1-\sin\varphi}\sigma_3$$

σ_1 and σ_3 are major and minor principal stress respectively.
φ = angle of internal friction
c = coefficient of cohesion
On comparing the above two equations, we get

$$\frac{2c\cos\varphi}{1-\sin\varphi} = 100$$

$$\text{And, } \frac{1+\sin\varphi}{1-\sin\varphi} = 5$$

Therefore, $\sin\varphi = 4/6$

$$\Rightarrow \varphi = \sin^{-1}(4/6) = 41.81^0 \approx 42^0$$

Putting the values, we get

$$\frac{2c\cos 41.81}{1-\sin 41.81} = 100$$

$$\Rightarrow c = 22.3 \text{ Mpa}$$

94. We have,

$$\text{Shear Strength (S)} = \frac{P}{A} = \frac{P}{\pi*d*t} [kg/cm^2]$$

Where, P = load applied to punch the specimen = 200 kg
A = circumferential shear area, cm^2
t = thickness of the disc = 3 cm
d = diameter of the puncher = 8cm

$$\text{Therefore, } S = \frac{200}{3.14*8*3} = 2.65 \, kg/cm^2$$

95. Given data
σ_x = 7.99 MPa
σ_y = 6.90 MPa
τ_{xy} = 0.07 MPa
We have, major and minor principal stresses

$$\sigma_1/\sigma_3 = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}} = \frac{7.99+6.90}{2} \pm \sqrt{\left(\frac{7.99-6.90}{2}\right)^2 + (0.07)^2}$$

$$= 7.445 \pm 0.555 = 8.0/6.89 \approx 8/6.9 \text{ MPa}$$

Therefore, Major Principal Stress (σ_1) = 8 MPa
Minor Principal Stress (σ_3) = 6.9 MPa

96. We have,

$$Q = \frac{RQD}{J_n} * \frac{Jr}{Ja} * \frac{Jw}{SRF}$$

Where, Q = rock mass quality index
RQD = rock quality designation
Jn = number ofjoint sets = 9
Jr = roughness of the most important joints = 1.5
Ja = wall rock condition and/or filling material = 2
Jw = water flow characteristics of the rock = 1
SRF = looseness and stress conditions = 2.5

Therefore, $Q = \frac{96.5}{9} * \frac{1.5}{2} * \frac{1}{2.5} = 3.21$

$$RQD = \frac{94.2 + 95.6 + 99.4 + 95.3 + 98.0}{5} = 96.5$$

97. We have,

$$Vp \text{ - P-wave (compressive wave) velocity} = \sqrt{\frac{\frac{4}{3}G + k}{\rho}} \qquad (1)$$

$$Vs\text{-S-wave (shear wave) velocity} = \sqrt{\frac{G}{\rho}} \qquad (2)$$

Where, E = elastic modulus
G = shear modulus
K = bulk modulus
ρ = density of rock

Therefore, $\frac{V_p^2}{V_s^2} = \frac{\frac{4}{3}G + K}{G}$ (3)

Again, we know,
2G (1 + v) = 3K (1–2v)
Where, v = Poisson's ratio

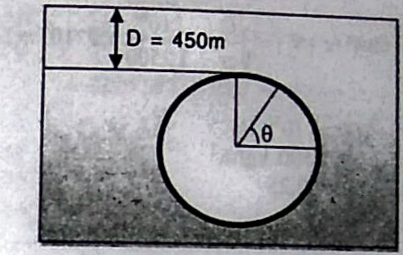
$$\Rightarrow K = \frac{2G(1+v)}{3(1-2v)}$$

Putting the value of K in eqn. (3), we get

$$\frac{V_p^2}{V_s^2} = \frac{\frac{4}{3}G + \left[\frac{2G(1+v)}{3(1-2v)}\right]}{G} = \frac{4(1-2v) + 2(1+v)}{3(1-2v)} = \frac{6(1-v)}{3(1-2v)}$$

$$= \frac{2(1-0.27)}{(1-2*0.27)} = 3.17 \approx 3.20$$

98.



Given data :
Depth of overburden (D) = 450m
Unit weight or rock (γ) = 6 kN/m^3
Uniaxial compressive strength (σ_c) = 25 MPa
Uniaxial tensile strength (σ_t) = 3 MPa
K (ratio of horizontal and vertical stress) = 0.3

We have,

$$\sigma_\theta = \frac{\sigma_v}{2}\left[(1+K)\left(1+\frac{a^2}{r^2}\right) + (1-k)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$\Rightarrow \sigma_\theta = \frac{\gamma D}{2}(1+K)(2) + (1-k)(4)\cos 2\theta$$

[At the boundary line, a = r]

$$\Rightarrow \sigma_\theta = \frac{6000*450}{2}(1+0.3)(2) + (1-0.3)(4)\cos 2\theta$$

$$\Rightarrow \sigma_\theta = 2.7*10^6(1.3 + 1.4\cos 2\theta)$$

At θ = 0,

$\sigma_\theta = 7.29$ MPa

At θ = 90^0,

σ_θ = –3.51 MPa (–ve sign indicates tensile)

Thus, σ_θ is greater than the tensile strength of rock at θ = 90^0, and thus failure occurs in tension.

100. Given data,
Uniaxial compressive strength (σ_1) = 90 MPa
Modulus of elasticity (E) = 30 GPa

We have, the stored strain energy per unit volume for uniaxial compression load is

$$U = \frac{\sigma_1 v_1}{2} = \frac{\sigma_1\left(\frac{\sigma_1}{E}\right)}{2} = \frac{90*10^6*90*10^6}{2*30*10^9} = 135*10^3\ N.m/m^3 = 135\ kN.m/m^3$$

Where, v_1 = Poisson's ratio = σ_1/E

101. Given data,
Shear modulus (G) = 10^{10} Pa
Rock density (ρ) = 2500 kg/m^3
We have,

Shear wave velocity $(V_s) = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{10^{10}}{2500}} = 2*10^3\ m/s$

102. Given data,
Shear modulus (G) = 10^{10} Pa
Rock density (ρ) = 2500 kg/m^3
Poisson's ratio (v) = 0.25
We have,

Compresive or P – wave velocity $(V_P) = \sqrt{\frac{\frac{4}{3}G+K}{\rho}} =$

Again, we know, 2G (1 + v) = 3K (1 – 2v)
$\Rightarrow K = 2G(1+v)/3(1-2v)$
Therefore,

$$V_p = \sqrt{\frac{\frac{4}{3}G+2G(1+v)/3(1-2v)}{\rho}} = \sqrt{\frac{10^{10}\left[\frac{4}{3}+\frac{2*(1+0.25)}{3*(1-2*0.25)}\right]}{2500}}$$

$$= \sqrt{\frac{10^{10}*9}{3*2500}} = 3.46*10^3\ m/s$$

106. Given data,
Diameter of the rock specimen (D) = 54mm
Young's modulus (E) = 68.97 GPa
Poisson's ratio (v) = 0.35
Lateral strain = 0.01%
Therefore, we have,
Longitudinal strain = Lateral strain/Poisson's ratio = 0.01/0.35 = 0.0286%
Again,

$$E = \frac{\text{stress}}{\text{longitudinal/ axial strain}} = \frac{F/A}{\Delta l/L}$$

Where, F = Axial load
A= area of cross section of sample = $\pi D^2/4$

$$\Rightarrow F = E*A*\left(\frac{\Delta l}{L}\right) = 68.97*10^9*(3.14*0.027^2)*0.000286$$

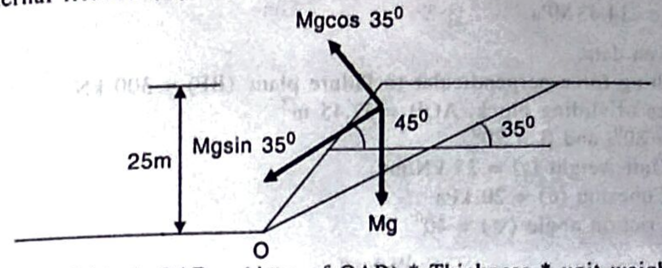
$$= 45.15*10^3\ N = 45.15kN$$

107. Given data,
Diameter of the rock specimen (D) = 54 mm
Length of the rock specimen (L) = 108 mm
Poisson's ratio (v) = 0.28
Axial strain (ε) = $2000*10^{-6}$
Therefore, we have,

$$\text{Volumetric strain}\left(\frac{\Delta V}{V}\right) = \varepsilon(1-2v) = 2000*10^{-6}*(1-2*0.28)$$

$$= 880*10^{-6} = 880 \text{ in micro strain}$$

109. Given data,
Unit weight of rock = 24.5 kN/m^3
Cohesion (c) = 0.02 MPa
Internal friction (φ) = 30^0



Weight of block OAB = (Area of OAB) * Thickness * unit weight
= 1/2*25*10.7*1*24.5 = 3278 kN
Therefore, force for failure, on the potential failure plane
= Wsin 35^0 = 1870kN = 1.87 MN

110. We have, factor of safety of slope $= \frac{\text{Force resist to slide}}{\text{Force for failure}}$

$$= \frac{cA + W.\cos\beta.\tan\varphi}{w\sin\beta}$$

$$= \frac{0.02*\left(\frac{25}{\sin 35^0}\right)*1+3.28.\cos 35^0.\tan 30^0}{3.28.\sin 35^0}$$

= (0.87 + 1.553)/1.88 = 1.3

112. Given relationship between the major and minor principal stresses is
$\sigma_1 = 50 + 3\sigma_3$
Again, we have,

$$\sigma_1 = \frac{2c\cos\varphi}{1-\sin\varphi} + \frac{1+\sin\varphi}{1-\sin\varphi}\sigma_3$$

σ_1 and σ_3 are major and minor principal stress respectively
φ = angle of internal friction
c = coefficient of cohesion
On comparing the above two equations, we get

$$\frac{2c\cos\varphi}{1-\sin\varphi} = 50$$

And, $\frac{1+\sin\varphi}{1-\sin\varphi} = 3$

There, $\sin\varphi = \frac{2}{4} = \frac{1}{2}$

$\Rightarrow \varphi = \sin^{-1}(1/2) = 30^0$

Putting the values, we get

$$\frac{2c\cos 30}{1-\sin 30} = 50$$

$\Rightarrow c = 14.43\,\text{MPa}$

114. Given data,

Bolting force perpendicular to failure plane (BF) = 300 kN

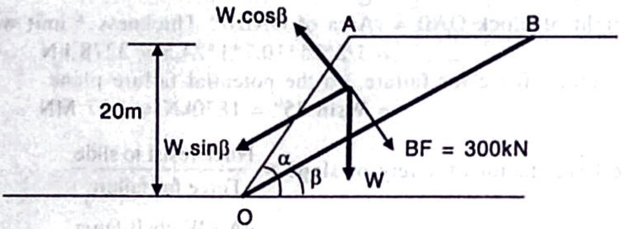
Area of sliding block, AOB = 37.45 m^2

$\alpha = 80^0$, and $\beta = 70^0$

Unit weight (γ) = 25 kN/m^3

Cohesion (c) = 20 kPa

Friction angle (φ) = 40^0



We have, factor of safety of slope

$$= \frac{\text{Force resist to slide}}{\text{Force for failure}} = \frac{cA + (W.\cos\beta + BF) * \tan\varphi}{W\sin\beta}$$

Surface area of sliding plane (A) = OB*1 [for unit width]

= 20/sinβ = 20/sin 70^0

Weight of block (W) = weight per unit volume * (area of ΔOAB*width)

=25*37.45*1 = 936.25kN

$$= \frac{20*\left(\frac{20}{\sin 70}\right)*1 + (936.25*\cos 70 + 300)*\tan 40}{936.25} = \frac{425.67 + 620.22*\tan 40}{936.25}$$

$$= \frac{425.67 + 520.42}{936.25} = \frac{946.09}{936.25} = 1.01$$

■

3.0 Mine Ventilation

3.1 Composition of Mine Air

Mine air mainly consists of following impurities :

a. Non-toxic but explosive in nature :
   CH_4, C_2H_2, H_2
b. Toxic gases :
   CO_2, Radon and its daughter products
c. Acutely poisonous gases :
   CO, NO_2, SO_2, H_2S

3.2 Layers of Gases in Mine Atmosphere

Mine atmosphere consists of following layers of gases from roof to floor :

Roof

H_2	S.G. = 0.070
CH_4	S.G. = 0.559
NH_3	S.G. = 0.590
N_2	S.G. = 0.967
CO	S.G. = 0.972
NO	S.G. = 1.037
O_2	S.G. = 1.105
H_2S	S.G. = 1.175
CO_2	S.G. = 1.529
N_2O	S.G. = 1.53
SO_2	S.G. = 2.264

Floor

3.3 Damps in u/g Mine Atmosphere

a. **Firedamp :** [CH_4 = 80-96 %; N_2 = 10-20 %; Higher hydrocarbons (C_2H_6, C_2H_4 etc.) = 5 %; CO_2 = 3-5 %].

b. **Blackdamp:** [CO_2 = 13 %; N_2 = 87 %]

c. **Afterdamp:** [Residual air = 30-50 %, N_2 = 44-58 %; CO_2 = 4-8 %; CO = 2-4 %]

d. **Stinkdamp:** [H_2S]

3.4 Permissible Concentrations of different Gases in Indian Underground Mines

CH_4 → < 0.75 % in the general body of return air
< 1.25 % in any parts of the mine
CO_2 → < 0.5 % (5000 ppm)
CO → < 0.005 % (50 ppm)
Oxides of nitrogen (NO, NO_2, N_2O_4) → < 0.0005 % (5 ppm)
O_2 → > 19 %
H_2S → < 0.001 % (10 ppm)
SO_2 → < 0.0005 % (5 ppm)

3.5 Degree of Gassiness of a Coal Mine

The degree of gassiness in coal mine are categorised as given in Table 3.1.

Table 3.1 : Degree of Gassiness

Degree of Gassiness	Percentage of inflammable gas in eneral body of return air	Rate of emission of gas m^3/te of coal output
I	< 0.1	< 1
II	> 0.1	1-10
III		> 10

3.6 Gas Monitoring Instruments

Different gas monitoring instruments used in mines are given in Table 3.2.

Table 3.2 : Gas monitoring instruments

Gas	Name of the instruments	Working principle/ chemical used for detection
CH_4	MSA D-6 Methanometer	Wheatstone bridge circuit
	Flame Safety Lamp	Length of the flame
	MSA Automatic firedamp Detectors (AFD-2)	Wheatstone bridge circuit
	Interference Methanometer	Refractive index of methane
CO	Draeger multigas Detector with P.S. Detector	Silica gel impregnated with light yellow potassium palladium sulphite
	Draeger multigas Detector with Hopcalite	Analysing cell containing MnO_2 + Cu_2O
	Draeger multigas Detector with Hoolamite tube	I_2O_5 + H_2SO_4

3.7 Coward Diagram

Coward diagram indicates the chances of fire damp explosion for different composition of methane air mixture.

The combustion of methane in air can be expressed as

$$CH_4 + 2\ O_2 + 8\ N_2 = CO_2 + 2\ H_2O + 8\ N_2$$

It is clear from the above equation that proper oxygen balance occurs when methane content of air is 1/ 11th or 9.5 % by volume. Thus the methane content of 9.5 % in air forms the most explosive mixture.

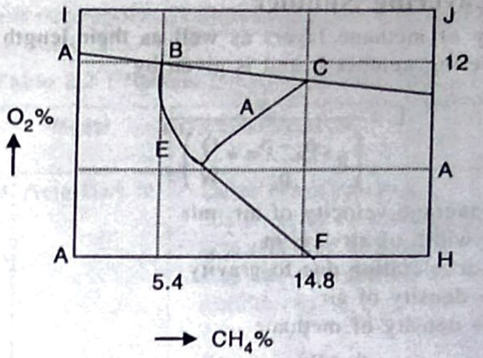


Figure 3.1 : Coward Diagram

There are four zones in the above Coward diagram shown in Figure 3.1. The characteristics of the different zones are as follows :

ABEFG : Not capable of forming explosive mixture with air

BCE : Highly explosive mixture

CEFHD : Capable of forming explosive mixtures with air contain too much methane to explode

ABCDJI : Mixture which cannot be produced from methane air

The above diagram clearly represents the limits of explosibility of the methane is 5.4% to 14.8%. The limits of explosibility expand, if methane is associated with other combustible gases such as C_2H_6, H_2 etc. depending on their relative concentration and individual limits of explosibility.

The limits of explosibility of a mixture of combustible gases can be computed from the Le Chatelier equation as :

Limits of explosibility of a mixture of combustible gases =

$$= \frac{100}{V_1 / x_1 + V_2 / x_2 + ...} \%$$

Where, V_1, V_2 + ... are the percent by volume of the different combustible components (V_1, V_2 + ...= 100%)

and x_1, x_2, ... are the limits of explosibility of the corresponding components.

3.8 Dealing with Firedamp in Mines

The gas is diluted by the ventilation current of air. The quantity and velocity of the air should be such that the gas is diluted to a safe limit. The general mechanism of dilution of gas in normal mine roadways or at the face where turbulent flow occurs is by turbulent diffusion. The distance required for complete mixing of methane in air by turbulent diffusion in a duct is given by

$$L = 22 * r / \sqrt{f}$$

Where, L = distance from source of gas
f = dimensionless resistance coefficient
r = radius of the duct

3.9 Methane Layering Number

The stability of methane layers as well as their length is indicated by a dimensionless layering number L and is given by

$$L = \frac{v}{\left(g * \frac{\rho_a - \rho_m}{\rho_a} * \frac{Q}{W}\right)^{1/3}}$$

Where v = average velocity of air, m/s
Q = width of airway, m
g = acceleration due to gravity
ρ_a = density of air
ρ_m = density of methane

$$g * \frac{\rho_a - \rho_m}{\rho_a} = 4.32$$

For L > 2, in a horizontal airway causes turbulent diffusion and breaks up the methane layer.

The index can also be determined by,

$$L = \left(\frac{24 * v^2}{c\sqrt{F}}\right)^{1/3}$$

Where v = average velocity of air, m/s
c = mean methand content of air current (% CH_4)
F = area of cross section at the measuring station, m^2

3.10 Graham's Ratio and its Importance

It is the ratio of CO produced to O_2 consumed in the mine atmosphere. The ratio indicates the spontaneous heating and its stage.

$$\text{Graham's ratio (GR)} = \frac{\text{CO produced}}{\text{Oxygen consumed}} * 100 = \frac{CO_f - CO_i}{O2_i - O2_f} * 100$$

$$= \frac{CO_i - CO_f}{\frac{20.95}{79.02} * N2_f - O2_f} * 100 = \frac{CO_i - CO_f}{0.265 * N2_f - O2_f} * 100$$

Where, O_{2i} = initial oxygen concentration (%)
N_{2f} = final nitrogen concentratior (%)
O_{2f} = final oxygen concentration (%
CO_i = initial CO concentration (%)
CO_f = final CO concentration (%)

GR → 0.1 – 0.5 % is normal to coal mine
GR → 1 % indicates existence of spontaneous heating

GR → 2 % indicates heating in advanced stage approaching active fire
GR → 3 % or more indicates active fire

3.11 Rescue/ Escape/ Reviving Apparatus

A person doing normal work breathes at the rate of 15 breath/ min and consumes 1.5 litres of O_2 /min. The rescue and reviving apparatus are also designed in such a way that it can supply 1.5 litres of O_2/min. The apparatus used in Indian mines are given in Table 3.3.

Table 3.3 : Rescue/Escape/Reviving Apparatus

Name of the apparatus	**Model**	**Characteristics**
Proto self contained compressed oxygen apparatus (S.C.B.A)	Proto Mark IV	· Closed circuit operation · 300 lit. of O_2 compressed at 150 kg/cm^2 · Feed rate 2 lit. of O_2/min · 2 kg CO_2 absorbent known as PROTOSORB [$Ca(OH)_2 + NaOH$] · 2 hrs. use · Weight 15.6 kg
	Proto Mark V	· Closed circuit operation · 300 lit. of O_2 compressed 150 kg/cm^2 · 1 hr. and 2 hrs. use capacity · Feed rate 2 lit. O_2/min in 2 hrs. apparatus and 2.5 lit. O_2/min in 1 hr. apparatus · Weight (14.5 kg for 1hr and 17.2 kg for 2 hrs.)
Drager self contained compressed oxygen apparatus (S. C. B. A	BG 174	· Closed circuit operation · Weight 12.8 kg · 4 hrs. use · 400 lit. of O_2 compressed 200 kg/cm^2 · Feed rate 1.5 lit./ min
Chemical Oxygen Breathing Apparatus	AUER/SAR - 30	· Closed circuit operation · 30 minutes use · It consists of a Canister where KO_2 is stored · O_2 needed for breathing produced from the chemical reaction as follows $2 KO_2 + H_2O \rightarrow 2 KOH + 3/2 O_2$ $2KO_2 + CO_2 \rightarrow K_2CO_3 + 3/2 O_2$
Self Rescue Breathing Apparatus (France)	SPIRAL 2C	· Closed circuit operation · Weight is 3 kg · 1 -4 hour use depends on the work performed · It consists of a Canister where KO_2 is stored · Working principle is the same as AUER/ SAR 30
Self Rescuer		· Open circuit operation i. e., O_2 is not supplied by the self rescuer

		· Weight is very low · 1 kg · It works on the principle of conversion of CO to CO_2 [Hopcalite catalyst is used] · It is used to escape during fire occurs · It can be used in place where maximum CO is 2% · Maximum 1 hr. use
Gas Mask		· Open circuit operation i. e., O_2 is not supplied by the self rescuer · Weight is very low · 1 kg · It works on the principle of conversion of CO to CO_2 [Hopcalite (MnO_2+Cu_2O) catalyst is used] · It is used to escape during fire occurs · It can be used in place where maximum CO is 2% · Maximum 1 -2 hr. use
Reviving Apparatus	Pulmometer	· It is used to supply pure O_2 to unconscious person

3.12 Air Flow through the Mine Openings:

3.12.1 Nature of Flow : There are two types of flow namely streamline & turbulent and the nature of flow is governed by a dimensionless parameter called the Reynolds number (Re).

$$Re = \frac{VD\rho}{\mu} = \frac{VD}{\nu}$$

Where, V = velocity of flow

D = diameter of the duct

ρ = density of the air

μ = viscosity of the air

$\nu = \mu/\rho$ = kinematics viscosity

For square shaped duct, D = 4A/P

A = area of cross section of the duct

P = perimeter of the duct

3.12.2 Pressure Drop in Mine Airway : Frictional pressure drop during the incompressible flow of air is given by

$$\Delta P_f = f\, LV^2\, \rho/\, 2D = f\, LPV^2\, \rho/\, 8A = k\, S\, V^2/\, A = k\, S\, Q^2/\, A^3 = R\, Q^2$$

f = Darcy-Weisbach resistance coefficient

L = length of the flow

V = velocity of the flow

ρ = density of the air

D = diameter of the duct = 4A/P

P = Perimeter of the duct

A = Area of cross section of the duct

k = coefficient of friction = f/8 [Ns^2m^{-4}]

S = Area of the rubbing surface (m^2) = L*P

Q = flow through the airway (m^3/s)

R = resistance of the path = k S/A^3

For the flow to be laminar, Re = 2000 and f is given by

$$f = 64/Re = 64^*/VD\rho$$

3.12.4 Shock Loss at Area Changes and Bends : The shock loss is given by

$$\Delta P_s = X * P_v = X^*\, \rho V^2 / 2$$

X = shock factor (dimensionless constant)

P_v = velocity pressure

3.12.4 Coefficient of Contraction : The coefficient of contraction during area changes is given by

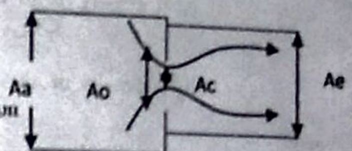
$$\text{Coefficient of contraction (C)} = \sqrt{\left(\frac{1}{Z - Z^*N_e^2 + N_e^2}\right)}$$

Z = contraction factor

NC = Aa / Ae = ratio of expansion

Ne = Ao / Ae = ratio of contraction

C = Ac / Ao = coefficient of contraction



3.13 Equivalent Resistance

o Equivalent resistance of a number of roadways in series (R_s) =

$$R_S = R_1 + R_2 + R_3 + \cdots\cdots\cdots R_n$$

o Equivalent resistance of a number of roadways in parallel (R_p)

$$\frac{1}{\sqrt{R_p}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}} + \frac{1}{\sqrt{R_3}} + \ldots\ldots + \frac{1}{\sqrt{R_n}}$$

Where,

R_S = equivalent resistance for n number of roadways situated in series

R_P = equivalent resistance for n number of roadways situated in parallel

R_1, R_2... Rn are the resistances of individual roadways

3.14 Air Power (AP)

Power of ventilation or air power is the rate at which work is done to maintain the air flow through a system and is given by

Air Power (AP) = $\Delta P * Q = R\, Q^2 * Q = R\, Q^3$ [Watt]

Where Q = quantity of air flow in m^3/s

ΔP = pressure drop in Pa.

3.15 Equivalent Orifice

The resistance of a mine airway can be expressed in terms of the equivalent orifice and is given by

$$A = \frac{Q\sqrt{\rho}}{0.6\sqrt{2P}} = \frac{1.29 * Q}{\sqrt{P}}$$

Where, A = area of equivalent orifice

Q = quantity of air flow, m^3/s

P = pressure of air flow

R = resistance of the roadways

ρ = density of air = 1.2 kg/m^3

3.16 Methods of Velocity Measurement:

- Single point measurement
- Continuous traversing
- Precise traversing

3.17 Natural and Mechanical Ventilation:

3.17.1 N.V.P – Air flow through the mine opening causes by natural means (difference in air densities in U. C and D. C. shaft) called the natural ventilation.

$$\text{N.V.P.} = D(\rho_d - \rho_u)g \qquad \text{(Pa)}$$

Where, D = depth of the shaft

ρ_d = density of air in D.C. shaft $= \dfrac{(B_d - 0.378e_d)*1000}{287.1*T_d}\text{kgm}^{-3}$

ρ_u = density of air in U.C. shaft $= \dfrac{(B_u - 0.378e_u)*1000}{287.1*T_u}\text{kgm}^{-3}$

Where,

B_d and B_u = mean barometer reading in kPa in the D.C and U.C. shafts respectively.

e_d and e_u = average vapour pressures of moisture in kPa in the D.C and U.C. shafts respectively.

Td and Tu = mean temperatures of the D.C and U.C. shafts respectively in K.

For mine shafts Bd = Bu = B and neglect the effect of moisture, the above equation can be rewritten as

$$\text{N.V.P.} = \frac{gDB(T_u - T_d)*10^3}{287.1*T_u*T_d} \text{ (Pa)}$$

3.17.2 Motive Column : Height of air column in the D.C. shaft which causes the N.V.P. termed as motive column.

$$\text{Motive Column (h)} = \frac{\text{N.V.P}}{\rho_d * g}$$

Where, h = height of motive column in m

N.V.P in Pa

ρ_d in kg/m^3

g in m/s^2

3.17.3 Head (He) Generation by Mine Fan : The velocity triangle at the inlet and outlet of a centrifugal impeller is shown in Figure 3.2. Euler's head (He) generated by the mine fan is

$$H_e = \frac{U_2(U_2 - V_{R2}\cot\beta_2)}{g} - \frac{U_1(U_1 - V_{R1}\cot\beta_1)}{g}$$

Where, U = the peripheral velocity of the impeller in m/s

V = the absolute velocity of flow in m/s

W = the relative velocity of flow of air to that of the impeller in m/s

V_R = radial component of the absolute velocity of flow

V_U = tangential component of the absolute velocity of flow

r_1 = radius of the impeller inlet in m

r_2 = radius of the impeller outlet in m

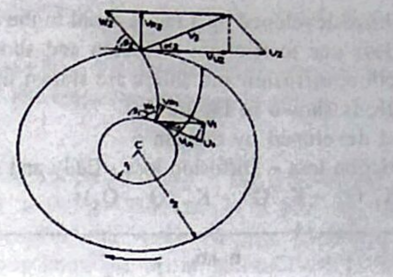


Figure 3.2 : Velocity triangles at the inlet and outlet of a centrifugal impeller

For the air has no pre-rotation before entering the impeller,

$$V_{U1} = 0 \text{ and } V_{R1} = V_1$$

Therefore the above equation can be rewritten as

$$H_e = H_{e2} = \frac{U_2(U_2 - V_{R2}\cot\beta_2)}{g}$$

In general, $H_e = \dfrac{U(U - V_R\cot\beta)}{g}$

The head characteristics curve and the Power characteristic curve are shown in Figure 3.3 and Figure 3.4.

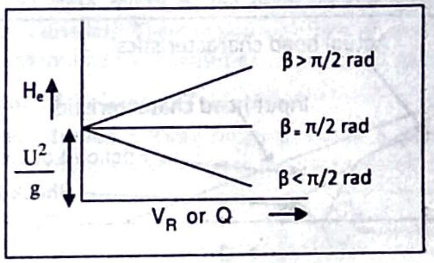


Figure 3.3 : Head vs. Quantity

Where, $U = 2\pi rn = \omega r$ [n = speed of the impeller in r.p.s]

Also, V_R = Q/ area of discharge of impeller = Q/ 2π r B

[B = width of impeller in m]

Again, Power $(P_e) = H_e*(\rho + g)*Q = H_e*K*V_R = \left(\dfrac{UV - UV_R\cot\beta}{g}\right)KQ$

[Where, K is constant = ρg and $V_R \infty Q$ for a particular impeller]

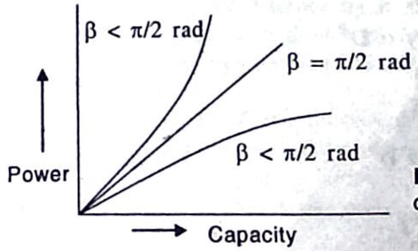


Figure 3.4 : Power vs. capacity characteristics

The actual head developed by a fan is equal to the theoretical head developed minus the head loss due to friction, diffusion and shock. The various hydraulic losses due to friction, diffusion and shock are shown in Figure 3.5 and the actual head characteristic is shown in Figure 3.6.

Actual head developed by the fan =

$H = H_i$ - Friction loss – Diffusion loss - Eddy and separation loss (shock loss)

$= H_i - K_1 Q^2 - K_2 Q^2 - K_3 (Q - Q_S)^2$

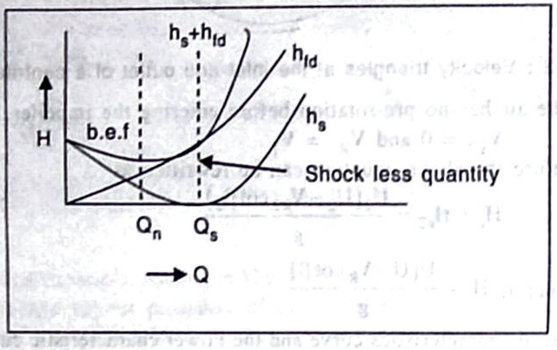


Figure 3.5 : Hydraulic losses in an impeller

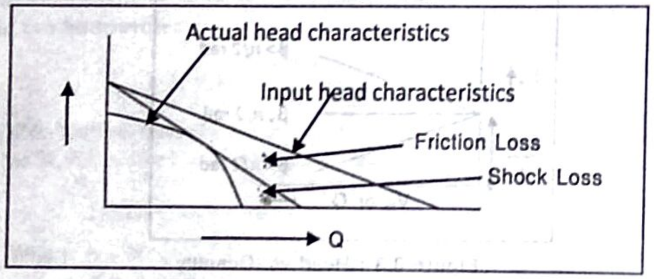


Figure 3.6 : Actual head characteristics of a centrifugal fan

3.17.4 Fan Laws: Fan laws are defined as

Law 1 : Head α D^2

- $\frac{H_1}{H_2} = \frac{n_1^2 * D_1^2}{n_2^2 * D_2^2}$

Law 2 : Quantity α D^3

- $\frac{Q_1}{Q_2} = \frac{n_1 * D_1^3}{n_2 * D_2^3}$

Law 3 : Power α D^5

- $\frac{P_1}{P_2} = \frac{n_1^3 * D_1^5}{n_2^3 * D_2^5}$

Where, H = head; Q = Quantity; P = power; n = speed, D = diameter of the impeller

[The above relations are true for geometrically similar impeller, i.e. the diameter and the width of the impeller changes in the same proportion]

3.17.5 Ventilation Survey

In underground mines regular monitoring of the quality and quantity of air is necessary to maintain the working conditions as it not only deteriorates the workers performance but also effect the workers health. A number of parameters required to monitor regularly to know the working conditions in the underground area. These are

- dry and wet bulb temperature
- velocity of air
- quantity of air
- presence of noxious gases in the mine atmosphere
- barometric pressure
- pathogenic dust concentration
- humidity

As par Indian mine regulations, in all mechanically ventilated mines, quantity measurements should be made every 14 days in case of gassy coal mines and every 30 days in case of non-gassy coal mines. The measurements should be made in the main intake and main return of the seam, in the splits near the starting points and in ventilation districts. There is also provision of monitoring temperature, humidity and other environmental conditions in every 30 days.

3.17.6 Measurement of Velocity of Air Flow

- **Smoke Cloud Method :** Time taken to cover a particular distance S, shown in Figure 3.7, by smoke has been recorded and velocity can be calculated by

$$V = S/T$$

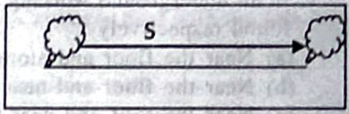


Figure 3.7 : Smoke Cloud Method

- Hot-wire Anemometer: This is a very sensitive instrument for measuring low air velocities.
- Vane Anemometer: This simply gives the distance travelled by the vanes set in the anemometer at a particular duration of time. The velocity thus calculated indirectly by dividing the distance by time.
- Velometer: The velometer consists of a double-pivoted spring-balanced aluminium vane mounted in a jeweled bearing bakelite case. This directly gives the velocity of air movement.
- The Pitot-static tube: It comprises of two concentric tubes, the outer of which has a few holes on the sides. The annular opening between the two tubes at the nose end is sealed so that the inner tube records the total pressure and the outer tube records the static pressure respectively. The

velocity pressure can be calculated by subtracting the static pressure from the total pressure. The velocity of the air can be calculated by the following formula

$$V = \sqrt{\frac{2P_v}{K\rho}}$$

Where, V = velocity of air in m/s
P_V = velocity pressure in Pa
ρ = density of air in kg/ m^3
K = correction factor [for standard design $K = 1$]

3.17.7 Measurement of Quantity of Air Flow : This requires the measurement of average air velocity at any point of mine airway and the cross-sectional area of the airway at that point. The quantity of air flow can be calculated by multiplying the above two factors. The average velocity can be calculated by multiplying the air velocity monitored at the centre of the duct with the method factor.

3.17.8 Measurement of Relative Humidity : This is the ratio of mass of water vapour present per unit volume of air to the mass of water vapour required to saturate the unit volume of air. This can be directly measured with the help of Hygrometer. It can also be calculated from the reading of dry bulb and wet bulb temperature.

Relative Humidity =
[100 – 7* (D.B.T. – W.B. T)] [For D.B.T. > 25°C]
[100 – 8* (D.B.T. – W.B. T)] [For 20°C < D.B.T. < 25°C]
[100 – 9* (D.B.T. – W.B. T)] [For D.B.T. < 20°C]

EXERCISE - 3

1. In an underground working area, CH_4 and CO_2 are normally expected to be found respectively
   (a) Near the floor and along the roadway
   (b) Near the floor and near the roof
   (c) Near the roof and near the floor
   (d) Along the roadway and near the floor
2. The oxygen concentration in u/g working should not be less than
   (a) 21 % (b) 20 %
   (c) 19 % (d) 18 %
3. In the Coward flammability diagram, the respective percentages of methane and oxygen at the nose limit are
   (a) 14.2, 0.0 (b) 14.1, 18.2
   (c) 5.8, 12.1 (d) 5.0, 19.2
4. The maximum wet bulb temperature in u/g mines is
   (a) 30°C (b) 30.5°C
   (c) 33°C (d) 33.5°C
5. The wet bulb temperature of air at a particular place in a mine is 31°C, then the velocity of air in that place should be

   (a) more than 1 m/s (b) less than 1 m/s
   (c) more than 2 m/s (d) less than 2 m/s
6. Effective temperature is estimated from
   (a) Wet-bulb temperature, relative humidity, and air velocity
   (b) Dry-bulb temperature, relative humidity, and air velocity
   (c) Dry-bulb temperature, Wet-bulb temperature, and air velocity
   (d) Dry-bulb temperature, Wet-bulb temperature, and relative humidity
   [GATE 2008]
7. Match the following

Mine gas	Principal constituent
P Stink damp	1 CO
Q White damp	2 H_2S
R Black damp	3 CH_4
S Fire damp	4 CO_2

   (a) P-1, Q-2, R-3, S-4 (b) P-3, Q-4, R-1, S-2
   (c) P-2, Q-1, R-4, S-3 (d) P-2, Q-1, R-3, S-4
   [GATE 2012]
8. Quantity of air to be circulated to u/g workings depends on
   (a) Manpower employed in a largest shift
   (b) Manpower employed in a day
   (c) Total production in a day
   (d) Both (a) and (c)
9. Quantity of air to be circulated to u/g workings where 150 worker employed in a largest shift and daily production 500 te will be
   (a) 900 m^3/ min (b) 1250 m^3/min
   (c) 900 m^3/ sec (d) 1250 m^3/sec
10. A Roadheader district produces 20 mg/m^3 of airborne dust with the following size distribution :

Size up to	Cumulative wt%
1 μm	1
5 μm	5
10 μm	10
20 μm	20
50 μm	50
>50 μm	100

The concentration of respirable fraction of dust in mg/m^3 is
a. 0.2 b. 2.0
c. 10.0 d. 1.0
[GATE 2008]

11. For a person working in an atmosphere containing 21 % O2, the exhaled air contains 4.5 % CO2 and 16 % O2. The respiratory quotient of breathing is
a. 0.21 b. 0.9
c. 0.28 d. 1.11
[GATE 2008]

12. An air sample produces 8 % CH_4 and 3 % H_2. If the lower flammability of CH_4 is 5 % and H_2 is 4 %, what is the lower flammability of the mixture?
(a) 9.50 % (b) 7.16 %
(c) 4.68 % (d) 3.90 %

13. 20 m^3/s of air flow through a longwall panel having 1000 tpd of production. Between the intake and return of the panel, the firedamp increases from 0 to 0.2 %. The rate of firedamp emission is
(a) 3.45 m3/day (b) 3.45 m3/s
(c) 3.45 m3/t (d) 3.45 l/g

14. A man exhales 0.002 m^3 of CO_2 per min when working in a confined space in underground. Assuming that the fresh air contains 0.03 % CO_2, the minimum quantity of fresh air to be supplied per person in m3/min, so that concentration of CO_2 in air never crosses 0.4 %, will be
(a) $0.5 * 10^{-3}$ (b) $5.4 * 10^{-3}$
(c) $540 * 10^{-3}$ (d) $66 * 10^{-3}$

15. A mine worker inhales normal air; whereas, the exhaled air contains 16.65% O_2 and 3.83% CO_2. The respiratory quotient of breathing for the worker is
(a) 0.23 (b) 0.89
(c) 0.99 (d) 1.13 [GATE 2013]

16. For Indian coal mines, the 'maximum allowable concentration' of respirable dust containing 7.5% free silica in mg/m3 is
(a) 2.0 (b) 2.2
(c) 2.5 (d) 2.7 [GATE 2014]

17. Air at dry bulb temperature of 27°C enters a 1200 m deep shaft. The specific heat of air-water vapour mixture is 1.03 KJ/kg °C. If the air goes through constant-moisture adiabatic auto compression, the air temperature at the bottom of the shaft in °C is
(a) 11.4 (b) 33.1
(c) 35.4 (d) 38.4

18. A gas mixture contains CH_4, C_2H_6 and H_2 with respective concentration of 75%, 15% and 10% by volume. The lower explosibility limit of CH_4, C_2H_6 and H_2 are 5.0%, 3.3% and 4.2% respectively. The lower explosibility limit of the gas mixture, in percentage, is
(a) 5.01 (b) 5.36
(c) 4.68 (d) 4.56

19. The mean velocity of air current through an airway of 9 m^2 cross-sectional area is found to be 1.5 m/s. If the mean methane content of air current is 0.6%, the layering index will be
(a) 3.10 (b) 2.71
(c) 2.15 (d) 1.80

20. Air of 20.8 % O_2 concentration enters a coal mine panel at 20 m^3/s and exits with 20.6 % O_2. The oxygen depletion is due to oxidation of carbon in spontaneous heating process only, assuming the density of oxygen as 1.3 kg/m3 and the calorific value of carbon as 34 MJ/kg, the heat generated in kW from the panel due to spontaneous oxidation is
(a) 533 (b) 640
(c) 663 (d) 693

21. A sealed area known to have black damp has 18 % O_2, 2 % CO_2, and 78% N_2. If fresh air has 21 % O_2, 0.03 % CO_2 and 78.97 % N_2, the percentage of black damp in the area is
(a) 10.31 (b) 12.28
(c) 12.31 (d) 14.28

22. Which one of the following composition of methane in air is most explosive in nature?
(a) 8.5 % by volume (b) 9.5 % by volume
(c) 10.5% by volume (d) 11.5% by volume

23. The percentage of inflammable gas in an u/g coal mine should not exceed
(a) 0.75 % in the general body of the return air of any ventilation district and 1.5 % in any place of the mine.
(b) 0.75 % in the general body of the return air of any ventilation district and 1.25 % in any place of the mine.
(c) 0.5 % in the general body of the return air of any ventilation district and 1.25 % in any place of the mine.
(d) 0.5 % in the general body of the return air of any ventilation district and 1.5 % in any place of the mine. [CIL 2001, 2007]

24. A flammable mixture has 70% CH4 and 30% CO. The lower flammability limits for these gases are 5% and 13% respectively. For the mixture, the lower flammability limit in % is
(a) 6.13 (b) 8.72
(c) 10.25 (d) 12.16 [GATE 2010]

25. An underground coal mine panel produces 520 tonnes per day deploying 220, 200 and 192 persons in three shifts. As per CMR 1957, the minimum quantity of air in m3/min to be delivered at the last ventilation connection of the panel is
(a) 1120 b) 1320
(c) 1220 (d) 1550 [GATE 2014]

26. If the CO/O_2 deficiency ratio in a mine is 2%, it indicates
(a) existence of spontaneous heating
(b) active fire (c) heating in advanced stage
(d) None of the above [CIL 2001]

27. An air sample taken from the return airway of a district contains the following gases. The Graham's ratio for the district is

Gas	CO_2	H_4	O_2	N_2	CO
Concentration %	0.4	1.17	19.92	78.49	0.02

(a) 5.6 (b) 4.8
(c) 3.0 (d) 2.3 [GATE 2011]

28. Intake air containing 0.2% methane enters a section of an underground mine where emission of methane is $0.05 m^3/s$. Assuming that the threshold limit value of methane is 1.25%, the minimum quantity of fresh air required in m^3/s is
(a) 4.70 (b) 8.70
(c) 6.70 (d) 1.50 [GATE 2015]

29. In an underground coal mine area the graham's ratio found 3% is the indication of
(a) existence of spontaneous heating (b) active fire
(c) heating in advanced stage (d) normal to the coal mine

30. The early indication of spontaneous heating of coal can be given by
(a) Sweating of strata (b) Formation of CO_2
(c) Rise of temperature (d) None of the above [CIL 2001]

31. Ignition temperature of methane is
(a) 100 – 1250°C (b) 923 – 1023°C
(c) 675 -975°C (d) 423 – 1000°C

32. The lower and upper limit of methane – air explosive mixture is
(a) 0.75%, 1.25% (b) 5.4%, 14.8%
(c) 5.0%, 15.0% (d) 5.9%, 12.5%

33. Geothermic gradient is
(a) Increase in temperature with depth
(b) Increase in pressure with depth
(c) Decrease in temperature with depth
(d) Decrease in pressure with depth

34. Which of the following instrument is used to measure the cooling power of the air?
(a) Anemometer (b) Velometer
(c) Manometer (d) Kata thermometer

35. Which one of the following is used as a reviving apparatus in u/g coal mine?
(a) Draeger BG 172 (b) Pulmometer
(c) Any open circuit rescue apparatus
(d) MSA self rescuer

36. Which one of the following is used as an escape apparatus in u/g coal mine?
(a) Draeger BG 172 (b) Pulmometer
(c) Any open circuit rescue apparatus (d) MSA self rescuer

37. The chemical Hopcalite used in
(a) Self contained breathing apparatus (b) Gas mask
(c) CO- detector (d) MSA- methanometer

38. The chemical protosorb used in
(a) Self contained breathing apparatus (b) Gas mask
(c) Reviving apparatus (d) MSA- methanometer

39. A gas mask does not include
a. Check valve b. Warning device
c. Face piece assembly d. Coolant canister [GATE 2009]

40. Bypass valve in a compressed oxygen type self-contained breathing apparatus is meant to
(a) release accumulated nitrogen in the breathing bag
(b) release excess pressure in the breathing bag
(c) supply oxygen directly to wearer in case pressure reducing valve does not function
(d) flush out the apparatus with oxygen on opening the cylinder valve [GATE 2014]

41. A coolant is desirable component in the design of a self- contained breathing apparatus since
(a) Surroundings can be hot and humid during rescue
(b) A rescue worker generates large amount of metabolic heat
(c) Exhaled air CO2 aboprtion is an exothermic reaction
(d) Exhaled air water vapour has been condensed. [GATE 2010]

42. Determine the correctness or otherwise of the following Assertion [a] and the Reason [r].

Assertion : Both intake and return side stoppings must closed simultaneously in the event of sealing off a coal mine panel with explosion hazard following a fire.

Reason : By continuously ventilating the area till simultaneously closure of the stoppings, the possibility of an explosion hazard due to gas built is avoided.

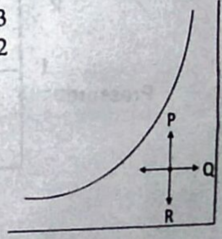
(a) [a] is true but [r] is false
(b) Both [a] and [r] are true and [r] is the correct reason for [a]
(c) Both [a] and [r] are true and [r] is no the correct reason for [a]
(d) Both [a] and [r] are false. [GATE 2010]

43. Given the following,

Rescue apparatus	**Characteristic**
P. Draeger BG-4	1. Open circuit chemical oxygen self-rescuer
Q. MSA IW-65	2. Filter type self-rescuer
R. Draeger Pulmotor	3. Self-contained breathing apparatus
S. Oxvoks	4. Resuscitation apparatus

The correct match is
(a) P-3, Q-2, R-1, S-4 (b) P-4, Q-1, R-2, S-3
(c) P-3, Q-2, R-4, S-1 (d) P-1, Q-4, R-3, S-2

44. Four psychrometric processes P, Q, R and S are shown in the psychrometric chart below.



These processes respectively represent

(a) dehumidification, humidification, sensible heating, sensible cooling
(b) sensible heating, humidification, dehumidification, sensible cooling
(c) dehumidification, sensible heating, sensible cooling, humidification
(d) humidification, sensible heating, dehumidification, sensible cooling

45. Match the following in the context of underground mine environment [GATE 2013]

Instrument	Measuring parameter
P. Haldane apparatus	I. Humidity
Q. Godbert-Greenwald apparatus	II. Air velocity
R. Hygrometer	III. Mine air composition
S. Anemometer	IV. Ignition point temperature

(a) P-II, Q-I, R-III, S-IV (b) P-III, Q-IV, R-I, S-II
(c) P-IV, Q-II, R-III, S-I (d) P-I, Q-III, R-IV, S-II [GATE 2014]

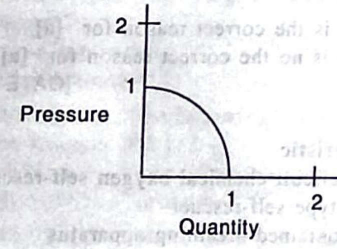
46. A Drager Gas Mask does not filter
(a) water vapour (b) nitrous fumes
(c) carbon monoxide (d) carbon dioxide [GATE 2012]

47. Match the followings

Component of flame safety lamp	**Purpose of component**
P- Asbestos ring	1. Dissipation of heat of flue gas
Q- Wire gauges	2. Formation of air-tight joints
R- Outer glass	3. Arrest of explosion inside the lamp
S- Combustion chimney	4. Separation of inlet air from flue gas

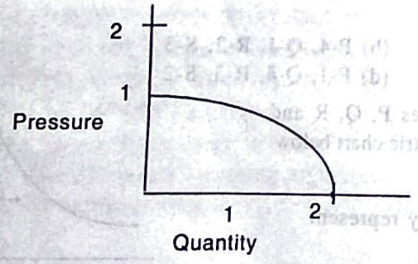
(a) P-2, Q-1, R-3, S-4 (b) P-4, Q-1, R-2, S-3
(c) P-2, Q-4, R-3, S-1 (d) P-1, Q-2, R-4, S-3 [GATE 2008]

48. Pressure-quantity characteristics of a mine fan is given below :

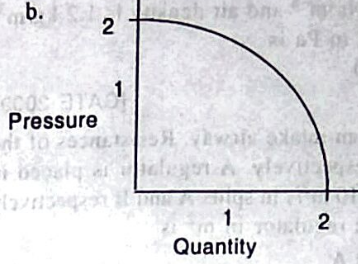


The combined characteristic of two such identical fans installed in parallel is

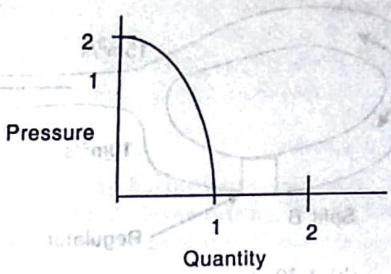
a.



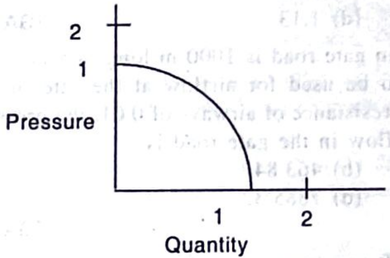
b.



c.



d.



[GATE 2008]

49. Moody diagram represents resistance coefficient in terms of
(a) Reynolds number and asperity ratio
(b) Viscosity and aspect ratio
(c) Surface tension and viscosity
(d) Reynolds number and surface tension [GATE 2009]

50. In a mine ventilation system, the resistances of two splits A and B are 0.5 Ns^2m^{-8} and 2 Ns^2m^{-8} respectively. Combined resistance of two shafts and trunk airways is 0.7 Ns^2m^{-8}. A quantity of 20 m^3/s of air passes through split A. The total air quantity passing the mine in m^3/s is
(a) 30 (b) 27
(c) 25 (d) 17 [GATE 2008]

51. In the above question, the total air power of the ventilation system in kW are
(a) 82.9 (b) 48.9
(c) 24.9 (d) 27.9 [GATE 2008]

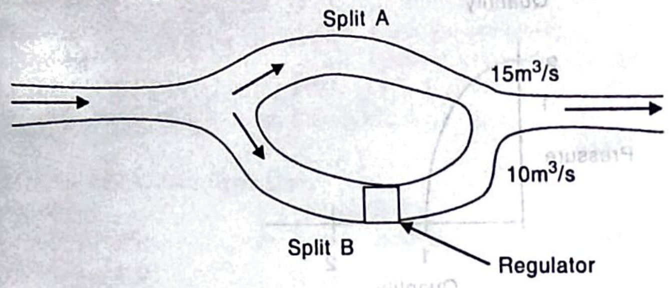
52. Air flows at 2 m^3/s through a forcing fan duct of 0.3 m^2 having uniform

cross-section. The duct resistance is 40 Ns^2m^{-8} and air density is 1.2 kg/m^3. The total pressure generated by the fan in Pa is

(a) 186.7 (b) 160.0

(c) 133.3 (d) 26.7 [GATE 2009]

53. Two splits A and B are ventilated from an intake airway. Resistances of the splits are 0.5 Ns^2m^{-8} and 0.8 Ns^2m^{-8} respectively. A regulator is placed in split B to maintain a flow of 15 m^3/s and 10 m^3/s in splits A and B respectively as shown in the figure. The size of the regulator in m^2 is



(a) 2.10 (b) 1.30

(c) 1.20 (d) 1.13 [GATE 2009]

★ 54. In a longwall panel, the main gate road is 1000 m long, 4.5 m wide and 2 m high. The gate road is to be used for airflow at the rate of 17 m3/s. Considering a coefficient of resistance of airways of 0.01, the pressure in Pa required to maintain the airflow in the gate road is

(a) 51.83 (b) 463.84

(c) 875.98 (d) 7885.32

(e) 7.73 [GATE 2012]

55. In a mine between upcast shaft and downcast shaft, two airways are connected in parallel and their resistances are 100 and 120 $Ns^{-2}m^{-8}$ respectively. The resistance of upcast shaft, downcast shaft and the fan drifts are 10, 20, and 5 $Ns^{-2}m^{-8}$ respectively. The fan drift air pressure is 15 MN/m^2. The rate of airflow through the mine in m^3

(a) 4.16 (b) 18.26

(c) 240.35 (d) 333.33

(e) 490.56 [GATE 2012]

56. In the above question, the rate of airflow through the split airway having resistance of 100 $Ns^{-2}m^{-8}$ in m^3/s is

(a) 0.42 (b) 0.79

(c) 2.19 (d) 7.90

(e) ~~270.06~~ 256.96 [GATE 2012]

★ 57. A pitot tube is inserted in a ventilation duct with the nose facing the air flow. A vertical U-tube manometer filled with alcohol (specific gravity 0.8) has been used for pressure measurements such that 10.2 mm is read as the total pressure and 8.8 mm as the static pressure. Given the density of air to be 1.2 kg/m^3, the air velocity at the nose of the pitot tube in m/s is

(a) 4.27 (b) 6.53

(c) 3.29 (d) 5.67 [GATE 2014]

58. The total pressure and the static pressure measured at a point in a ventilation duct are 20 mm and 10 mm of water gauge respectively. If density of air is 1.2 kg/m^3, the velocity of the air in m/s is

(a) 14.08 (b) 12.78

(c) 8.53 (d) 6.24 [GATE 2011]

59. Air samples collected from the intake and the return gates of a retreating longwall face show methane concentration values of 0.1% and 0.8% respectively. The production from the longwall face is 2000 tonne/day and the air quantity circulating the face is 15 m^3/s. The rate of methane emission in m3 per tonne of coal produced is

(a) 11.0 (b) 9.5

(c) 5.5 (d) 4.5 [GATE 2011]

60. A mine airway having cross-section of 2.2 m × 2.2 m and length 500 m contains a bend. Given that the airway friction factor is 0.01 Ns^2m^{-4}, shock loss factor for the bend is 0.07 and density of air is 1.2 kg/m^3, the equivalent length of the airway in m is

(a) 502.31 (b) 622.51

(c) 534.51 (d) 512.31 [GATE 2014]

61. Ascensionally ventilated coal mine inclines ideally should have a higher methane layering number when compared to descensionally ventilated inclines. The reason is

(a) In ascensionally ventilated inclines density of air is higher

(b) Ascensionally ventilated inclines created conditions for improved turbulent mixing of methane layer.

(c) Methane drainage is not practiced in ascensionally ventilated incline

(d) Descensionally ventilated incline creates conditions for improved turbulent mixing of methane layer.

[GATE 2010]

62. In order to estimate the NVP in a mine, measurements are made at the main fan as shown below.

Fan speed (RPM)	Fan drift pressure (Pa)	Fan quantity (m^3/s)
800	655	82.2
950	730	85.5

The NVP in Pa is

(a) 156.84 (b) 256.84

(c) 236.84 (d) 246.84 [GATE 2014]

63. Mean air temperature of a 450 m deep downcast shaft is 29°C and that of the upcast shaft is 37°C. The height of the motive column in m is

(a) 8.2 (b) 9.5

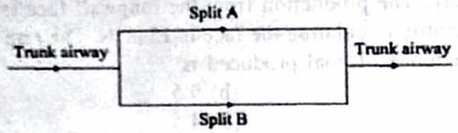
(c) 11.6 (d) 12.8 [GATE 2011]

64. The resistances of two splits A and B are 0.35 Ns^2m^{-8} and 0.05 Ns^2m^{-8} respectively. The combined resistance of the shafts and trunk airways is 0.4

Ns^2m^{-8}. A booster fan is planned to be installed in split A to increase the quantity flowing through it. Assuming that the surface fan continues to operate at a constant pressure of 1000 Pa, the critical pressure of the booster fan in Pa is

(a) 675 (b) 875
(c) 825 (d) 625 [GATE 2014]

65. A mine ventilation system consists of two splits A and B with resistances of 0.8 Ns^2m^{-8} and 3.2 Ns^2m^{-8}, respectively as shown in figure. Trunk airways have resistance of 0.2 Ns^2m^{-8}. The main mine fan is generating pressure of 500 Pa.



The air quantities in m3/s circulated in the splits A and B respectively are

(a) 20 and 30 (b) 30 and 20
(c) 20 and 10 (d) 40 and 10 [GATE 2011]

66. In the above question, the flows in the two splits are equalized by placing a booster fan in split B. Assume that the fan pressure does not change after installation of the booster fan. The size of the booster fan in Pa is

(a) 749.05 (b) 850.08
(c) 950.02 (d) 1000.50 [GATE 2011]

67. Oxygen flow rate in the self contained breathing apparatus Proto Mark IV is

(a) 1 L/ min (b) 1.5 L/ min
(c) 2 L/ min (d) 2.5 L/ min

68. Samples of air collected in the intake and return gate roads of an advancing longwall face show 0.25 % and 0.75 % CH_4, respectively. The panel has a production of 400 t in an 8 hour shift when an air quantity of 18 m^3/ s passes through it. Then the methane emission in m^3/ te of coal mined is

(a) 0.02 (b) 0.81
(c) 2.25 (d) 6.48

69. A toxic gas flows into a mine working place at the rate of 2.52 m^3/min. The concentration of the gas in the intake air is 0.25%. The minimum quantity of intake air in m3/min required to dilute the gas to its threshold limit value of 1.0 % is

(a) 123 (b) 252
(c) 295 (d) 336 [GATE 2011]

70. For the same roadway roughness, area, length and velocity condition the ratio of resistances of a square roadway to that of a rectangular roadway having width and height ratio, 2 : 1 is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2\sqrt{2}}{3}$

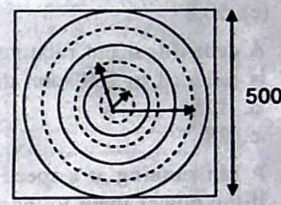
(c) $\frac{3\sqrt{2}}{2}$ (d) $\frac{\sqrt{2}}{3}$

71. In a coal mine district, air flows at a rate of 30 m^3/ s at a pressure of 1200 Pa. The density of air is 1.2 kg /m^3. The mine management decided to reduce the flow rate of air to 20 m^3/s using a regulator without changing the ventilation pressure. The area of regulator expressed in m^2 will be

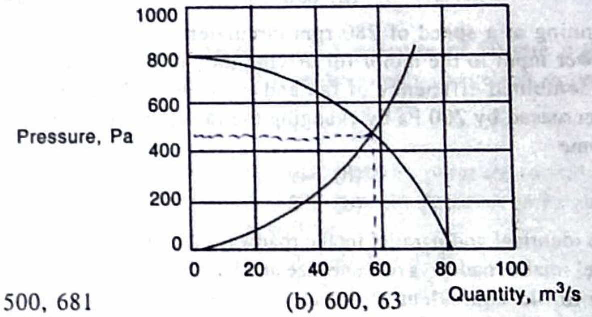
(a) 0.99 (b) 0.77
(c) 0.71 (d) 0.60 [GATE 2004]

72. While measuring average air velocity of air flowing through a circular duct using precise traversing with equal area method, the radii r_1, r_2 and r_3, in mm, as shown in the figure of the three circles should be

(a) 83, 167 and 240
(b) 36, 107 and 261
(c) 102, 177 and 228
(d) 108, 157 and 216



73. The main fan operating point of a ventilation system is shown in the figure below. If an NPV of 200 Pa assists the ventilation system, the resultant pressure (Pa) and quantity (m^3) generated by the



(a) 500, 681 (b) 600, 63
(c) 640, 55 (d) 400, 63
(e) 640, 66 [GATE 2010]

74. Air at a density of 1.2 kg/m3 flows in a straight duct such that the velocity at the centre is 12.5m/s. the method factor for the velocity profile is known to be 0.80. The velocity pressure value in the duct in Pa is

(a) 31 (b) 47
(c) 60 (d) 83 [GATE 2010]

75. In the above question, the air flow encounters a symmetric expansion such that the cross- sectional are of the duct becomes double. The static pressure value at the inlet and outlet of the expansion are 60 Pa and 90 Pa respectively. Neglecting friction the shock pressure loss on account of expansion in Pa is

(a) 15 (b) 22
(c) 38 (d) 46 [GATE 2010]

76. Pressure characteristic of a mine fan is given by, $P = -0.06Q^2 + 400$, where P is the pressure in Pa and Q is the quantity in m3/s. The resistance of the mine is 0.19 Ns^2/m^8. The mine quantity in m^3/s is
(a) 160.0 (b) 56.57
(c) 45.9 (d) 40.0 [GATE 2013]

77. Pressure characteristic of a mine fan is given by, $P = -0.06Q^2 + 400$, where P is the pressure in Pa and Q is the quantity in m^3/s. The resistance of the mine is 0.19 Ns^2/m^8. An identical fan is installed in the mine to operate in series with the existing fan. The new mine quantity in m^3/s is
(a) 75.6 (b) 56.7
(c) 50.8 (d) 30.2 [GATE 2013]

78. A centrifugal fan rotating at 500 rpm delivers 70 m^3/s of air. If the speed is reduced to 200 rpm, the quantity of air delivered in m^3/s will be
a. 175 b. 55
c. 28 d. 11 [GATE 2011]

79. A fan running at a speed of 280 rpm circulates 105 m^3/s of air in a mine. If the power input to the motor for driving the fan is recorded to be 75 kW, with the combined efficiency of fan and motor at 70%, the fan pressure in Pa is
(a) 50 (b) 350
(c) 500 (d) 650 [GATE 2009]

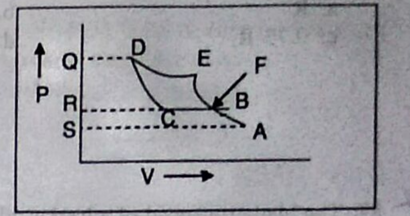
80. A fan running at a speed of 280 rpm circulates 105 m^3/s of air in a mine. If the power input to the motor for driving the fan is recorded to be 75 kW, with the combined efficiency of fan and motor at 70%. If the fan pressure is to be increased by 200 Pa by changing the fan speed, the fan speed in rpm will become
(a) 768 (b) 549
(c) 392 (d) 332 [GATE 2009]

81. When two identical and parallel intake roadways are changed to three identical and parallel intake roadways of same size and length, the ratio of new equivalent resistance to old equivalent resistance will be
(a) 0.44 (b) 0.67
(c) 1.50 (d) 0.82

82. A mine fan delivers 50 m^3/s of air at a fan drift of 10 mm of w.g. It is proposed to increase the quantity to 60 m^3/s of air, the required w.g. shall be
(a) 12.0 mm (b) 14.4 mm
(c) 13.6 mm (d) 20.2 mm [CIL 2001]

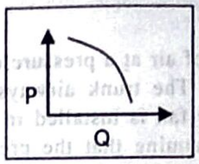
83. The figure shows PV (pressure-volume) diagram of a mine ventilated by both natural means and an exhaust fan. The total ventilation energy is represented by

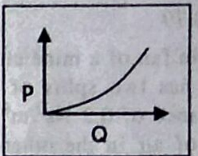
(a) area ABF + area CDEF
(b) area ABRS + area CDEF
(c) area ABRS + area QDCR
(d) area ABF + area QDCR

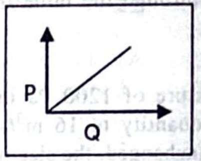


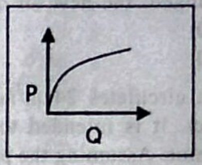
[Gate 2004]

84. The plot of theoretical power consumed against quantity delivered by a forward bladed centrifugal fan is best represented by

a.


b.


c.


d.


[Gate, 04]

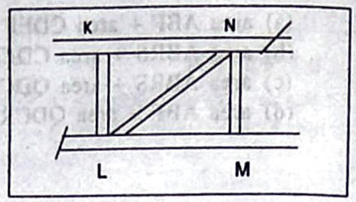
85. A fan running at 200 rpm develops a pressure of 50 mm water gauge; the fan speed is increased to 300 rpm. the water gauge developed will be
a. 75 mm b. 112.5 mm
c. 164 mm d. 97 mm

86. A mine fan produces 30 m^3/s of air at 240 Pa pressure running at 1400 rpm. If the fan speed is increased to 2100 rpm, the pressure in Pa and the quantity in m^3/s generated by the fan will be
a. 540 and 67.5 b. 540 and 45.0
c. 360 and 45.0 d. 540 and 30.0 [GATE, 04]

87. The fan drift pressure readings and fan quantities are measured to be 500 Pa and 65 m^3/s respectively at the time when NVP aids fan pressure. On reversal, the fan delivers a quantity of 50 m^3/s while acting against NVP with fan drift pressure of 600 Pa. Assuming that the NVP remains constant in both cases the total resistance of the mine in Ns^2/m^8 will be
a. 0.9487 b. 0.2750
c. 0.4105 d. 0.1635 [GATE, 04]

88. The resistances of each branches KL, LM, MN, NK is R and that of LN is 1.414 R. the equivalent resistance between K and M is

a. R b. 0.5 R
c. 0.75 R d. 2 R



89. The blade tip speed of a backward bladed centrifugal fan running at 300 rpm delivers 120 m3/s of air. If the tip speed of the blade is 47.12 m/s and width of the fan is 1.3 m then the radial velocity of air at the periphery of the impeller in m/s is
a. 9.79 b. 12.73
c. 30.79 d. 40.22 [GATE, 04]

90. A main fan of a mine circulates 40 m3/s of air at a pressure of 500 Pa. the mine has two splits of equal resistance. The trunk airways have a total resistance of 0.2 Ns^2/m^8. When a booster fan is installed in one split, the flow of air in the other splits stops. Assuming that the pressure remains unchanged, the new air quantity flowing through the mine in m^3/s is
a. 20 b. 30
c. 40 d. 50 [GATE, 06]

91. A fan circulates 24 m^3/s of air at a pressure of 1200 Pa in a ventilation district. It is intended to reduce the air quantity to 16 m^3/s by placing a regulator. Assuming the pressure remains unchanged, the size of the regulator in m^2 is
a. 1.48 b. 0.74
c. 0.37 d. 0.18 [GATE 2011]

92. An exhaust fan attached to an evasee of 18 m^2 cross-sectional area at the outlet circulates 150 m^3/s of air at the pressure of 1000 Pa in a mine ventilation system. The ratio of the inlet to outlet area of the evasee is 1:4 and the density of air is 1.2 kg/m^3. The quantity of air circulated in the mine in absence of evasee is 120 m^3/s. The evasee efficiency in % is
a. 57.6 b. 43.2
c. 39.06 d. 37.7 [GATE 2011]

93. Given the following,

Equation/formula/law	**Application**
P. Bernoulli equation	1. Pressure loss in laminar flow of fluid
Q. Poiseuille equation	2. Drag loss due to regular obstructions in fluid flow
R. Bromilow's formula	3. Energy conservation in ideal fluid flow
S. Stokes law	4. Terminal settling velocity of fine particles in fluid

the correct match is
a. P-3, Q-1, R-2, S-4 b. P-1, Q-3, R-2, S-4
c. P-2, Q-3, R-4, S-1 d. P-3, Q-1, R-4, S-2
[GATE, 2013]

94. The type of fire extinguisher that must NOT be used in case of fire in an electric substation located in an underground metal mine is
a. multi-purpose dry chemical extinguisher
b. CO_2 snow extinguisher
c. dry chemical powder extinguisher
d. foam extinguisher s[GATE, 2011]

95. Proximate analysis of 50 g of a coal sample shows the following:
Moisture = 0.80 g
Ash = 7.85 g
Volatile matter = 15.90 g
The fixed carbon in percentage on a dry, ash free basis is
a. 83 b. 66
c. 55 d. 45 e. 61.55 [GATE 2011]

Answer

1. c	2. c	3. c	4. d	5. a	6. c
7. c	8. d	9. ~~d~~ b	10. d	11. b	12. c
13. c	14. c	15. b	16. a	17. d	18. a
19. a	20. c	21. [illegible] b	22. b	23. b	24. a
25. b	26. c	27. d	28. a	29. b	30. a
31. b	32. b	33. a	34. d	35. b	36. c
37. b	38. a	39. d	40. c	41. a	42. c
43. c	44. d	45. b	46. b	47. a	48. a
49. a	50. a	51. c	52. a	53. a	54. e
55. e	56. e	57. a	58. b	59. d	60. a
61. d	62. b	63. c	64. b	65. c	66. a
67. c	68. d	69. d	70. b	71. a	72. c
73. e	74. c	75. a	76. d	77. b	78. c
79. c	80. d	81. a	82. b	83. a	84. b
85. b	86. b	87. d	88. b	89. a	90. d
91. b	92. a	93. a	94. d	95. e	

Solution of Exercise 3

4. Refer CMR No. 133(IV)
Wet bulb temperature should be 33.5°C

5. Refer CMR No. 133(IV)
If the wet bulb temperature in a mine exceeds 30°C, the air velocity should be more than 1 m/s

8. Refer CMR No. 133(I)
Quantity of air to be circulated in a mine should be the largest of 6 m^3 per minute per person or 2.5 m^3 per minute per day tonne output.

9. Given data,
Number of workers employed (N) = 150

Daily production (Q) = 500 te
Therefore, According to CMR 133(I)

Quantity of air require to circulate on the basis of number of workers (q_1) = N*6

= 150*6 = 900 m^3/min

Again, quantity of air require to circulate on the basis of number of workers (q_2) = Q*2.5 = 500*2.5 = 1250 m^3/min

Therefore, q = maximum (q_1, q_2) = 1250 m^3/min

10. Respirable dust size considered as < 5 μm in size to be sampled by an NCB/MRE 113 A or equivalent gravimetric sampler. But, in general, the respirable size considered as <10 μm in ambient atmosphere.

The cumulative weight percentage of < 5 μm = 5 %

Again, total dust emission = 20 mg/m^3

Therefore, the concentration of respirable dust fraction = 20*5/100 = 1 mg/m^3

11. We have,

$$\text{Respiratory quotient of breathing (RQ)} = \frac{CO_2 \text{ exhaled}}{O_2 \text{ consumed}} = \frac{4.5V}{(21-16)V} = 0.9$$

12. Given data,
Volume of methane (V_m) = 8 %
Volume of H_2 (V_h) = 3 %
Lower flammability of methane (x_m) = 5 %
Lower flammability of H_2 (x_h) = 4 %
Therefore, lower flammability of the mixture

$$X = \frac{V_m + V_h}{\left(\frac{V_m}{x_m} + \frac{V_h}{x_h}\right)}\% = \frac{11}{\frac{8}{5}+\frac{3}{4}}\% = 4.68\%$$

13. Given data, Rate of air flow (Q) = 20 m^3/s

Panel production (M) = 1000 tpd
Therefore, Rate of firedamp emission

$$(q) = \frac{Cr - Ci}{100} * 20 = \frac{0.2-0}{100} * 20 = 0.04\, m^3/s$$

Where, c_r and c_i are the firedamp percentage in return and intake points respectively

⇒ q = 0.04*24*60*60/1000 m^3/t = 3.45 m^3/t

14. Given data,
Rate of emission of CO_2 from man exhale (q) = 0.002 m^3/min
Concentration of CO_2 in fresh air = 0.03 %
Assuming, fresh air supplied to man = Q m^3/min

Therefore,

$$\text{Percentage of } CO_2 \text{ in return air} = \frac{\left[q+(Q-q)*\frac{0.03}{100}\right]}{Q} * 100$$

$$= \frac{\left[0.002+(Q-0.002)*\frac{0.03}{100}\right]}{Q} * 100 \le 0.4 \text{ (given)}$$

[(Q−q) m^3 of air flow without inhale by man and q-m^3 inhaled by man]

$\Rightarrow 0.2 + 0.03Q - 0.00006 \le 0.4\,Q$

$\Rightarrow 0.37Q \geqslant 0.19994$

$\Rightarrow Q \geqslant 0.54\, m^3/min$

15. We have,

$$\text{Respiratry quotient of breating (RQ)} = \frac{CO_2 \text{ exhaled}}{O_2 \text{ consumed}} = \frac{3.83v}{(21-16.65)v} = 0.88$$

Assuming, fresh air contains 21% O_2 and 0% CO_2

16. We have,

$$\text{Maximum Allowable Concentration (MAC)} = \frac{15}{\text{Percentage of free silica}}\, mg/m^3$$

$$= \frac{15}{7.5} = 2 mg/m^3$$

17. Given data,
Air temperature at the top of the shaft (T_1) = 27°C = 300 K
Depth of the shaft (H) = 1200 m
Specific heat of air-water vapour mixture (Cp) = 1.03 kJ/kg °C = 1030 J/kg °C
From general energy balance equation, we have
dQ = dW + dH + dPE + dKE
where, dQ = heat added to or removed from the system
dW = external work done on or by the air
dH = change in enthalpy
dPE = change in potential energy of the air
dKE = change in kinetic energy of air
As there is no heat transfer, dQ = 0
For adiabatic compression, no work is done, dW = 0
Since the flow is non accelerative, dKE = 0
Therefore the above equation can be written as:
dPE = −dH
H*g = C_p*(T_2 − T_1) [−ve indicate increase in temperature]
[Let the temperature at the bottom of the shaft is T_2]
=> 1200*9.81= − 1030*(T_2 − 300)
=> T_2 = 300 + 11.43 = 311.43 K = 38.43°C

18. Give data :
Concentration of CH_4 (V_1) = 75 % by volume
Concentration of C_2H_6 (V_1) = 15 % by volume
Concentration of H_2 (V_1) = 10 % by volume
Lower explosibility limit of CH_4 (X_1) = 5 %
Lower explosibility limit of C_2H_6 (X_2) = 3.3 %
Lower explosibility limit of H_2 (X_3) = 4.2 %
The lower explosibility limit of the gas mixture can be determined as

$$LEL_{mixture} = \frac{100}{V_1/X_1 + V_2/X_2 + V_3/X_3} * 100\%$$

$$LEL_{mixture} = \frac{100}{\frac{75}{5}+\frac{15}{3.3}+\frac{10}{4.2}}\% = \frac{100}{15+2.54+2.38}\% = 5.01\%$$

19. The index can be determined by,

$$L = \left(\frac{24*v^2}{c\sqrt{F}}\right)^{1/3} = 3.10$$

where v = average velocity of air, m/s
c = mean methane content of air current (% CH_4)
F = area of cross section at the measuring station, m_2

$$\Rightarrow L = \left(\frac{24*1.5^2}{0.6\sqrt{9}}\right)^{1/3} = 3.10$$

20. Given data,
Rate of air flow (Q) = 20 m^3/s
Percentage of O_2 at inlet (C_i) = 20.8 %
Percentage of O_2 at exit (C_o) = 20.6 %
Density of oxygen (ρ_o) = 1.3 kg/m^3
Calorific value of Carbon (H) = 34 MJ/kg = 34*106 J/kg

Therefore, rate of volumetric oxygen consumed in spontaneous heating process

$V = Q*(C_i - C_o)/100 = 20*0.2/100 = 0.04\ m^3/s$

Mass of oxygen consumed rate (M) = V* ρ_o = 0.04*1.3 = 0.052 kg/s

Now, we have

$C + O_2 = CO_2$

Therefore, from above equation we can say,

32 unit of O_2 consume by 12 unit of C

$\Rightarrow$ 0.052 kg of O_2 consume uses the carbon mass = m_c = 12*0.052/32= 0.0195 kg/s

Again, calorific value of carbon = $34*10^6$ J/kg

Therefore, rate of heat emission= $0.0195*34*10^6$ J/s = $663*10^3$ J/s = $663*10^3$ W

= 663 kW

21. Composition of fresh air:
O_2 = 21 %; CO_2 = 0.03 %; N_2 = 78.97 %
Composition of mine air :
O_2 = 18 %; CO_2 = 2 %; N_2 = 78 %

Air equivalent to $O_2 = \frac{18}{21}*100 = 85.71\%$

Nitrogen equivalent to $O_2 = \frac{18}{21}*78.97 = 67.69\%$

CO_2 equivalent to $O_2 = \frac{18}{21}*0.03 = 0.026\%$

Therefore,
Excess N_2 = 78 − 67.69 = 10.31%, Excess of CO_2 = 2 − 0.026 = 1.974
Black Damp = Sum of excess percentage of N_2 and CO_2 = 10.31+~~0.026~~ 1.974
= ~~10.336~~
= 12.284

23. Refer CMR No. 133(III)

24. Given data,
Volume of methane (V_m) = 70%
Volume of CO (V_c) = 30%
Lower flammability of methane (x_m) = 5%
Lower flammability of CO (x_c) = 13%
Therefore, lower flammability of the mixture

$$X = \frac{V_m + V_c}{\frac{V_m}{X_m} + \frac{V_c}{X_c}}\% = \frac{100}{\frac{70}{5}+\frac{30}{13}}\% = \frac{100}{16.31} = 6.132\%$$

25. Given data,

Number of workers employed (N) in three shift = 220, 200, and 192
Daily production (Q) = 520 te
According to CMR No. 133(I) :
Quantity of air to be circulated in a mine should be the largest of 6 m^3 per minute per person in a largest shift or 2.5 m^3 per minute per day tonne output.
Quantity of air require to circulate on the basis of number of workers (q_1)
= N*6
= 220*6 = 1320 m^3/min

[Largest shift means maximum number of manpower and hence N = 220]
Again, quantity of air require to circulate on the basis of number of workers (q_2)
= Q*2.5 = 520*2.5 = 1300 m^3/min

Therefore, q = maximum (q_1, q_2) = 1320 m^3/min

26. Refer section 3.10 for Graham's ratio.

27. Graham's Ratio

$$= \frac{\text{CO produced}}{O_2\text{ consumed}} = \frac{CO_f - CO_i}{0.265*N2_f - O2_f}*100 = \frac{0.02-0}{0.265*78.49-19.92}*100$$

= 2.27% ≃ 2.3%

28. Given data,

Percentage of toxic gas in air at intake point (C_i) = 0.2 % = 0.002
Maximum percentage of toxic gas in air at return point is (C_r) = 1.25 % = 0.0125
Let the quantity of fresh air flow required is Q m^3/sec
Rate of toxic gas emission (q) = 0.05 m^3/sec
Applying mass balance equation, we get
$Q * C_i + q = (Q + 1) * C_r$
$\Rightarrow Q*0.002 + 0.05 = (Q + 0.05)*0.0125$
$\Rightarrow 0.0105*Q = 0.05*0.9875$
$\Rightarrow Q = 4.7\ m^3/s$

50. Given data,

Resistant of split A (R_a) = 0.5 Ns^{2}m^{-8}
Resistance of split B (R_b) = 2 Ns^{2}m^{-8}
Quantity of air flow through split A (Q_a) = 20 m^3/s
Quantity of air flow through split B = Q_b
We have, $P = R_aQ_a^{\ 2} = R_bQ_b^{\ 2}$
$\Rightarrow 0.5*20^2 = 2*Q_b$
$\Rightarrow Q_b^{\ 2} = 100$
$\Rightarrow Q_b = 10m^3/s$
Therefore, Total air flow through mine (Q)= $Q_a + Q_b$ = 20 + 10 = 30m^3/s

51. Let, equivalent resistance of the two splits is R

Therefore $1/\sqrt{R} = 1/\sqrt{R_a} + 1/R_b$ **[Splits are in parallel]**

$\Rightarrow 1/\sqrt{R} = 1/\sqrt{0.5} + 1/\sqrt{2}$

$\Rightarrow R = 0.22\,Ns^2m^{-8}$

Total resistance of the mine (R_e) = Equivalent resistance of the splits + combined resistance of trunk roadways and shaft **[for series connection]**
We have, total air power of the system (Power) = $R_eQ^3 = 0.92*30^3$
= 24.9 kW

52. Given data,

Quantity of air flow through duct (Q) = 2 m^3/s
Area of cross section of the duct (A) = 0.3 m^2
Duct resistance (R) = 40
Air density (ρ) = 1.2 kg/m^3

We have,
Total pressure generated by the fan = Velocity pressure + Flow pressure
$= (½\ \rho v^2) + RQ^2 = ½\ \rho\ (Q/A)^2 + RQ^2$
$= 1/2 * 1.2*(2/0.3)^2 + 40*2^2$
$= 26.67 + 160 = 186.67$ Pa

53. Given data,

Resistance of split A (R_a) = 0.5 Ns^{2}m^{-8}
Resistance of split B (R_b) = 0.8 Ns^{2}m^{-8}
Quantity of air flow through split A (Q_a) = 15 m^3/s
Quantity of air flow through split B (Q_b) = 10 m^3/s
Assuming, R_r = resistance of the regulator

We have,
$P = R_aQ_a^{\ 2} = (R_b + R_r)\ Q_b^{\ 2}$
$\Rightarrow 0.5*152 = (0.8 + R)*10^2$
$\Rightarrow R_r = 0.325$ Ns^{2}m^{-8}
Therefore, size of the regulator $= 1.2/\sqrt{R_r} = 1.2/\sqrt{R_r} = 1.2/\sqrt{0.325} = 2.10m^2$

54. Given data,

Length of gate road (L) = 1000 m
Width (W) = 4.5 m
Height (H) = 2 m
Air flow through gate road (Q) = 17 m^3/s
Coefficient of resistance (f) = 0.01

$$P = RQ^2 = \frac{kS}{A^3}*Q^2 = \frac{\left(\frac{f\rho}{8}\right)*\{2(H+W)L\}}{A^3}*Q^2 = \frac{2f\rho(H+W)LQ^2}{8A^3}$$
$$= \frac{2*0.01*1.2*(6.5)*1000*17*17}{8*(2*4.5)^3} = 7.73\,\text{Pa}$$

55. Given data,

Resistance of 1st parallel (R_1) = 100 Ns^{2}m^{-8}
Resistance of 2nd parallel (R_2) = 120 Ns^{2}m^{-8}
Resistance of upcast shaft (Ru) = 10 Ns^{2}m^{-8}
Resistance of downcast shaft (Rd) = 20 Ns^{2}m^{-8}
Resistance of fan drift (R_{fd}) = 5 Ns^{2}m^{-8}
Fan drift air pressure (P) = 15 MN/m^2 = 15*10^6 N/m^2
Assuming, equivalent resistance of two parallels is R

Therefore,
$$\frac{1}{\sqrt{R}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}} = \frac{1}{\sqrt{100}} + \frac{1}{\sqrt{120}}$$
$\Rightarrow R = 27.33$

Again, upcast shaft, downcast shaft, fan drift are in series with the equivalent resistance of two parallel.
Equivalent resistance of the mine (Re) = $R + R_u + R_d + R_{fd}$
= 27.33 + 10 + 20 + 5 = 62.33

Therefore,

Rate of air flow through the mine (Q) $= \sqrt{\frac{P}{R_e}} = \sqrt{\frac{15*1000000}{62.33}} = 490.56m^3$

56. Again, we have,

$R_1 * q_1^2 = R_2 q_2^2$

And, $Q = q_1 + q_2$

$\Rightarrow q_2 = Q - q_1$

Where, q_1 and q_2 are the air flow through split 1 and 2 respectively.

Putting the value of q_2 in above equation, we get

$R_1 q_1^2 = R_2(Q - q_2)^2$

$\Rightarrow R_1 * q_1^2 = 150*(490.56 - q_1)^2$ $R_2 = 120$

$\Rightarrow 100*q_1^2 = 150*(490.56 - q_1)^2$

$\Rightarrow q_1^2 = 1.50*(490.56 - q_1)^2$

$\Rightarrow q_1 = \pm 1.224*(490.56 - q_1)$

$\Rightarrow q_1 = 1.224*\dfrac{490.56}{2.224}$ [–ve sign is not feasible]

$\Rightarrow q_1 = 270.06\ m^3/s \rightarrow 258.46$

57. Given data

Specific gravity of alcohol (γ) = 0.8 [Therefore, density of alcohol (ρ_{al}) = 800 kg/m^3]

Density of air (P_a) = 1.2 kg/m^3

Total pressure (P_t) = 10.2 mm of alcohol

$= 10.2*10^{-3}*800*9.81\ N/m^2 = 80.05$ Pa

Static pressure (P_s) = 8.8 mm of alcohol

$= 8.8*10^{-3}*800*9.81\ N/m^2 = 69.06$ Pa

Velocity pressure = P_v

We have,

$P_t = P_s + P_v$

$\Rightarrow 80.05 = 69.06 + P_v$

$\Rightarrow P_v = 10.99$

$\Rightarrow \frac{1}{2}\rho_a v^2 = 10.99$

$\Rightarrow v^2 = 10.99*\frac{2}{1.2} = 18.31$

$\Rightarrow v = 4.27\ m/s$

58. We have,

Total Pressure (P_t) = Static Pressure (P_s) + Velocity Pressure (P_v)

$\Rightarrow h_t * \rho_w * g = h_s * \rho_w * g + P_v$

$\Rightarrow P_v = (20 - 10) * 10^{-3} * 1000 * 9.81 = 98.1$

$\Rightarrow \frac{1}{2}\rho_a v^2 = 98.1$

$\Rightarrow v^2 = 98.1*\frac{2}{1.2} = 163.5$

$\Rightarrow v = 12.78$ m/s

59. Given data,

Methane percentage in air at intake point (c_i) = 0.1 % = 0.001

Methane percentage in air at return point (c_o) = 0.8 % = 0.008

Coal Production from face (W) = 2000 te/day

Quantity of air circulating (Q) = 15 m^3/s

Therefore, rate of methane emission (q) = $Q*(c_o - c_i) = 15*(0.008-0.001)$
= 0.105 m^3/s = 0.105*24*3600 m^3/day = 9072 m^3/day
= 9072/W m^3/te = 9072/2000 = 4.536 m^3/te of coal produced

60. Given data,

Cross section of air way (W × H) = 2.2 × 2.2 m^2

Length of airway (L) = 500 m

Friction factor (k) = 0.01

Shock loss factor (x) = 0.07

Density of air (ρ) = 1.2 kg/m^3

Let equivalent length of the airway (Le) = L + L_b

We have,

Shock loss pressre (P_s) $= x*\frac{1}{2}*\rho*V^2$

Substituting, V = Q/A, we get

$$P_s = x*\frac{1}{2}*\rho*(Q/A)^2 = \frac{x*\rho}{2*A^2}*Q^2 = R_s*Q^2$$

Where,

Q = Quantity of airflow (m^3/s)

V = velocity of air (m/s)

R_s = Shock resistance

Therefore, we get

$$R_s = \frac{x*\rho}{2*A^2} = \frac{0.07*1.2}{2*(2.2*2.2)^2}$$

Again, from resistance equation, we get

$$R_s = \frac{kS}{A^3} = \frac{k*2(H+W)}{(H*W)^3} = \frac{0.01*2(2.2+2.2)*L_b}{(2.2*2.2)^3}$$

[S = surface rubbing area]

Equating the above two equations we get,

$$R_s = \frac{0.07*1.2}{2*(2.2*2.2)^2} = \frac{0.01*2(2.2+2.2)*L_b}{(2.2*2.2)^3}$$

$\Rightarrow L_b = 2.31$ m

Therefore equivalent legnth,

$L_e = L + L_b = 500 + 2.31 = 502.31$m

62. Given data,

The two different fan speeds are N_1 = 800 rpm and N_2 = 950 rpm.

Fan-drift pressure at speed N_1 of the fan (P_{f1}) = 655 Pa

Fan-drift pressure at speed N_2 of the fan (P_{f2}) = 730 Pa

Fan-drift quantity at speed N_1 of the fan (Q_1) = 82.2 m^3/s
Fan-drift quantity at speed N_2 of the fan (Q_2) = 85.5 m^3/s
Let the resistance of the mine is R and the NVP is P_n

Therefore, we have,

$$(P_{f1}+P_n)=R*Q_1^2$$

And,

$$(P_{f2}+P_n)=R*Q_1^2$$

From the above two equations, we get

$$\frac{P_{f1}+P_n}{P_{f2}+P_n}=\frac{R*Q_1^2}{R*Q_2^2}$$

$$\Rightarrow \frac{655+P_n}{730+P_n}=\frac{82.2^2}{85.5^2}=0.924$$

$$\Rightarrow P_n=19.52/0.076=256.84\,m^3/s$$

63. Given data,
Depth of the shaft (D) = 450m
Mean temperature of downcast shaft (T_d) = 29^0C = 302 K
Mean temperature of upcast shaft (T_u) = 37^0C = 310 K
Therefore,

$$\text{Height of Motive Column (h)}=\frac{NVP}{g*\rho_d}$$

Where, NVP is natural ventilation pressure in Pascal
ρ_d is the density of air in the downcast shaft (kg/m^3)
g is acceleration due to gravity (m/s^2)

$$\text{Motive Column (h)}=\frac{(\rho_d*D*g-\rho_u*D*g)}{g*\rho_d}=\frac{(\rho_d-\rho_u)*D}{\rho_d}$$

$$=\left(1-\frac{\rho_u}{\rho_d}\right)*D=\left[1-\frac{\dfrac{(B_u-0.378e_u)*1000}{287.1*T_u}}{\dfrac{(B_d-0.378e_d)*1000}{287.1*T_d}}\right]*D$$

Where,
B_d and B_u = mean barometer reading in kPa in the D.C. and U.C. shafts respectively.
e_d and e_u = average vapour pressures of moisture in kPa in the D.C. and U.C. shafts respectively.
For a mine shafts, $B_d = B_u = B$, and negelecting the effect of moisture, we can write

$$B_u - 0.378e_u = B_d - 0.378e_d = B$$

Therefore,

$$\text{Motive Column (h)}=\left(1-\frac{B/T_u}{B/T_d}\right)*D=\left(1-\frac{T_d}{T_u}\right)*D=\left(1-\frac{302}{310}\right)*450=11.6m$$

64. Given data,
Resistance of split A (R_a) = 0.35 Ns^2m^{-8}
Resistance of split B (R_b) = 0.05 Ns^2m^{-8}
Combined resistance of shaft and trunk roadways (R_s) = 0.4 Ns^2m^{-8}
Fan Pressure after and before installation of booster fan = P = 1000 Pa
Now, after installation of booster fan in split A,

$$P=R_sq^2+R_bq_b^2$$

$$\Rightarrow 1000=0.4q^2+0.05q_b^2$$

Again,

$$P+P_b=R_sq^2+R_aq_a^2$$

$$\Rightarrow 1000+P_b=0.4q^2+0.35q_a^2$$

Where, q = total air flow through the mine.
P_b = critical pressure of the booster fan
q_a = flow through split A
q_b = flow through split B

At critical pressure of booster fan, reversal just commences in split B and under this condition no air passes in split B. Then, flow in split B = q_b = 0. Under critical condition all the quantity of air passes through Split A. And hence, $q = q_a$
Therefore from the above two equations, we get

$$\Rightarrow P_b=875\,Pa$$

65. Given data,
Resistance of split A (R_a) = 0.8 Ns^2m^{-8}
Resistance of split B (R_b) = 3.2 Ns^2m^{-8}
Resistance of trunk airways (R_t) = 0.2 Ns^2m^{-8}
Fan Pressure (P) = 500 Pa
Let the equivalent resistance of two splits is Re and the equivalent resistance of the mine is R_m
Let the quantity of air flow through split A = Q_a
And, the quantity of air flow through split B = Qb
Air flow through mine/trunk airways = Q = Q_a + Q_b
Before installation of booster fan, we have,

$$P=R_tQ^2+R_aQ_a^2=R_tQ^2+R_bQ_b^2$$

$$\Rightarrow 500=0.2*(3Q_b)^2+3.2Q_b^2=5Q_b^2$$

$$\Rightarrow Q_a=2Q_b$$

Again, $Q = Q_a + Q_b = 3Q_b$
Putting the value of Q in above equation, we get

$$\Rightarrow 500=0.2*(3Q_b)^2+3.2Q_b^2=5Q_b^2$$

$$\Rightarrow Q_a = 2Q_b$$

Again, $Q = Q_a + Q_b = 3Q_b$

Putting the value of Q in above equation, we get

$\Rightarrow 500 = 0.2*(3Q_b)^2 + 3.2Q_b^2 = 5Q_b^2$

$\Rightarrow Q_b = 10 m^3/s$

Hence, $Q_a = 2*10 = 20\ m^3/s$

66. Let the pressure of the booster fan = P_b

Now, after installation of booster fan in split B without changing the fan pressure, the quantity of air flow in both the splits are equal.

That is, $q_a = q_b$

Let quantity of air flow through trunk airways after installation of booster fan = q

Therefore, $q = 2q_a = 2q_a = 2q_b$

We have,

$$P = R_t q^2 + R_a q_a^2$$

$\Rightarrow 500 = 0.2q^2 + 0.8q_a^2 = 0.4q^2$

$\Rightarrow q = 35.35 m^3/s$

Again,

$$P + P_b = R_t q^2 + R_b q_b^2$$

$\Rightarrow 500 + P_b = 0.2q^2 + 3.2q_b^2 = 1q^2$

Putting the value of q in the above equation, we get

$\Rightarrow 500 + P_b = 1250$

$\Rightarrow P_b = 750$ pa

68. Given data,

Methane percentage in air at intake point (c_i) = 0.25 % = 0.0025

Methane percentage in air at return point (c_o) = 0.75 % = 0.0075

Production from face (W) = 400 te/ 8 hour shift

Quantity of air circulating in the face (Q) = 18 m^3/s

Therefore, rate of methane emission (q) = $Q*(c_o - c_i)$ = 18*(0.0075–0.0025)

= 0.09 m^3/s = 0.09*8*3600 m^3/shift = 2592 m^3/day

= 2592/W m^3/te = 2592/400 = 6.48 m^3/te of coal produced

69. Given data,

Percentage of toxic gas in air at intake point (c_i) = 0.25% = 0.0025

Let percentage of toxic gas in air at return point is c_o (= 1%) for Q m^3/min of air flow

Rate of toxic gas emission (q) = 2.52 m^3/min

Therefore, rate of toxic gas emission (q) = $Q*(c_o - c_i)$

$\Rightarrow$ 2.52= Q*(0.01-0.0025)

$\Rightarrow$ Q = 2.52/0.0075 = 336 m^3/min

70. Let, length of both the roadway = L

Height and width of the square roadway = X

Height and Width of the rectangular roadway are H and W respectively

Friction coefficient of both the roadways = k

Area of cross section of square roadway = $A_1 = X^2$

Area of cross section of rectangular roadway = A_2 = W*H

Surface roughness of square roadway = S_1 = 2(X + X)*L

Surface roughness of rectangular roadway = S_2 = 2(W + H)*L

Resistance of the roadway having square area of cross section = R_s

Resistance of the roadway having rectangular area of cross section = R_r

Therefore, $R_s = \frac{kS_1}{A_1^3} = \frac{k*2(x+x)*L}{(x*x)} = \frac{4kLx}{x^6}$

[For square area of cross section, H = W]

Similarly,

$$R_r = \frac{kS_2}{A_2^3} = \frac{k*2(H+W)*L}{(H*W)^3} = \frac{k*2(H+2H)*L}{(H*2H)^3} = \frac{3kLH}{4*H^6}$$

[Given that, W/H = 2]

For equal area of cross section we have,

$A_1 = A_2$

$\Rightarrow X^2 = W*H = 2H^2$

$\Rightarrow X = \sqrt{2H}$

Therefore,

$$\frac{R_s}{R_r} = \left(\frac{\frac{4kLx}{x^6}}{\frac{3kLH}{4*H^6}}\right) = \frac{16H^5}{3X^5} = \frac{2\sqrt{2}}{3}$$

71. Given data,

Air flow before installation of regulator = Q_1 = 30 m^3/s

Air flow after installation of regulator = Q_2 = 20 m^3/s

Fan pressure before and after installation of regulator = P = 1200 Pa

Air density = ρ = 1.2 kg/m^3

Let resistance of mine is R_m and resistance of regulator is R_r

Before installation of regulator, we have

$$P = R_m Q_1^2$$

$\Rightarrow 1200 = R_m * 30^2$

$\Rightarrow R_m = \frac{4}{3} Ns^2m^{-8}$

After installation of regulator, we have

$P = (R_m + R_r)Q_2^2$

$\Rightarrow 1200 = \left(\frac{4}{3} + R_r\right) * 20^2$

$\Rightarrow R_r = 3 - \frac{4}{3} = 5/3\ Ns^2m^{-8}$

Therefore, size of the regulator $(A) = \frac{1.29}{\sqrt{R_r}} = \frac{1.29}{\sqrt{5/3}} = 0.99\ m^2$

72. Diameter of the duct (D) = 500 mm
Radius of the duct (R) = 250 mm
Since the duct is divided into three equal area of cross sections having radii x_1, x_2, and R for measuring the air velocity.
Therefore, we have

$$\pi x_1^2 = \pi\left(x_2^2 - x_1^2\right) = \pi\left(R^2 - x_2^2\right)$$

$$\Rightarrow x_1^2 = \left(x_2^2 - x_1^2\right) = \left(R^2 - x_2^2\right)$$

$$\Rightarrow X_2 = \sqrt{2}\ x_1 \text{ and } R = \sqrt{3}\ x_1$$

Therefore, $X_2 = \sqrt{(2/3)} * R = 204.12$

$X_1 = 144.34$

Let the velocity be measured at radii r_1, r_2 and r_3 rexpectively.

$$\pi r_1^2 = \pi x_1^2 / 2$$

$$\Rightarrow r_1 = \frac{x_1}{\sqrt{2}} = \frac{R}{\sqrt{6}} = \frac{250}{2.45} = 102$$

Again, $\pi r_2^2 = \pi x_1^2 + \frac{\pi x_2^2 - \pi x_1^2}{2}$

$$\Rightarrow \quad r_2^2 = \frac{x_2^2 + x_1^2}{2} = \frac{\frac{R^2}{3} + \frac{2R^2}{3}}{2} = \frac{3R^2}{6}$$

$$\Rightarrow \quad r_2^2 \frac{\sqrt{3}R}{\sqrt{6}} = \frac{250}{\sqrt{2}} = 176.77 \approx 177$$

similarly, $\pi r_3^2 = \pi x_2^2 + \frac{\pi R^2 - \pi x_2^2}{2}$

$$\Rightarrow \quad \pi r_3^2 = \frac{R^2 + x_2^2}{2} = \frac{R^2 + \frac{2R^2}{3}}{2} = \frac{5R^2}{6}$$

$$\Rightarrow \quad r_3 = \frac{\sqrt{5}R}{\sqrt{6}} = 228.22 \approx 228$$

[Note : In general, for n number of measurement or n number of equal area of cross section, we have]

$$r_n = \frac{\sqrt{2n-1}}{\sqrt{2n}} R$$

Where, r_n is the distance from centre for n^{th} measurement and R is the radius of the duct.]

73. Main fan operating point is
Air flow = Q = 55 m^3/s
Pressure generated by fan = P = 440 Pa
Let the resistance of the mine is R_m

Therefore, $R_m = \frac{P}{Q^2} = \frac{440}{55*55} = 0.145\ Ns^2m^{-8}$

If the natural ventilation pressure (P_n) assist the main fan, then this can be treated in series with the main fan.
Therefore,

Resultant presure = P_r = P + P_n = 440 + 200 = 640 Pa

Resultant Quantity $= Q_r = \sqrt{\frac{P_r}{R_m}} = \sqrt{\frac{640}{0.145}} = 66\ m^3/s$

74. In single point velocity measurement, average velocity can be determined as
Average velocity (V_{av}) = method factor (x) * velocity measure at the centre point (V)
Therefore,
Average velocity (V_{av}) = 0.8* 12.5 = 10 m/s
Densit of air (ρ) = 1.2 kg/m^3

Velocity pressure $(P_v) = \frac{1}{2} * \rho * V_{av}^2 = \frac{1}{2} * 1.2 * 100 = 60\ m/s$

75. Given data :
Static presure at the inlet point (Psi) = 60 Pa
Static pressure at the outlet point (Pso) = 90 Pa
Let the area of cross-section at the inlent and outlet points are A_i and [illegible] respectively.
It is given that, Ao = $2A_i$
Applying energy conservation laws at inlet and outlet point, we have,
Sum of pressure energy at inlet point
= Sum of pressure energy at outlet point
+ Shock Pressure Loss

$$\Rightarrow \text{Psi} + \text{Pvi} = \text{Pso} + \text{Pvo} + \text{Psl}$$

$$\Rightarrow 60 + \frac{1}{2} * \rho * V_{avi}^2 = 90 + \frac{1}{2} * \rho * V_{avo}^2 + P_{sl}$$

$$\Rightarrow 60 + \frac{1}{2} * 1.2 * 100 = 90 + \frac{1}{2} * \rho * \left(A_i * \frac{V_{avi}}{A_0}\right)^2 + P_{sl}$$

$$\Rightarrow 60 + 60 = 90 + \frac{1}{2} * 1.2 * \left(A_i * \frac{10}{2A_i}\right)^2 + P_{sl}$$

$$\Rightarrow 30 = \frac{1}{2} * 1.2 * 25 + P_{sl}$$

$$\Rightarrow P_{sl} = 30 - 15 = 15\ Pa$$

76. Given data,

Resistance fo the mine (R) = 0.19 Ns^2m^{-8}

Pressure characteristics of a mine fan is

$P = -0.06Q^2 + 400$

[Where, P is pressure in Pa and Q is quantity in m^3/s]

$\Rightarrow RQ^2 = -0.06Q^2 + 400$

$\Rightarrow 0.19Q^2 = -0.06Q^2 + 400$

$\Rightarrow 0.25Q^2 = 400$

$\Rightarrow Q = 40 m^3/s$

77. Given that an identical fan is installed in the mine to operate in series and thus pressure generation will be doubled.

Pressure generation by the single fan = $P = RQ^2$ = 0.19*40*40 = 304 Pa

Resultant pressure after installation of similar fan (P_r) = 2P = 2*304 = 608 Pa

Let the resultant air flow is Q_r

Therefore,

$$P_r = RQ_r^2$$

$$\Rightarrow Q_r = \sqrt{\frac{P_r}{R}} = \sqrt{\frac{608}{0.19}} = 56.57 m^3/s$$

78. Given data,

Condition 1 :

Fan speed (N_1) = 500 rpm

Quantity (Q_1) = 70 m^3/s

Condition 2 :

Fan speed (N_2) = 200 rpm

Quantity (Q_2) =?

We know, Quantity of flow is directly proportional to the speed of fan. Thus,

$Q \alpha N$

$\Rightarrow$ Q = k*N [k is constant]

Therefore,

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2}$$

$$\Rightarrow \frac{70}{Q_2} = \frac{500}{200}$$

$$\Rightarrow Q_2 = 28 m^3/s$$

79. Given data,

Fan speed (N) = 280 rpm

Air quantity (Q) = 105 m^3/s

Power input = 75 kW

Combined efficiency of the fan and motor (η) = 70 %

Let the fan pressure is P

We have,

Output Power = P * Q

$\Rightarrow$ Input Power * η = P * Q

$\Rightarrow 75*10^3*0.7 = P*105$

$$\Rightarrow P = \frac{75*10^3*0.7}{105} = 500 Pa$$

80. Given that the fan pressure is increased by 200 Pa by changing the fan speed.

Resultant Pressure (Pr) = 500 + 200 = 700 Pa

Let the speed of the fan increased to N_r

We know, pressure of fan is directly proportional to square of the speed of fan.

That is,

$P \alpha N^2$

$\Rightarrow$ P = k*N^2 [k is constant]

Therefore,

$$\frac{P}{P_r} = \frac{N^2}{N_r^2}$$

$$\Rightarrow \frac{500}{700} = \frac{280^2}{N_r^2}$$

$$\Rightarrow N_r = \sqrt{\frac{700}{500}}*280 = 331.3 \text{ rpm}$$

81. Let the resistance of each roadway is R

The equivalent resistances are R_{e1} and R_{e2} respectively for two and three identical parallel roadways connected.

Therefore, we have

$$\frac{1}{\sqrt{R_{e1}}} = \frac{1}{\sqrt{R}} + \frac{1}{\sqrt{R}} = \frac{2}{\sqrt{R}}$$

$$\Rightarrow R_{e1} = R/4$$

Similarly,

$$\frac{1}{\sqrt{R_{e2}}} = \frac{1}{\sqrt{R}} + \frac{1}{\sqrt{R}} + \frac{1}{\sqrt{R}} = \frac{3}{\sqrt{R}}$$

Thus,

$$\frac{R_{e2}}{R_{e1}} = \frac{4}{9} = 0.44$$

82. Given data,

Initial fan drift pressure = P_1 = 10mm of w.g.

Initial quantity = Q = 50m^3/s

Final fan drift pressure = P_2 = ?

Final air quantity = Q_2 = 60 m^3/s

ow, pressure of fan is directly proportional to square of the quantity of air flow. That is,

P α Q^2

⇒ P = R*Q^2 [k is constant]

Therefore,

$$\frac{P_1}{P_2} = \frac{Q_1^2}{Q_2^2}$$

$$\Rightarrow \frac{10}{P_2} = \frac{50^2}{60^2}$$

$$\Rightarrow P_2 = \frac{60^2}{50^2} * 10 = 14.4 \text{ mm of w.g.}$$

85. Given data,

Initial fan drift pressure = P_1 = 50mm of w.g.

Initial speed = N_1 = 200 rpm

Final fan drift pressure = P_2 = ?

Final speed = N_2 = 300 rpm

We know, pressure of fan is directly proportional to square of the speed of fan. That is,

P α N^2

P = k*N^2 [k is constant]

Therefore,

$$\frac{P_1}{P_2} = \frac{Q_1^2}{Q_2^2}$$

$$\Rightarrow \frac{50}{P_2} = \frac{200^2}{300^2}$$

$$\Rightarrow P_2 = \frac{300^2}{200^2} * 50 = 112.5 \text{ mm of w.g.}$$

86. Given data,

Initial pressure = P_1 = 240 Pa

Initial speed = N_1 = 1400 rpm

Initial quantity of air flow = Q_1 = 30m^3/s

Final speed = N_2 = 2100 rpm

Final pressure = P_2 = ?

Final quantity of air flow = Q_2 = ?

We know, pressure of fan is directly proportional to square of the speed of fan and quantity is directly proportion to speed of fan. That is,

P α N^2 and Q α N

⇒ P = k*N^2 [k is constant] and Q = cN [c is constant]

Therefore,

$$\frac{P_1}{P_2} = \frac{N_1^2}{N_2^2}$$

$$\Rightarrow \frac{240}{P_2} = \frac{1400^2}{2100^2}$$

$$\Rightarrow P_2 = \frac{2100^2}{1400^2} * 240 = 540 \text{ pa}$$

Again,

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2}$$

$$\Rightarrow \frac{30}{Q_2} = \frac{1400}{2100}$$

$$\Rightarrow Q_2 = \frac{2100}{1400} * 30 = 45 \text{ m}^3/\text{s}$$

87. When the NVP aids the fan presure, the measured data are

Quantity of flow (Q_1) = 65 m^3/s

Fan drift pressure (P_1) = 500 Pa

When the NVP resist the fan pressure, the measure data are

Quantity of flow (Q_2) = 50m^3/s

Fan drift pressure (P_2) = 600 Pa

Let the total resistance of the mine is R and the NVP is P_n

Therefore we have,

$$P_1 + P_n = RQ_1^2$$

$$\Rightarrow 500 + P_n = R * 65^2$$

$$\Rightarrow P_n = R * 65^2 - 500$$

Similarly, for second case,

$$P_2 + P_n = RQ_2^2$$

$$\Rightarrow 600 - P_n = R * 50^2$$

$$\Rightarrow P_n = 600 - R * 50^2$$

Equating the above two equations, we get

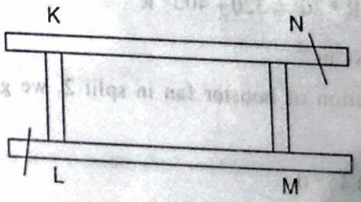
R * 65^2 – 500 = 600 – R * 50^2

$$\Rightarrow R * 65^2 + R * 50^2 = 600 + 500$$

$$\Rightarrow R = \frac{1100}{4225 + 2500} = 0.163 \text{ Ns}^2\text{m}^{-8}$$

88. Applying Kirchoff's law, we have,

Air flow through split LN is equal to zero. Thus, the resultant network is



Equivalent resistance of splits KN and NM is given by

$R_{KNM} = R_{KN} + R_{NM} = R + R = 2R$

Similarly, equivalent resistance of splits KL and LM is given by

$R_{KLM} = R_{KL} + R_{LM} = R + R = 2R$

Equivalent resistance across KM is

$$\frac{1}{\sqrt{R_e}} = \frac{1}{\sqrt{R_{KNM}}} + \frac{1}{\sqrt{R_{KLM}}} = \frac{1}{\sqrt{2R}} + \frac{1}{\sqrt{2R}} = \sqrt{\frac{2}{R}}$$

$$\Rightarrow R_e = \frac{R}{2} = 0.5\,R$$

89. Given data,

Angular velocity of the blade (ω) = 300 rpm = 300*2*3.14/60 rad/s = 31.4 rad/s

Air quantity deliver by fan (Q) = 120 m^3/s

Width of the fan (B) = 1.3 m

Tangential velocity of the blade (U) = 47.12 m/s

We have,

Radius of the fan (r) = U/ω = 47.12/ 31.4 = 1.5 m

Therefore, radial velocity of air at the periphery of the impeller = Q/π(2r)B = 120/(3.14*2*1.5*1.3) = 9.79 m/s

90. Let the resistance of both the splits is R

Resistance of trunk airways = R_t = 0.2

Fan pressure before and after installation of booster fan = P = 500 Pa

Quantity of air circulate before installation of booster fan = Q = 40 m^3/s

Since the resistance of both the splits are equal, the quantity of air in both the split before installation of booster fan are also equal. That is,

$Q_1 = Q_2 = Q/2 = 20\ m^3/s$

Let air flow through the mine after installation of booster fan in split 2 is q. Installation of booster fan in one split leads to zero air flow in other split. Therefore, air flow in trunk airways is equal to air flow in the split, where booster fan is installed.

That is,

Air flow in split 1 = q_1 = 0

Air flow in split 2 = q_2 = q

Before installation of booster fan, we get,

$$P = R_tQ^2 + RQ_1^2 = R_tQ^2 + RQ_2^2$$

$$\Rightarrow 500 = 0.2 * 40^2 + R * 20^2 = 320 + 400 * R$$

$$\Rightarrow R = \frac{180}{400} = 0.45\,Ns^2m^{-8}$$

Again, after installation of booster fan in split 2, we get

$$P = R_tq^2 + R * q_1^2$$

$$\Rightarrow 500 = 0.2 * q^2 + 0.45 * 0$$

$$\Rightarrow q^2 = \frac{500}{0.2} = 2500$$

$$\Rightarrow q = 50\,m^3/s$$

91. Given data,

Air flow before installation of regulator = Q_1 = 24 m^3/s

Air flow after installation of regulator = Q_2 = 16 m^3/s

Fan pressure before and after installation of regulator = P = 1200 Pa

Let resistance of mine is R_m and resistance of regulator is R_r

Before installation of regulator, we have

$$P = R_mQ_1^2$$

$$\Rightarrow 1200 = R_m * 24^2$$

$$\Rightarrow R_m = 2.08\,Ns^2m^{-8}$$

After installation of regulator, we have

$$P = (R_m + R_r)Q_2^2$$

$$\Rightarrow 1200 = (2.08 + R_r) * 16^2$$

$$\Rightarrow R_r = 4.69 - 2.08 = 2.61\,Ns^2m^{-8}$$

Therefore, size of the regulator (A) $= \frac{1.2}{\sqrt{R_r}} = \frac{1.2}{\sqrt{2.61}} = 0.74\,m^2$

[1.29 is for air density of 1.2 kg/m^3. General value considered for this is 1.2]

92. Given data,

Area of cross section of evasee at the outlet = A_0 = 18m^2

Area of cross section of evasee at the inlet = A_i = 18/4 = 4.5m^2

Pressure generated by fan with evasee = 1000 Pa

Quantity of air flow in mine with evasee (Q) = 150 m^3/s

Quantity of air flow in mine without evasee (q) = 120 m^3/s

Density of air = 1.2 kg/m^3

Let v_i and v_0 are velocity of air at inlet and cutlet point of evasee respectively.

Pressure at inlet point of evasee $= \frac{1}{2}\rho * v_i^2 = \frac{1}{2} * 1.2 * \left(\frac{Q}{A_i}\right)^2$

$$= \frac{1}{2} * 1.2 * (150/4.5)^2 = 666.67\,Pa$$

Pressure at outlet point of evasee $= \frac{1}{2}\rho * v_0^2 = \frac{1}{2} * 1.2 * \left(\frac{Q}{A_0}\right)^2$

$$= \frac{1}{2} * 1.2 * (150/18)^2 = 41.67\,Pa$$

Therefore,

Pressure gain in the evasee (Δp) $= (666.67 - 41.67) * \eta = 625\eta$

[η = efficiency of evasee]

Again, we have

$$P = RQ^2$$

$$\Rightarrow R = \frac{P}{Q^2} = \frac{1000}{150^2} = 0.044\,Ns^2m^{-8}$$

[R = resistance of mine]

$P_l = Rq^2 = 0.044*120^2 = 640$ Pa

Therefore, change in pressure with installation of evasee

$= 1000 - 640 = 360$ Pa

And this is equal to evasee pressure gian.

Thus, $625\eta = 360$

$$\Rightarrow \eta = \frac{360}{625} = 0.576 = 57.6\%$$

95. Given data,

weight of sample (W) = 50g

Moisture (M) = 0.80g

Ash (A) 7.85g

Volatile matter (VM) = 15.90g

Therefore, weight of residual sample

= Initial weight of sample – Moisture – Ash = 50 – 0.8 – 7.85 = 41.35g

Percentage of VM $= \frac{15.9}{41.35} * 100 = 38.45$

Percentage of FC on a dry ash free basis = 100-%VM = 100 – 38.45 = 61.55

■

4.0 Mine Environment

4.1 Air Pollution

4.1.1 Definition

Air pollution is the excessive concentration of foreign matter in the air which adversely affects the well being of the individual or causes damage to the property. The Suspended Particulate Matter (SPM), Respirable Particulate Matter (RPM), Sulphur Dioxide (SO_2) and Oxides of Nitrogen (NOX) concentration in downwind direction considering predominant wind direction, at a distance of 500 m from the following dust generating source (Mines) shall not exceed the standard specified in the table 4.1.

Table 4.1 : Standards for Coal Mines (Stipulated by Ministry of Environment and Forests (MoEF), Vide Notification No. GSR 742(E), dt. 25.09.2000)

Pollutant	Time weighted Average	Concentration in Ambient Air	
		New Coal Mines (Commenced after 25.09.2000)	Existing Coal Mines (commenced prior to 25.09.2000
Suspended Particulates Matter (SPM)	Annual*	$360 \mu g/m^3$	$430 \mu g/m^3$
	24 hours**	$500 \mu g/m^3$	$600 \mu g/m^3$
Respirable Particulate Matter (Size less than $10 \mu m$) (SPM)	Annual*	$180 \mu g/m^3$	$215 \mu g/m^3$
	24 hours**	$250 \mu g/m^3$	$300 \mu g/m^3$
Sulphur dioxide (SO_2)	Annual*	$80 \mu g/m^3$	$80 \mu g/m^3$
	24 hours**	$120 \mu g/m^3$	$120 \mu g/m^3$
Oxides of Nitrogen (NO_2)	Annual*	$80 \mu g/m^3$	$80 \mu g/m^3$
	24 hours**	$120 \mu g/m^3$	$120 \mu g/m^3$

Note : (i) Annual arithmatic mean of 24- hourly / 8 hourly values shall be met 92% of the time in a year. However, 8% of the time it may exceed but not on two consecutive days.

(ii) In case of residential of commercial or industrial place falls within 500 metres of any dust generating sources, the Ntional Ambient Air Quality Standards (Shown in Table 4.2) shall be made applicable.

4.1.2 Air Pollutants Standard

The revised air pollution standards as per CPCB, New Delhi published in November, 2009 are given in Table 4.2.

Table 4.2 : National Ambient Quality Standards

Pollutant	Time weighted Average	Concentration in Ambient Air	
		Industrial, Residential, Rural and other area	Ecologically Senstive area (notified by Central Government)
Particulate Matter (Size less than 10µm) or PM_{10}	Annual*	60µg/m³	60µg/m³
	24 hours**	100µg/m³	100µg/m³
Particulate Matter (Size less than 2.5µm) or $PM_{2.5}$	Annual*	40µg/m³	40µg/m³
	24 hours**	60µg/m³	60µg/m³
Sulphur Dioxide (SO_2)	Annual*	50µg/m³	20µg/m³
	24 hours**	80µg/m³	80µg/m³
Oxides of Nitrogen (NO_2)	Annual*	40µg/m³	30µg/m³
	24 hours**	80µg/m³	80µg/m³
CO	8 hours**	2 mg/m³	2 mg/m³
	1 hour**	4 mg/m³	4 mg/m³
Ozone (O_3)	8 hours**	100 µg/m³	100 µg/m³
	1 hour**	180 µg/m³	180 µg/m³
Lead (Pb)	Annual*	0.5 µg/m³	0.5 µg/m³
	24 hours**	1.0 µg/m³	1.0 µg/m³
Ammonia (NH_3)	Annual*	100 µg/m³	100 µg/m³
	24 hours**	400 µg/m³	400 µg/m³

* Annual arithmetic mean of minimum 104 measurements in a year taken twice a week 24 hourly at uniform interval.

**24 hourly or 8 hourly or 1 hourly monitored values, as applicable, shall be complied with 98 % of the time in a year, 2 % of the time, that may exceed the limit but not on two consecutive days of monitoring.

4.1.3 Sources of Air Pollution in Mining Industries

The main sources of dust generation in the mining industries are

- Haul road
- Drilling operation
- Loading plant
- Crushing plant

4.1.4 Air Pollution Control Devices:

☐ **Gravity Settler :** The simplest kind of air cleaning device is the settling chamber, which is shown below in Figure 4.1. It is basically an enlarged section or compartment in a flue in which the velocity of the carrier gas is reduced. When the gas velocity is slowed sufficiently, coarse particles (> 40 µm in diameter) can settle out by gravity. A settling chamber of this type usually serves as a pre-cleaner, to prevent clogging of the more efficient small particle collectors that must follow it. Baffles are usually added inside the chamber to increase settling and particulate removal efficiency.

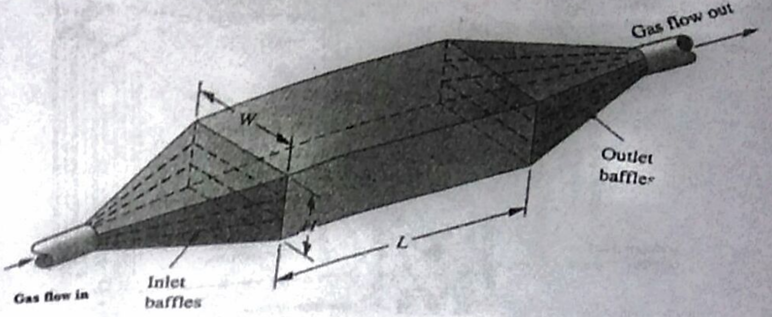


Fig. 4.1 Gravity Settler

To compute the efficiency-partical diameter relationship, we get

$$\eta = \frac{L\left(gD^2\rho_P/18\mu\right)}{HV_{avg}}$$

Where, V_{avg} = Horizontal velocity of the gas in the chamber $= \frac{Q}{WH}$

D = diameter of the particulate matter.

ρp = density of the particulate matter

µ = coefficient of viscosity

H = Height of the gravity settling chamber

L = length of the gravity settling chamber

W = width of the gravity settling chamber

☐ **Electrostatic Precipitator :** The electrostatic precipitator (ESP) is like a gravity settler or centrifugal separator, but electrostatic force drives the particles to the wall. The working mechanism and components of an ESP is shown in Figure 4.2. It is effective on much smaller particles than the previous two devices.

In all three kinds of devices, the viscous (Stoke's law) resistance of the particle to being driven to the wall is proportional to the particle diameter. In ESPs the resisting force is proportional to the diameter.

To compute the efficiency-particle diameter relationship, we get

$$\eta = 1 - \exp\left(\frac{V_t A}{Q}\right) \Rightarrow \text{Deutsche – Anderson equation}$$

Where, Vt = terminal settling velocity of the particulate matter

A = Area of cross section of the plates

Q = Rate of flow of polluted gas.

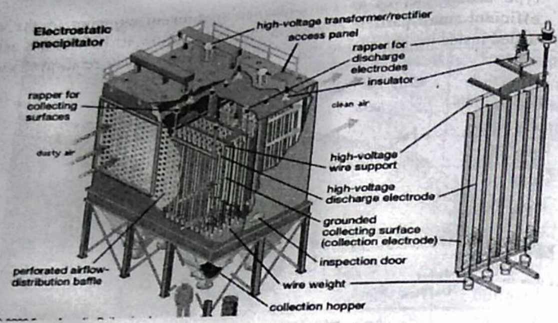

Electrostatic precipitator
high-voltage transformer/rectifier
access panel
rapper for discharge electrodes
insulator
rapper for collecting surfaces
clean air
dusty air
high-voltage wire support
high-voltage discharge electrode
grounded collecting surface (collection electrode)
inspection door
perforated airflow-distribution baffle
wire weight
collection hopper

Fig. 4.2 Electrostatic Precipitator

☐ *Cyclone Separator: It is probably the most widely used particle collection device in the world. A cyclone consists of a vertical cylindrical body, with a dust outlet at the conical bottom as shown in Figure 4.3. The gas enters through a rectangular inlet, normally twice as high as it is wide, arranged tangentially to the circular body of the cyclone, so that the entering gas flows around the circumference of the cylindrical body, not radially inward. The gas spirals around the outer part of the cylindrical body with a downward component, then turns and spiral upward, leaving through the outlet at the top of the device. During the outer spiral of the gas the particles are driven to the wall by centrifugal force, where they collect, attach to each other, and form larger agglomerates that slide down the wall by gravity and collect in the dust hopper in the bottom.*

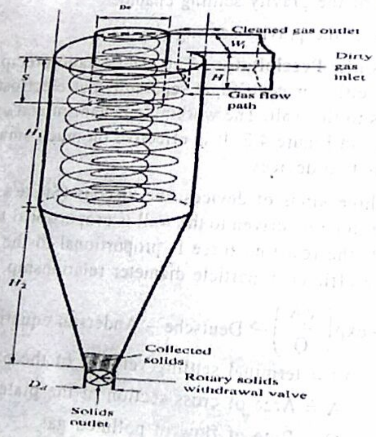

De
Cleaned gas outlet
W_i
Dirty gas inlet
H
S
Gas flow path
H_1
H_2
Collected solids
Rotary solids withdrawal valve
D_d
Solids outlet

Fig. 4.3 Cyclone Separator

Efficiency of particulate matter removal in a cyclone separator can be determined by

$$\eta = \frac{N\pi V_c D^2 \rho_P}{9 W_i \mu}$$

Where, N = effective number of turns a gas makes in traversing the cyclone.
ρ_p = density of the particulate matter
μ = coefficient of viscosity
D = diameter of the particulate matter.
Wi = width of the inlet
H = height of the inlet
Vc = centrifugal velocity

Cut Diameter : Cut diameter is the diameter of a particle for which the efficiency curve has the value of 0.5, i.e., 50 percent.

Cut diameter for block flow model is

$$D_{cut} = \left(\frac{9 W_i \mu}{2\pi N V_c \rho_P} \right)^{1/2}$$

Schematic of a cyclone separator : Dimensions are typically based on the overall diameter D_o.

Taken as ratios to that dimensions,

$W_i = 0.25$ $H = 0.5$ $H_1 = 2$ $H_2 = 2$
$D_e = 0.5$ $S = 0.625$ $D_d = 0.25$

The principal differences are that high efficiency cyclones have smaller values of W_i.

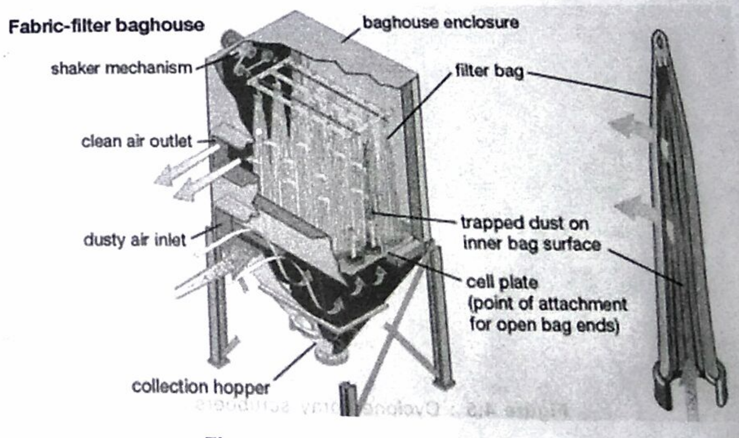

Fabric-filter baghouse
baghouse enclosure
shaker mechanism
filter bag
clean air outlet
dusty air inlet
trapped dust on inner bag surface
cell plate (point of attachment for open bag ends)
collection hopper

Figure 4.4 : Fabric-filter Baghouse

☐ **Fabric Bag Filter :** In a fabric filter system shown in Figure 4.4, the particulate- laden gas stream passes through a woven or felted fabric that filters out the particulate matter and allows the gas to pass through. It typically consists of a tubular bag or an envelope, suspended or mounted in such a manner that the collected particles fall into a hopper when dislodged from the fabric. Small particles are initially retained on the fabric by direct interception, inertial impaction, diffusion, electrostatic attraction, and gravitational settling. After a dust mat has formed on the fabric, more efficient collection of submicron particles (+99%) is accomplished by sieving.

Filter bags, usually tubular or envelope- shaped, are capable of removing most particles as small as 0.5 µm and will remove substantial quantities of particles as small as 0.1 µm. Filter bags ranging from 1.8 to 9 m long, can be utilized in a bag-house filter arrangement similar to shown below.

- **Wet Scrubber :** Scrubbers effectively divide the flow of particle-laden gas by sending many small drops through it. In air pollution control engineering, the term scrubber originally meant a device for collecting fine particles on liquid drops. Most fine particles will adhere to a liquid drop if they contact it. So if we can make the drop and the particle touch each other, the particle will caught on the drop. A typical Cyclone spray scrubbers is shown in Figure 4.5.

There are several types of wet scrubbers as

- Spray-chamber scrubbers
- Cyclone spray scrubbers
- Orifice scrubbers
- Wet-impingement scrubbers
- Venturi scrubbers

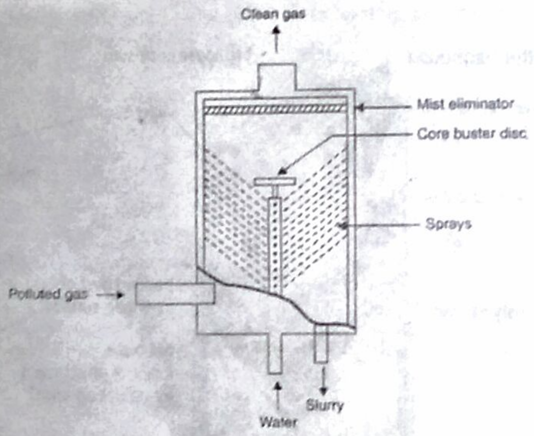


Figure 4.5 : Cyclone spray scrubbers

4.1.5 Miner's Diseases and Dust Hazards :

Mine occupational health hazards leads to the following diseases due to poor environmental working conditions :

A. **Nystagmus :** This is a type of disease in which muscles and nerves of the eyes are affected and there is an abnormal movement of the eyeballs. This disease is caused by working in places of insufficient light.

B. **Ankylostomiasis :** This refers to the disease like anemia or hookworm disease and is caused by blood sucking worm enters to the body during working in insanitary conditions or standing in dirty water with bare foot.

C. **Pneumoconiosis :** This is a type of lungs diseases caused by working in the dusty environment for a long period.

Different types of dusts causes the same type of disease but can be classified as :

i. Fibrosis producing : Silicosis (caused by Silica or Quartz dust) and Asbestosis (Silicate of magnesium) are the most important in this group.

ii. Non-Fibrosis producing : These dusts are inert and become encapsulated in the tissues or absorbed.

iii. Toxic producing : This group comprises lead, mercury and other heavy metals dust to cause the disease.

4.1.6 Dust Sampling Methods

- **Filtration :** In this method, dust is collected in the filter paper and on analyzing this dust concentration can be calculated. This method is adopted in general for particles whose diameter is less than 10 µ.
- **Sedimentation :** This method is adopted in general for particles whose diameter exceeds about 10 µ.
- **Impingement Method :** Separation of particulates from an air stream by abrupt collision against a flat surface forms the basis of this method.

4.2 Noise Pollution

4.2.1 Definition

Sound (or noise) is the result of pressure variations, or oscillations, in an elastic medium (e.g., air, water, solids), generated by a vibrating surface, or turbulent fluid flow. Sound propagates in the form of longitudinal (as opposed to transverse) waves, involving a succession of compressions and rarefactions in the elastic medium. When a sound wave propagates in air (which is the medium considered in this document), the oscillations in pressure are above and below the ambient atmospheric pressure. The unit of sound pressure level is Pa. Sound waves which consist of a pure tone only are characterised by the following parameters given in Figure 4.6.

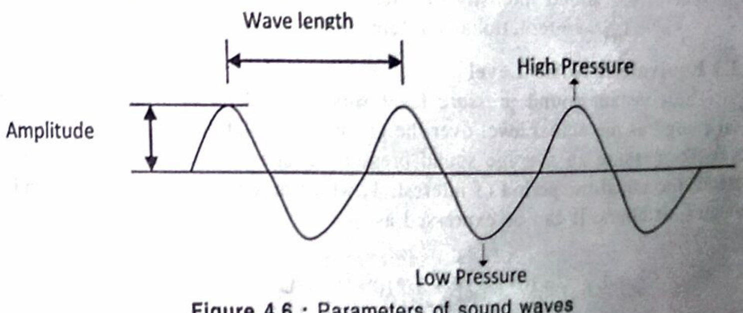


Figure 4.6 : Parameters of sound waves

The speed of a wave is equal to the product of its wavelength and frequency.

$v = f * \lambda$

Where, v = speed of sound, m/s
f = frequency, Hz
λ = wavelength, m

4.2.2 Measurement of Noise

The human ear is a remarkably sensitive organ. The audible frequency range for the human ear is 20 – 20000 Hz. The average person with normal hearing can detect a sound of pressure amplitude as small as 0.0002 µbar and in the higher range without causing pain up to 1000 µbar. The range of pressure covers a very wide spectrum (from 0.0002 – 1000 µbar). Thus measurement of sound levels with pressure units that can vary over a wide range of values is impractical and inconvenient. To avoid this, a logarithmic scale called decibel scale is used for the noise measurement. The measurement unit is called decibels (dB).

The magnitude of volume or a sound expressed in decibels is called a sound pressure level (SPL). An SPL is defined mathematically as

$$SPL = 20*\log_{10}\left(\frac{P}{P_0}\right)$$

Where, SPL = sound pressure level, dB
P = r.m.s sound pressure, µbar
P_0 = reference pressure, µbar $(=2*10^{-5})$

Similarly, Sound power level can be represented by

$$L_w = 10*\log_{10}\left(\frac{W}{W_{ref}}\right)\text{dB}$$

Where, ➢W = acoustic power of interest (Watt)
➢Wref = internationally accepted reference = 10^{-12} watt

Sound Intensity Level : The acoustical power passing through a unit area is defined as the sound intensity. Symbolically it can be represented as

$I = W/\ A\ [W/m^2]$

In logarithmic scale, it can be defined as

$$L_i = 10*\log\left(\frac{I}{I_{ref}}\right)\text{dB}$$

Where, I = sound intensity of interest (W/ m^2)
I_{ref} = internationally reference for sound intensity = 10^{-12} W/m^2

4.2.3 Equivalent Noise Level

The constant sound pressure level which would have produced the same total energy as the actual level over the given time is defined as equivalent noise level. To measure an average sound pressure level the meter averaging time is extended to equal the period of interest, T, which may be an interval in seconds, minutes, or hours. It can be expressed as

$$L_{Aeq.T} = 10*\log_{10}\left[\sum_{i=1}^{n} t_i * 10^{L_i/10}\right]$$

Where, $L_{aeq,T}$ = equivalent sound pressure level
L_i = sound pressure level for the time duration of t_i
t_i = fraction of time for the sound pressure level L_i

Day-Night Equivalent Noise Levels (Ldn): The day night equivalent noise levels of a community can be expressed as –

$$L_{dn} = 10 \times \log_{10}\ [15/24\ (10^{Ld/10}) + 9/24\ (10^{(Ln\ +\ 10)/10})]\ \text{dB (A)}$$

Where, L_d = day-equivalent noise levels (from 6AM - 9 PM), dB (A)
L_n = night equivalent noise levels (from 9 PM - 6 AM), dB (A)

The day hours in respect to assessment of noise levels, is fixed from 6 AM - 9 PM (i.e., 15 hrs) and night hours from 9 PM - 6 AM (i.e., 9 hrs). A sound level of 10 dB is added to Ln due to the low ambient sound levels during night for assessing the Ldn values.

[Note : Indian standard defined. 6 Am – 10PM as day time and 10 PM to 6Am as night time.]

4.2.4 Frequency Spectrum

The second parameter, in addition to sound pressure level, required to characterize the noise is frequency. The noise level for the particular source varies over the frequency range. The basic concept here lies in the definition of bandwidth (BW) :

$$f_{n+1} - f_n = \text{bandwidth}$$

Where, f_{n+1} & f_n are successive frequency band limits i.e. Upper and Lower bands respectively.

Now these bands are geometrically related by the following recursive relationship :

$$\frac{f_{n+1}}{f_n} = 2^K$$

The index K is a positive integer or fraction depends on the octave band. For 1/ 1 octave band the value of K is 1 and for 1/3 octave band K is 1/3 and so on.

Finally, for all types of octave band analysis, there is a central frequency (fc) and it is given by

$$f_c = \sqrt{f_n * f_{n+1}}$$

4.2.5 Frequency Weighting

A weighted frequency scale is simply a value of corrections which is applied to sound pressure levels on a power basis as a function of frequency. The frequency weighting network alters the frequency response of the sound level meter. These frequency corrections are designed to match the frequency weighting scales. Therefore, when selected, frequency band level corrections need not be applied when measuring A- and C- weighted values. Statistically, the A scale follows closely the frequency response of the human ear to sound; that is the ear is less sensitive to frequencies below 1000 Hz. The frequency response characteristics for the various weighting circuits are given below in Figure 4.7.

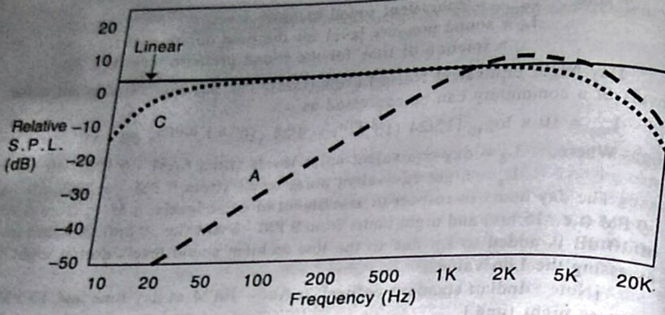


Fig. 4.7 Frequency response characteristics for weighting networks of sound level meters

4.2.6 Sound Wave Propagation

4.2.6.1 Point Source : *The relationship between the sound pressure level (Lp) and sound power level (Lw) is given by*

$$L_P = L_W - 20*\log_{10}(r) - 11 \quad [dB]$$

Where, L_w *= sound power level of the point source*

L_P *= sound pressure level of the point source at a adial distance of r from the noise source*

r = radial distance from source in m

Thus, the relationship between the two sound pressure levels at a radial distance of r_1 *and* r_2 *can be obtained from the above equation is*

$$L_{P1} - L_{P2} = L_w - 20*\log_{10}(r_1) - 11 - L_w - 20*\log_{10}(r_2) - 11$$

$$\Rightarrow L_{P1} - L_{P2} = 20*\log_{10}(r_2) - \log_{10}(r_1)$$

where L_{p1} *&* L_{p2} *= sound pressure level at a radial distance of* r_1 *and* r_2 *respectively.*

4.2.6.2 Line Source : *Similarly, the relationship between the two sound pressure levels at a radial distance of* r_1 *and* r_2 *can be obtained from the above equation is*

$$L_{p1} - L_{p2} = 10*[\log_{10}(r_2) - \log_{10}(r_1)]$$

[Note : The variation is due to the sound propagation characteristics of the point and line sources. In case of point source $Lp \propto 1/r^2$ *whereas for line sources* $Lp \propto 1/r$*]*

4.2.7 Noise Pollution Standards

4.2.7.1 Worksite Standards : *Based on the study conducted in the different parts of the world with respect to harmful consequence of noise exposure, it was realized by the Directorate General of Mines Safety (DGMS) to define noise limits for the mining areas in order to achieve the following objectives :*

- To prevent a risk of hearing impairment
- To prevent interference of communication essential for safety purposes, and
- To eliminate nervous fatigue with due consideration to the work to be done.

Accordingly following standards and guidelines, based on ILO Code of Practice have been recommended [DGMS Circular No.18 (Tech), 1975].

- A warning limit of 85-dB (A) may be set as the level below which very little risk to an unprotected ear of hearing impairment exists for an eight-hour exposure.
- The danger limit value shall be 90-dB (A) above that the danger of hearing impairment and deafness may result from an unprotected ear.
- A worker should not be allowed to enter, without appropriate ear protection, an area in which the noise level is 115-dB (A) or more.
- Personal protective equipment shall be worn, if there are single isolated outbursts of noise, which can go above 130-dB (A) "Impulse", or 120-dB (A) 'Fast'.
- No worker shall be allowed to enter an area where noise level exceeds 140-dB (A).

4.2.7.2 Ambient Standards : The ambient noise standards as per CPCB is given in Table 4.3

Table 4.3 : Ambient Air Quality Standards in respect of Noise

Area Code	Category of Area/Zone	Leq, Limits in dB(A)	
		Day Time	Night Time
(A)	Industrial Area	75	70
(B)	Commercial area	65	55
(C)	Residential area	55	45
(d)	Silence Zone	50	40

Note :

1. Day time shall mean from 6.00 a.m. to 10.00 p.m.
2. Night time shall mean from 10.00 p.m. to 6.00 a.m.
3. Silence zone is defined as an area comprising not less than 100 m around hospitals, educational institutions, courts, religious places or any other area which is declared as such by the competent authority.
4. Mixed categories of areas may be declared as one of the four above-mentioned categories by the competent authority.

4.3 Water Pollution

4.3.1 Water Pollution Parameters

4.3.1.1 **pH :** The pH is a dimensionless number that indicates the strength of an acidic or a basic solution. It can be defined as the negative logarithm of the hydrogen ion concentration. The pH scale ranges from 0 – 14. Pure water is neutral (pH = 7) because it contains the same number of hydrogen ions (H^+ ions) as hydroxyl ions (OH^- ions).

4.3.1.2 **Total Suspended Solids :** *Solids occur in water either in solution or in suspension. These two types of solids are distinguished by passing the water sample through a glass fiber filter. By definition, the suspended solids are retained on top of the filter and the dissolved solids pass through the filter with the water.*

Total suspended solids (TSS) can be calculated by the following equation

$$TSS = (A - B) / C$$

Where,
A = *weight of the filter plus retained solids*
B = *weight of the clean filter*
C = *volume of sample filtered*

4.3.1.3 **Bio-chemical Oxygen Demand (BOD) :** The amount of oxygen required to decompose the biodegradable organics present in the waste water is called the bio-chemical oxygen demand. BOD is used as an indirect measure of the total amount of biodegradable organics in the water. The amount of oxygen required to decompose all the biodegradable organics in a given volume of water is called the ultimate BOD or BoD_L.

BOD at any time of a given solution can be determined by the following equation

$$BOD_t = BOD_L * \underline{(1 - 10^{-kt})} \; \xrightarrow{} (1-e^{-Kt})$$

Where,
BOD_t = *BOD at any time t, mg/L*
BoD_L = *ultimate BOD, mg/L*
k = *a constant representing the rate of the BOD reaction*
t = *time, day*

4.3.1.4 **Chemical Oxygen Demand (COD) :** *There also might be non-biodegradable substances present in the water which cannot be decomposed by the microbes to use as food. In COD test a strong oxidizing agent ($K_2Cr_2O_7$) is used to oxidize the organics rather that relying on microorganisms to do the job. COD test indicates the oxygen required decomposing the organics (biodegradable + non-biodegradable) present in the waste water and thus it is always greater than the BOD. The result of the COD test can be obtained in just 2 hour whereas for BOD test standard time required is 5 days.*

4.3.2 Standards of Effluents Discharge from Coal Mines

The standard of effluents discharge from coal Mines is given in Table 4.4.

Table 4.4 : Standards of Effluents Discharge from Coal mines

Parameters	Maximum permissible value
pH	5.5 - 9.0
Total Suspended Solids	100 mg/L [200 mg/L in case of discharge into Land for Irrigation]
Bio-chemical Oxygen Demand (BOD)	250 mg/L
Chemical Oxygen Demand (COD)	30 mg/L
Oil & Grease (O & G)	10 mg/L

4.4 Illumination

4.4.1 Definitions of Important terms of Photometry

4.4.1.1 **Intensity :** Intensity of light is the relative amount of luminous energy given by any source and is measured in candles or candle power or in candelas.

4.4.1.2 **Horizontal Illuminance :** The measure of brightness from a light source, usually measured in foot candles or lumens, which is taken through a light meter's sensor at a horizontal position on a horizontal surface.

4.4.1.3 **Vertical Illuminance :** The measure of brightness from a light source, usually measured in foot candles or lumens, which is taken through a light meter's sensor at a vertical position on a vertical surface.

4.4.1.4 **Mean Spherical Candle Power (m. s. c. p) :** It is the average candle power of a lamp in all directions, or the candle power of a uniform source giving the same total flux of light. It is directly proportional to the total light given by the lamp and is measured by taking intensity readings in all directions.

4.4.1.5 **Illumination :** It is the total amount of visible light illuminating (incident upon) a point on a surface from all directions above the surface as shown in Figure 4.8. This can be measured in foot candles or in metre candle. One meter candle is the intensity of illumination on a surface of 1 m distant from a source of one candela. Illumination at a surface is inversely proportional to the square of the distance of the surface from the source of light, and directly proportional to cos(angle between the normal to the surface and the direction of light rays).

Illumination of a surface (meter candle) = (Candela of source)* $\cos\theta$ / (distance in m)^2

Where, θ is the angle between the normal to the surface and the direction of light rays.

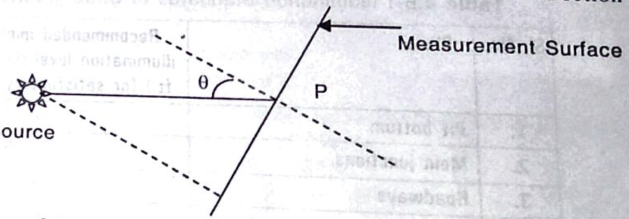


Figure 4.8 : Illumination measurement at a plane

4.4.1.6 **Lumen (lm) :** This is the unit of light (luminous flux) emitted by a light source.

Lumen emitted by a lamp = mean spherical candle power * 4π

4.4.1.7 **Lux :** It is the unit of illumination in S.I. units. A lux is an illumination of 1 lumen/ m^2.

4.4.1.8 **Footcandle** (fc) : A unit of measure of illumination. A unit of illumination on a surface that is one foot from a uniform point source of light of one candle and equal to one lumen per square foot. Foot candle values can be measured directly with handheld incident light meters. One foot candle is equal to 1 lumen per sq. ft. of surface.

$$\mathbf{1\ fc = 10.764\ lx}$$

4.4.2 Illumination Standards in Mines

4.4.2.1 **Opencast Mining Areas :** The illumination standards in opencast mines are given in Table 4.5.

Table 4.5 : Illumination Standards in Opencast Mines

Sl. No	Location	Minimum illumination in Lux	Level in which illumination is to be provided
1.	Operational area of draglines and shovels	5	Horizontal
		10	Verticals
2.	Operational area of drills	10	Vertical
3.	Operater's cabin of Shovel, Dragline, Drill etc.	30	Horizontal
4.	Dumper haul road	0.5 - 3.0	Horizontal
5.	Overburden dumps and coal dups	3	Horizontal
6.	Roadways and footpaths from bench to bench	3	Horizontal
7.	Coal handling plant, workshop and service building	As per the BIS specification	

4.4.2.2 **Underground working areas :** The illumination standards in opencast mines are given in Table 4.6.

Table 4.6 : Illumination Standards in Underground Mines

Sl. No.	Place	Recommended minimum average illumination level (in lumens per sq. ft.) for satisfactory light condition
1.	Pit bottom	1.5 - 3.0
2.	Main junctions	1.25
3.	Roadways	0.4
4.	Haulage engines, control gear and haulage drum	1.5

EXERCISE - 4

1. *An air quality parameter required to be monitored under the Indian National Ambient Air Quality Standards is*
   a. *As* b. *Pb*
   c. *Hg* d. *Silica*
   [GATE 2012]

2. *In an area within a surface mine, under static condition the following gases are found:* NO_2, CO_2, O_3, *and* SO_2. Assuming no diffusion, reaction and bonding of the gases, the concentration of the gases from bottom upwards will be in the order of
   a. NO_2, CO_2, O_3, and SO_2 b. SO_2, NO_2, CO_2, and O_3
   c. SO_2, O_3, NO_2, and CO_2 d. NO_2, CO_2, SO_2 and O_3
   [GATE 2009]

★

3. A spherical droplet of water, with density 1000 kg/m^3 and diameter of 1 μm, is falling in air. The viscosity of air is 1.85×10^{-5} kg/m.s. Neglecting air density and assuming that the settling of droplet in air follows Stokes' Law, the settling velocity in m/s is
   a. 0.98×10^{-5} b. 2.95×10^{-5}
   c. 8.04×10^{-5} d. 53.03×10^{-5}
   [GATE 2012]

4. In ambient air of industrial area, the permissible concentration of suspended particulate matter less than 10 μm size measured on 24 hours average in μg/m^3 is
   a. 200 b. 300
   c. 100 d. 400

5. In ambient air of industrial area, the permissible concentration of suspended particulate matter less than 10 μm size measured on annual average in μg/m^3 is
   a. 50 b. 60
   c. 80 d. 100

6. In ambient air of industrial area, the permissible concentration of particulate matter less than 2.5 μm size measured on 24 hours average in μg/m^3 is
   a. 20 b. 40
   c. 60 d. 100

7. The size of the respirable particulate matter (RPM) considered in the range of
   a. < 5 μm b. > 5 μm
   c. < 10 μm d. < 1 μm

8. At any particular place the relation between the suspended particulate matter (SPM) and respirable particulate matter (RPM) is
   a. SPM > RPM b. SPM < RPM
   c. SPM = RPM d. There is no relation

9. In ambient air of an School locality, the permissible concentration of PM_{10} measured on 24 hours average in μg/m^3 is
   a. 50 b. 150
   c. 75 d. 100

10. In ambient air of Hospital locality, the permissible concentration of $PM_{2.5}$ measured on 24 hours average in μg/m3 is
    a. 75 b. 50
    c. 40 d. 60

11. During 24 hours sampling of mass concentration of suspended particulate matter (SPM) by the High Volume Sampler, the following data are obtained :
    Air flowing through the filter : 1.7 m^3/ min
    Air flowing through filter at the end of the test : 1.5 m^3/ min
    Mass of the clean filter : 5.0 g
    Mass of the filter after 24 hours of exposure : 5.4 g
    Assuming the flow rate through the sampler decreases linearly, the average concentration of suspended particulates in μg/m^3 is

a. 163.40 b. 173.61 [GATE 2006]
c. 185.18 d. 190.20

12. A high volume air sampler is operated for 8 hrs in a mine with the flow rate of air varying from 1.5 m^3/min to 1.3 m^3/min. the empty weight of the filter paper is 2.30 g and the final weight is 2.65 g. The mean concentration of the suspended particulate matter (SPM) during the study period in μg/ m^3 is
a. 591 b. 550 [GATE 2010]
c. 545 d. 521

13. To minimize the air pollution problem, residential buildings in the mining areas should be located in
a. Downwind direction of predominant wind direction
b. Upward direction of predominant wind direction
c. Perpendicular to predominant wind direction
d. At any place away from the mines

14. The most wholesome ventilation of buildings is provided by
a. Natural ventilation b. Plenum system of ventilation
c. A.C. system of ventilation d. Vacuum system of ventilation

15. The important gaseous pollutants, contributing to acid rains, are
a. SO_2 and NOx b. CO_2 and H_2S
c. NOx and O_3 d. None of these

16. The device, which can be used to control gaseous as well as particulate pollutants, in the industrial emissions, is known as
a. Cyclone separator b. Spray tower
c. Dynamic precipitator d. Fabric filter

17. Electrostatic precipitators remove
a. SO_2 b. NOx
c. Particulate matter d. None of the above

18. Electrostatic precipitator works on the principle of
a. Capacitance change b. Ionisation of the particles
c. Electro heating of gases d. Centrifuging the gaseous molecules
[GATE 2008]

19. Cyclone, bag filter and scrubber can be used for control of
a. water pollution b. air pollution
c. soil pollution d. noise pollution [GATE 2014]

20. Which one of the followings is secondary pollutant?
a. SO_2 b. NOx
c. O_3 d. Hydrocarbons

21. Hairs of our nose can remove all particles of size, greater than
a. 1 mm b. 1 micron
c. 10 micron d. 100 micron

22. The sound pressure level is measured as 5×10^{-4} N/m^2. What is the noise level in dB?
a. 27.9 b. 30.7
c. 28.7 d. 25.0

23. The noise levels measured during the operation of three machines are 80 dBA, 75 dBA and 85 dBA respectively. Determine the equivalent noise level for the above three machines.
a. 81.73 b. 86.57
c. 80.00 d. 240.0

24. At a surface mine office the independent Sound Pressure Level (SPL) measured in dB(A) on account of 3 drill machines are 85,88 and 85. If all the three machines work simultaneously, the combined SPL, in dB (A) is
a. 91 b. 90
c. 92 d. 94
[GATE 2010]

25. It is required to find out the day-night equivalent noise levels at a location. The three-hourly day average values in dB are 48, 54, 56, 52, 61 and three-hourly night average values in dB are 36, 42, and 48. What is value of Ldn?
a. 43.9 b. 56.9
c. 55.7 d. 45.8

26. The noise level measured during the operation of a jackhammer are given below :

Noise Level	Duration
70 dBA	30 min
80 dBA	15 min
85 dBA	10 min

The equivalent noise level (Leq) in dBA calculated from these data is
a. 88.50 b. 79.55
c. 71.45 d. 69.55
[GATE 2005]

27. A worker is exposed to 86 dB for six hours and 92 dB for three hours, giving a nine hour working day. Determine the dose % and TWA.
a. 92.3%, 89.4 b. 89.4% 69.4
c. 89.4%, 86.4 d. 92.3%, 88.4

28. If the measured sound pressure level at a distance of 10 m from a source is 120 dB, the sound pressure level in dB at a distance of 60 m from the source is
a. 104.44 b. 121.55
c. 127.78 d. 135.56
[GATE, 2006]

29. On a flat terrain noise received from a source 100 m away is 82 dBA. If the receiver is 160 m away from source, what is the noise level in dBA?
a. 72 b. 65
c. 81 d. 78
[GATE, 2001]

30. What is the center frequency of the band number whose lower and upper bands are 1414 and 2828 Hz respectively?
a. 3000 Hz b. 1814 Hz
c. 2000 Hz d. 1500 Hz

31. What is the center frequency for the one-third (1/3) octave band whose lower and upper frequency limits are 7079 and 8913 Hz, respectively?
a. 7000 Hz b. 7943 Hz
c. 8000 Hz d. 7852 Hz

32. What is the sound power level of a sound source radiating energy at a rate of 0.2 W?
    a. 110 dB
    b. 112 dB
    c. 113 dB
    d. 120 dB

33. What is the acoustical power of a 100 hp air compressor whose rated sound power level is 130 dB
    a. 13 W
    b. 9 W
    c. 10 W
    d. 12 W

34. The sound intensity level of a particular mine machine at a radial distance of 10 m is 2.4 * 10-7 W/ m2. Determine the sound power level of the noise source in dB. Assume that the sound waves are radiating uniformly in all the directions.
    a. 84.7 dB
    b. 95.4 dB
    c. 80.0 dB
    d. 75.4 dB

35. A small source whose sound power level Lw is 110 dB is hanging freely outdoors. What is the sound pressure level at 20 m from the source?
    a. 73 dB
    b. 75 dB
    c. 77 dB
    d. 76 dB

36. The noise level at 10 m from a long conveyor belt was 95 dBA. What is the noise level at 100 m?
    a. 85 dBA
    b. 90 dBA
    c. 95 dBA
    d. 80 dBA

37. As par the DGMS norm, warning level of noise for the worksite area is
    a. 85 dBA
    b. 90 dBA
    c. 95 dBA
    d. 100 dBA

38. A jackhammer operates at a corner of a square field of side 50 m. At the diagonally opposite corner, the SPL sensed is 82.3 dB. The SPL at any of the other two corners of the field in dB is
    a. 86.3
    b. 85.3
    c. 83.6
    d. 81.2

[GATE, 2007]

39. In a surface mine, sound pressure level at a location generated by operation of a dozer and a drill respectively are 80 dBA and 60 dBA, when operated independently. The sound pressure generated by the dozer compared to the drill is higher by a factor of
    a. 10
    b. 20
    c. 100
    d. 200

[GATE, 2012]

40. Two machines A and B while operating simultaneously produce a sound pressure level of 85 dBA at a point. When the machine A stops, the sound pressure level at that point reduces to 80 dBA. The sound pressure level at the same point due to machine A operating alone in dBA is
    a. 70.0
    b. 75.0
    c. 80.0
    d. 83.3

[GATE, 2014]

41. The concentration of OH- ion in a mine water sample is 10^{-11} mol/L. The pH of the sample is

    a. 2
    b. 3
    c. 4
    d. 11

[GATE 2009]

42. Precipitation of metallic ions in mine water drainage is carried out by
    a. $CaSO_4$ and $MgSO_4$
    b. $CaCO_3$ and $MgCO_3$
    c. $Ca(OH)_2$ and NaOH
    d. $CaCO_3$ and $MgSO_4$

[GATE 2008]

43. A sample of sewage from a coal mines is found to have a BOD after 5 d (BOD_5) of 180 mg/L. Estimate the ultimate BOD (BOD_L) of the sewage. Assume that k = 0.1 /d for this waste water.
    a. 240 mg/ L
    b. 350 mg/ L
    c. 400 mg/ L
    d. 450 mg/ L

44. A 6 ml sample of mine water is diluted to 300 mL with distilled water in a standard BOD bottle. The initial DO in the bottle is determined to be 8.5 mg/L, and the dissolved after 5 d at 20 °C is found to be 0.5 mg/ L. Determine the BOD_5 of the mine water.
    a. 240 mg/ L
    b. 350 mg/ L
    c. 400 mg/ L
    d. 450 mg/ L

45. The BOD_5 of an effluent from a coke oven plant is 25 mg/L, and the effluent discharge is 4 ML/d. The receiving stream has a BOD_5 of 2 mg/L and the stream flow is 40 ML/d. What will be the BOD_5 in the stream just below the mixing zone?
    a. 4.5
    b. 5.2
    c. 4.1
    d. 4.8

46. An effluent sample is diluted with fresh water to make up a solution of 300 ml. The DO of the solution initially is 8 mg/l and the value falls to 3 mg/l after 5 days. If the 5 day BOD of the original effluent is known to be 50 mg/l, the amount of fresh water added in ml to the solution is
    a. 270
    b. 160
    c. 54
    d. 30

[GATE, 2007]

47. Mine water flowing at 1.5 m^3/s with 2 mg/l dissolved oxygen, joins river water flowing at 7 m^3/s containing 6 mg/l dissolved oxygen. The dissolved oxygen concentration of the mixture in mg/l is
    a. 5.3
    b. 4.8
    c. 4.2
    d. 3.9

[GATE 2008]

48. In the above question, the saturated value of the dissolved oxygen in the mixture is given to be 9.3 mg/l. On the basis, the initial oxygen deficit of the mixture in mg/l is
    a. 2.4
    b. 4.0
    c. 6.8
    d. 14.6

[GATE 2008]

49. The value of COD is related to BOD as
    a. COD > BOD
    b. COD = BOD
    c. COD < BOD
    d. There is no relation

50. The permissible value of oil and grease discharge from the mine workshop is
    a. 10 mg/ L
    b. 20 mg/L
    c. 25 mg/L
    d. 15 mg/L

51. The permissible value of BOD in the effluent discharge from a coal mines is
   a. 120 mg/L
   b. 240 mg/L
   c. 150 mg/L
   d. 250 mg/L

52. The ultimate BOD value of a waste
   a. Increases with temperature
   b. Decreases with temperature
   c. Remains the same at all temperatures
   d. Doubles with every 10°C rise in temperature

53. When a mine water is disposed off in a river, the rate of depletion of dissolved oxygen of the river mainly depends on
   a. BOD of the mine water
   b. COD of the mine water
   c. Total organic carbon present in the mine water
   d. Dissolved oxygen present in the mine water

54. Which of the following pairs is correctly matched?
   a. $\frac{BOD}{COD} = 0$: Mine water is taxic
   b. $\frac{BOD}{COD} \le 0.2$: Acclimatization of seed is necessary
   c. $\frac{BOD}{COD} \ge 0.6$: Mine water is non-biodegradable
   d. BOD = COD = 0 : Mine water is devoid of organic matter

55. The correct statement of comparison of ultimate BOD, COD, Theoritical Oxygen Demand and 5-day BOD (BOD_5) is
   a. $BOD_u > COD > ThOD > BOD_5$
   b. $COD > ThOD > BOD_u > BOD_5$
   c. $ThOD > COD > BOD_u > BOD_5$
   d. $COD > BOD_u > BOD_5 > ThOD$

56. A mine waste dump of pH 5.2 can be neutralized by adding
   a. urea
   b. calcium carbonate
   c. sulphuric acid
   d. sodium chloride

[GATE 2014]

57. A waste water effluent has BOD_5 of 80 mg/L and the reaction rate constant is 0.16 per day. The ultimate BOD in mg/L is
   a. 85
   b. 100
   c. 120
   d. 145

[GATE 2014]

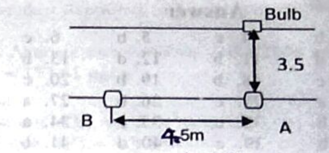
58. 100 ml of waste water is allowed to evaporate in a dish weighing 48.6232 g. The weight of the dish with dry solids is 48.6432 g. The concentration of dry solids in waste water in mg/l is
   a. 200
   b. 220
   c. 260
   d. 320

[GATE 2011]

59. Nystagmus is a miner's disease associated with
   a. Lever
   b. Lung
   c. Eye
   d. Stomach

[GATE 2012]

60. A filament bulb produces 30 lux illumination at point A as shown in the figure. Assuming that no other source of lighting present in the roadway, the illumination level in lux at point B will be



   a. 4.88
   b. 1.63
   c. 0.35
   d. 7.00

[GATE, 2002]

61. Floor illumination at a point directly below a light source in an underground garage of height 4 m is 40 lux. What is the floor illumination in lux at a point 8 m away from the light source?
   a. 2
   b. 5
   c. 10
   d. 20

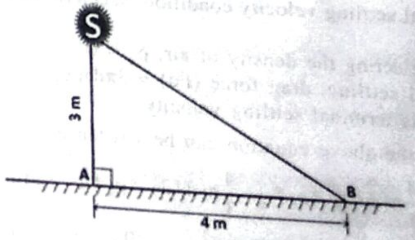
62. The minimum illumination standard at the coal mine pit bottom is
   a. 1.5 lux
   b. 1.5 lm/ ft^2
   c. 0.4 lm/ ft^2
   d. 2.4 lux

[GATE, 1997]

63. A single lamp placed centrally at the roof provides 40 lux illumination vertically below, at the floor of an underground workshop. The workshop of dimensions 20 m × 20 m with height 4m. Assuming uniform spherical dispersion of luminous intensity, the floor level illumination in lux at any corner of the workshop is
   a. 23.2
   b. 10.9
   c. 3.0
   d. 0.8

[GATE, 2007]

64. An illumination source S shown in the figure emits light equally in all directions. At a point A on the floor, the illuminance is 5.0 lux. The illuminance at point B o



   a. 1.08
   b. 3.58
   c. 0.76
   d. 2.78

[GATE, 2014]

65. An incandescent headlight of a mining vehicle is of spot beam type with a beam angle of 30°. The spherical surface in m2 subtended by the lighted beam at a distance of 5 m from the headlight is

[GATE, 2011]

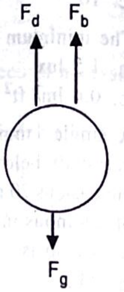
a. 7.5 b. 15
c. 21 d. 25

Answer

1. c	2. c	3. b	4. c	5. b	6. c	7. c
8. a	9. d	10. d	11. b	12. d	13. b	14. c
15. a	16. b	17. c	18. b	19. b	20. c	21. c
22. a	23. b	24. a	25. c	26. b	27. a	28. a
29. d	30. c	31. b	32. c	33. c	34. a	35. a
36. a	37. a	38. b	39. a	40. d	41. b	42. d
43. d	44. c	45. c	46. d	47. a	48. b	49. a
50. a	51. d	52. c	53. a	54. d	55. c	56. b
57. d	58. a	59. c	60. d	61. b	62. b	63. d
64. a	65. c					

Solution of Exercise 3

3. Given data,
Density of water (ρ_d) = 1000 kg/m^3
Diameter of the droplet (d) = 1 μm
Viscosity of air (η) = 1.85*10^{-5} kg/m^{-s}
Let the density of air is ρ_a



Resultant Force acting on the particle (F_r) =
Gravityforce (F_g) – Buoyant force (F_b) – Drag force (F_b)

$$F_r = 0 = mg - V\rho_a g - F_d = \frac{4}{3}\pi(d/2)^3 * \rho_d * g - \frac{4}{3}\pi(d/2)^3 * \rho_a * g - F_d = ma$$

At terminal settling velocity condition, acceleration (a) of the water droplet is zero.
Again, neglecting the density of air, $\rho_a = 0$
At terminal settling, drag force (Fd) = $3\pi d\eta\, v_t$
Where, v_t is terminal settling velocity.

Therefore, the above equation can be written as :

$$\frac{4}{3}\pi(d/2)^3 * \rho_d * g - \frac{4}{3}\pi(d/2)^3 * \rho_a * g - 3\pi d\eta v_t$$

$$\Rightarrow v_t = \frac{1}{18\eta} g * d^2(\rho_d - \rho_a) = \frac{g * d^2}{18\eta} * \rho_d = \frac{9.81*10^{-12}*1000}{18*1.85*10^{-5}} = 2.95*10^{-5}\ m/s$$

11. Given data,
Initial flow rate (Q_i) = 1.7 m^3/min
Final flow rate (Q_f) = 1.5 m^3/min
Mass of clean filter (m_i) = 5g

Mass of filter paper after sampling (m_f) = 5.4 g
Average flow rate (Q_{av}) = (Q_i + Q_f)/2 = (1.7 + 1.5)/2 = 1.6 m^3/min
Total air flow in 24 hours = 1.6*24*60 = 2304 m^3
Mass of the dust deposited in filter paper in 24 hours = $m_f - m_i$ = 5.4 – 5 = 0.4g
Therefore, Average concentration = 0.4/2304 g/m^3 = 400000/2304 μg/m^3
= 173.61 μg/m^3

12. Given data,
Initial flow rate (Qi) = 1.5 m^3/min
Final flow rate (Qf) = 1.3 m^3/min
Mass of clean filter (m_i) =2.30 g
Mass of filter paper after sampling (m_f) = 2.65 g
Average flow rate (Q_{av}) = (Q_i + Q_f)/2 = (1.5 +1.3)/2 = 1.4 m^3/min
Total air flow in 8 hours = 1.4*8*60 = 672 m^3
Mass of the dust deposited in filter paper in 8 hours = $m_f - m_i$ = 2.65 – 2.30
= 0.35 g

Therefore, Average concentration = 0.35/672 g/m^3 = 350000/672 μg/m^3
= 520.83 μg/m^3

22. Given that,
Sound pressure level = 5 *10^{-4} N/m^2

We have, $L_P = 20 \log \frac{P}{P_{ref}}$

Where L_P = sound pressure level in dB
P = measured sound pressure leve in N/m^2
P_{ref} = reference sound pressure level; 2*10^{-5} N/m^2

Therefore, $L_P = 20 \log \frac{5*10^{-4}}{2*10^{-5}} = 20 \log 25 = 27.9$ dB

23. Noise level of three machines are
L_1 = 80 dBA
L_2 = 75 dBA
L_3 = 85 dBA

Equivalent noise level generated by three machines can be determined as

$$L_{Aeq,t} = 10 \log_{10} \sum_{i=1}^{n} t_i 10^{(L_i/10)} = 10 \log_{10}\left(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}} + 10^{\frac{L_3}{10}}\right)$$

$$= 10 * \log_{10} 10^{7.5} + 10 * \log_{10}(14.16) = 75 + 11.51 = 86.51\,\text{dBA}$$

[Since, all the machines generating noise simultaneously, fraction of tim (t_i) for the sound pressure level Li is equal to 1]

24. Noise level of three drill machines are
L_1 = 85 dBA
L_2 = 88 dBA
L_3 = 85 dBA

Equivalent noise level generated by three machines can be determined as

$$L_{Aeq,t} = 10\log_{10}\sum_{i=1}^{n} t_i 10^{(L_i/10)} = 10\log_{10}\left(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}} + 10^{\frac{L_3}{10}}\right)$$

$$= 10\log_{10}\left(10^{\frac{85}{10}} + 10^{\frac{88}{10}} + 10^{\frac{85}{10}}\right) = 10 * \log_{10} 10^{8.5}\left(1 + 10^{0.3} + 1\right)$$

$$= 85 + 10 * \log_{10}(3.99) = 85 + 6 = 91$$

Alternative method :

If two equal sources generating noise simultaneously, then the equivalent level is increased by a level of 3 dBA.

That is, equivalent level of L_1 and L_3 is

$Le = 85 + 3 = 88$ dBA

Now, this equivalent level is equal to the source L_2. Therefore, equivalent noise level for all the three sources is

$= 88 + 3 = 91$ dBA

25. *Day equivalent noise levels :*

$$Lde = 10 * \log_{10}\left(10^{48/10} + 10^{54/10} + 10^{56/10} + 10^{52/10} + 10^{61/10}\right)/5$$

$$= 56.3 \text{ dB}$$

Night-equivalent noise levels :

$$Lne = 10 * \log_{10}\left(10^{36/10} + 10^{42/10} + 10^{48/10}\right)/3$$

$$= 44.4 \text{ dB}$$

Therefore, day-night equivalent noise level, L_{dn} :

$$Ldn = 10 * \log_{10}\left[\frac{15}{24}\left(10^{Lde/10}\right) + \frac{9}{24}\left(10^{Lne+10/10}\right)\right]$$

$$Ldn = 10 * \log_{10}\left[\frac{15}{24}\left(10^{5.63}\right) + \frac{9}{24}\left(10^{4.44}\right)\right]$$

$$L_{dn} = 55.68$$

26. We have, $L_{Aeq.T} = 10 * \log_{10}\left[\sum_{i=1}^{n} t_i * 10^{L_i/10}\right]$

$$L_{Aeq,t} = 10 * \log_{10}(30/55) * 10^{70/70} + (15/55) * 10^{80/10} + (10/55) * 10^{85/10}$$

$$= 10 * \log_{10}(10^7/55)[30 + 15 * 10 + 10 * 10^{1.5}]$$

$$= 10 * [\log_{10} 10^7 - \log_{10} 55 + \log_{10}(496.2)]$$

$$= 10 * (7 - 1.74 + 2.7) = 10 * 7.96 = 79.6 \text{ dBA}$$

27. We have,

$$\text{Dose} = 100 * \left(\frac{C1}{T1} + \frac{C2}{T2}\right)$$

Again, we have, $Tn = \dfrac{8}{2(L-90)/5}$

From the above equation, $T_1 = 13.9$ and $T_2 = 6.1$

Thus, Dose = 100*(6/13.9 + 3/6.1) = 92.3%

Now we can calculate the TWA using the following equation

$TWA = 16.61 \log_{10}(D/10) + 10$

$= 16.61 \log_{10}(92.3/10) + 10$

$= 89.4$ dB

28. Given data,

Sound pressure level at distance r_1 (=10 m) = Lp_1 = 120 dB

Let the sound pressure level at distance r_2 (= 60 m) is Lp_2

We have,

$L_{P1} - L_{P2} = 20 * [\log_{10} r_2 - \log_{10} r_1]$

$\Rightarrow 120 - L_{P2} = 20 * [\log_{10} 60 - \log_{10} 10] = 20 * \log_{10}(60/10) = 20 * 0.78 = 15.5$

$\Rightarrow L_{P2} = 120 - 15.56 = 104.43$ dB

29. Given data,

Sound pressure level at distance r1 (=100 m) = L_{p1} = 82 dBA

Let the sound pressure level at distance r_2 (= 160 m) is L_{p2}

For a point source, we have,

$L_{P1} - L_{P2} = 20 * [\log_{10} r_2 - \log_{10} r_1]$

$\Rightarrow 82 - L_{P2} = 20 * [\log_{10} 160 - \log_{10} 100] = 20 * \log_{10}(160/100) = 20 *$

$0.204 = 4.08$

$\Rightarrow L_{P2} = 82 - 4.08 = 77.91$ dB

30. Given data,

Frequency of upper band (f_u) = 2828 Hz

Frequency of lower band (f_l) = 1414 Hz

Since, $f_u/f_l = 2$, these are the 1/1 octave band frequency.

We have,

Central frequency $(f_c) = \sqrt{f_l * f_u} = \sqrt{2828 * 1414} = \sqrt{3998792} = 2000$ Hz

31. Given data,

Frequency of upper band (f_u) = 8913 Hz

Frequency of lower band (f_l) = 7079 Hz

We have,

Central frequency $(f_c) = \sqrt{f_l * f_u} = \sqrt{7079 * 8913} = 7943$ Hz

32. Given that,

Sound radiating energy (w) = 0.2 W

We have, $L_w = 10 \log \dfrac{w}{w_{ref}}$

Where, L_w = sound power level in dB

w = measured radiating energy in watt

W_{ref} = reference sound energy level; 10^{-12} watt

Therefore, $L_w = 10 \log \dfrac{0.2}{10^{-12}} = 10 \log\left(2 * 10^{11}\right) = 110 + 3 = 113$ dB

33. Given that,

Sound power level (L_w) = 130 dB

Let the acoustical power of the machine is w watt

We have, $L_w = 10\log\frac{w}{w_{ref}}$

$$\Rightarrow 130 = 10\log\frac{w}{10^{-12}}$$

$$\Rightarrow \frac{130}{10} = \log\frac{w}{10^{-12}}$$

$$\Rightarrow \frac{w}{10^{-12}} = 10^{13}$$

$\Rightarrow$ w = 10 watt

34. Given that, sound intensity level of a machine at radial distance r (= 10m) = I = $2.4 * 10^{-7}$ W/m^2

Since, the sound waves radiating uniformly in all directions, the surface area covers at radial distance r is equal to $4\pi r^2$

We have,

Sound Power (W) = $I * 4\pi r^2 = 2.4 * 10^{-7} * 4 * 3.14 * 10^2$

$= 30.144 * 10^{-5}$ W

$$\text{Therefore, } L_w = 10\log\frac{w}{w_{ref}} = 10\log\frac{30.114*10^{-5}}{10^{-12}} = 10\log\left(30.114*10^7\right)$$

$= 70 + 14.7 = 84.7$ dB

35. Given data,

Sound power level of the source (L_w) = 110 dB

Let the sound pressure level at distance r (=20m) is L_P

We have, $L_P = L_W - 20 * \log_{10} r - 11 = 110 - 20 * \log_{10} 20 - 11$

$= 110 - 26 - 11 = 72.98$ dB $\approx$ 73 dB

[The above formula is valid for a point source]

36. Given data,

Noise level at distance r_1 (= 10m) from a conveyor belt is L_{p1} = 95 dB

Let the noise level at distance r_2 (=100 m) from a conveyor belt is L_{p2}

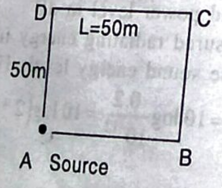
The conveyor belt generated noise all along its length. It must be treated as line source. For a line source, we have,

$L_{p1} - L_{p2} = 10* [\log_{10} r_2 - \log_{10} r_1]$

$\Rightarrow 95 - L_{p2} = 10*[\log_{10} - \log_{10} 10] = 10* \log10\ (100/10) = 10*1 = 10$

$\Rightarrow L_{p2} = 95 - 10 = 85$ dB

38.



Let the sound source is located at corner A and sound pressure level at corners B or D is L_{p1}

Sound pressure level at diagonally opposite corner at C = L_{p2} = 82.3 dB

Length of the side of square = r_1 = 50m

Therefore, length of the diagonal (AC or BD) = r_2 = $50\sqrt{2}$ = 70.71m

For a point source, we have,

$L_{p1} - L_{p2} = 20*[\log_{10} r_2 - \log_{10} r_1]$

$$\Rightarrow L_{p1} - 82.3 = 20*[\log_{10} 70.71 - \log_{10} 50] = 20*\log_{10}\left(\frac{70.71}{50}\right) = 20*0.15 = 3$$

$\Rightarrow L_{p2} = 82.3 + 3 = 85.3$ dB

39. Given data,

Sound pressure level of dozer = L_{p1} = 80 dBA

Sound pressure level of drill machine = L_{p2} = 60 dBA

$$\text{We have, } L_{p1} = 20\log\frac{P_1}{P_{ref}}$$

$$\text{and, } L_{p2} = 20\log\frac{P_2}{P_{ref}}$$

Where, P_1 and P_2 are the respective sound pressure of dozer and drill machine in N/m^2

P_{ref} = reference sound pressure level; $2*10^{-5}$ N/m^2

$$\text{Therefore, } 80 = 20\log\frac{P_1}{2*10^{-5}}$$

$$\Rightarrow \log\frac{P_1}{2*10^{-5}} = 4$$

$\Rightarrow$ $P_1 = 2*10^{-5}*10^4$ [$\log_{10}X$ = a can be reduce to x = 10^8]

$\Rightarrow$ P_1 = 0.20 N/m^2

Similarly, $P_2 = 2*10^{-5}*10^3 = 0.02$ N/m^2

Therefore, P_1/P_2 0.2/0.02 = 10

$P_1 = 10P_2$

40. Let the sound pressure level of machine A and B are L_{pa} and L_{pb} respectively

Given that,

Combined sound pressure level of A and B = L_e = 85 dBA

L_{pb} = 80 dBA

$$\text{We have, } L_e = 10\log_{10}\sum_{i=1}^{n} t_i 10^{(L_i/10)} = 10\log_{10}\left(10^{\frac{L_{pa}}{10}} + 10^{\frac{L_{pb}}{10}}\right)$$

$$\Rightarrow 85 = 10\log_{10}\left(10^{\frac{L_{pa}}{10}} + 10^{\frac{80}{10}}\right)$$

$$\Rightarrow \log_{10}\left(10^{\frac{L_{pa}}{10}} + 10^{\frac{80}{10}}\right) = 8.5$$

$$\Rightarrow \left(10^{\frac{L_{pa}}{10}} + 10^{\frac{80}{10}}\right) = 10^{\frac{85}{10}}$$

$$\Rightarrow 10^{\frac{L_{pa}}{10}} = 10^{8}\left(10^{0.5} - 1\right) = 2.16 * 10^{8}$$

Taking logarithm both sides, we get

$$\log 10^{\frac{L_{pa}}{10}} = \log\left(2.16 * 10^{8}\right)$$

$$\Rightarrow \frac{L_{pa}}{10} = 8 + 0.33 = 8.33$$

$$\Rightarrow L_{pa} = 83.3\,\text{dBA}$$

41. Given that,

Concentration of OH - ion = $P_{OH-} = 10^{-11}$

Let the concentration of H+ ion = P_{H+}

We have,

$P_{OH-} * P_{H+} = 10^{-14}$ mol/ℓ

$\Rightarrow P_{H+} = 10^{-3}$ mol/ℓ

Therefore, pH = negative of logarithm ion concentration of H+ ion

$= -\log 10^{-3} = 3$

43. Given data,

BOD level after 5 days = BOD_5 = 180 mg/l

Reaction rate constant = k = 0.1/d

We have,

$BOD_L = BOD_t\,(1 - e^{-kt})$ [t = number of days]

$\Rightarrow 180 = BOD_L\,(1 - e^{-0.1*5})$

$$\Rightarrow BOD_L = \frac{180}{\left(1 - e^{-0.1*5}\right)} = \frac{180}{1 - 0.60} = 450\,\text{mg/l}$$

44. Given data,

Initial DO (DO_i) = 8.5 mg/l

Final DO after 5 days (DO_f) = 0.5 mg/l

Volume of sample (V_s) = 6 ml

Volume of solution (V_d) = 300 ml

Therefore dilution factor (DF) = V_d/V_s = 300/6 = 50

Now, we have

BOD_5 of the sample = $(DO_i - DO_f)$ * DF = (8.5 – 0.5) * 50 = 400 mg/l

45. Given data,

BOD_5 of coke oven effluent = c_1 = 25mg/l

Rate of effluent discharge = V_1 = 4 ML/d = $4*10^6$ L/d

BOD_5 of receiving stream = c_2 = 2 mg/l

Rate of stream flow = V_2 = 40 ML/d = $40*10^6$ L/d

Therefore,

BOD_5 of mixture in the mixing zone

$$= \frac{c_1 v_1 + c_2 v_2}{v_1 + v_2} = \frac{25*4*10^6 + 2*40*10^6}{4*10^6 + 40*10^6} = \frac{100+80}{4+40}$$

$$= \frac{180}{44} = 4.1\,\text{mg/l}$$

46. Initial DO of the solution (DO_i) = 8 mg/l

Final DO of the solution after 5 days (DO_f) = 3 mg/l

Let V_1 volume of effluent mix with V_2 volume of fresh water to make 300 ml solution.

i.e., $V_1 + V_2 = 300$

Therefore, dilution factor (DF) = $300/V_1$

Now, we have

BOD_5 of the sample (c) = $(DO_i - DO_f)$ * DF = (8 – 3)*300/V_1 = $\frac{1500}{V_1}$ mg/l

$\Rightarrow 50 = 1500/V_1$ [BOD_5 of the original effluent is 50 mg/l]

$\Rightarrow V_1 = 30$ ml

47. Given data,

Dissolved oxygen in mine water = c_1 = 2mg/l

Rate of mine water discharge = V_1 = 1.5 m^3/s

Dissolved oxygen in river water = c_2 = 6 mg/l

Rate of river water flow = V_2 = 7 m^3/s

Therefore,

$$\text{Dissolved oxygen of the mixture} = \frac{c_1 V_1 + c_2 V_2}{V_1 + V_2} = \frac{2*1.5 + 6*7}{1.5 + 7} = \frac{45}{8.5} = 5.3\,\text{mg/l}$$

48. Initial oxygen deficit of mixture

= Saturated level of oxygen concentration – Oxygen dissolved in the mixture

= 9.3 – 5.3= 4 mg/l

49. COD indicate the amount of oxygen required for degrading the organic and inorganic waste present in the wastewater; whereas, BOD indicate the amount of oxygen required to degrade organic waste only. Hence, COD is always greater than BOD.

57. Given data,

BOD level of wastewater effluent after 5 days = BOD_5 = 80 mg/l

Reaction rate constant = k = 0.16/d.

Let the ultimate BOD is BODL

We have,

$BOD_L = BOD_t(1-e^{-kt})$ [t number of days]

$\Rightarrow 80 = BOD_L(1-e^{-0.16*5})$

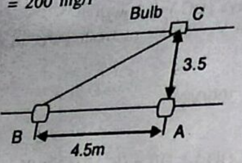
$\Rightarrow BOD_L = \frac{80}{(1-e^{-0.16*5})} = \frac{80}{1-0.45} = 145$ mg/l

58. Weight of dry solids = (Weight of the dish + dry solid) – Weight of the dish
= 48.6432 – 48.6232 = 0.02 g

Thus, 0.02 g of dry solid present in 100 ml of wastewater

Concentration of dry solids in wastewater = 0.02/100 g/ml
= 200 mg/l

60.



Given that,
AB = 4.5m
AC = 3.5m
Therefore,

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{4.5^2 + 3.5^2} = 5.7m$$

Illumination at point A = I_a = 30 lux
Let illumination at point B is I_b and source candle power is P
We have,

$$I_a = \frac{P*\cos\theta_a}{AC^2}$$

$$I_b = \frac{P*\cos\theta_b}{BC^2}$$

[Where, θ_a and θ_b are the angles between the normal to the surface at A and B respectively and the direction of light rays. Therefore, $\theta_a = 0$ and θ_b = <ACB]

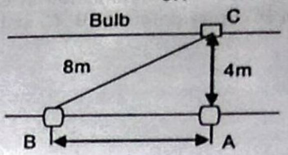
From the above two equations,

$$\frac{I_a}{I_b} = \frac{P*\cos\theta_a}{AC^2} * \frac{BC^2}{P*\cos\theta_b}$$

$$\Rightarrow \frac{30}{I_b} = \frac{\cos\theta}{3.5^2} * \frac{5.7^2}{(AC/AB)} = \frac{1}{3.5^2} * \frac{5.7^2}{(3.5/5.7)}$$

$$\Rightarrow I_b = \frac{30*3.5^2}{5.7^3} = 7 \text{ lux}$$

61.



Given that,
AC = 4m
BC = 8m
Illumination at point A = I_a = 40 lux
Let illumination at point B is I_b and source candle power is P
We have,

$$I_a = \frac{P*\cos\theta_a}{AC^2}$$

$$I_b = \frac{P*\cos\theta_b}{BC^2}$$

[Where, θ_a and θ_b are the angles between the normal to the surface at A and B respectively and the direction of light rays. Therefore, $\theta_a = 0$ and θ_b = <ACB]

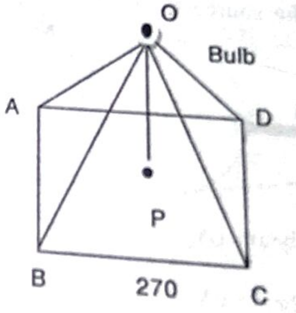
From the above two equations,

$$\frac{I_a}{I_b} = \frac{P*\cos\theta_a}{AC^2} * \frac{BC^2}{P*\cos\theta_b}$$

$$\Rightarrow \frac{40}{I_b} = \frac{\cos\theta}{4^2} * \frac{8^2}{(AC/AB)} = \frac{1}{4^2} * \frac{8^2}{(4/8)}$$

$$\Rightarrow I_b = \frac{40*4^3}{8^3} = 5 \text{ lux}$$

63.



Given that,
AB =BC=CD=AD = 20 m
OP = 4 m

Illumination at vertically below point P = Ip = 40 lux
Since the area ABCD is square in shape and thus the distance of each corner from the bulb location is equal. Therefore the illumination at each corner point is also equal. Let illumination at corner points A, B, C, and D is I and source candle power is P.

We have,

$$I_p = \frac{P*\cos\theta_p}{OP^2}$$

$$I = \frac{P*\cos\theta}{OA^2}$$

$[\theta_P = 0 \text{ and } \theta = <AOP]$

From the above two equations,

$$\frac{I_p}{I} = \frac{P*\cos\theta_p}{OP^2} * \frac{OA^2}{P*\cos\theta}$$

$$\Rightarrow \frac{40}{I} = \frac{\cos\theta}{4^2} * \frac{14.7^2}{(OP/OA)} = \frac{1}{4^2} * \frac{14.7^2}{(4/14.7)}$$

$$\left[OA = \sqrt{OP^2 + AP^2} = \sqrt{4^2 + \left(\frac{AC}{2}\right)^2} = \sqrt{4^2 + \frac{(AB^2 + BC^2)}{4}} = \sqrt{16 + \frac{2*20^2}{4}}\right.$$

$$\left. = \sqrt{216} = 14.7\text{m}\right]$$

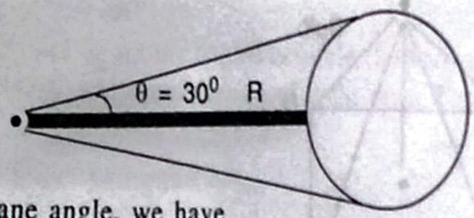
$$\Rightarrow I = \frac{40*4^3}{14.7^3} = 0.8 \text{ lux}$$

64. Let illumination at point B is I_b and source candle power is P
Using the same technique as above, we get

$$I_b = \frac{5*3^3}{5^3} = 1.08 \text{ lux}$$

[Distance of point B from the source is 5m]

65.



For plane angle, we have
Surface area covered by the beam (A_s) = W R^2
For solid angle, we have
Surface area covered by the beam (A_s) = 2p (1–cos θ)R^2
For θ = 30° and R = 5 m,
As = 2*3.14*(1–cos30°)*5^2 = 21 m^2

■

5.0 Mine Surveying

5.1 Introduction

Mine surveying is the way of determining the relative positions of points on the surface of the earth and underground.

Surveying primarily classified into two categories :

- Geodetic Surveying/ Trigonometrical Surveying: In this type of surveying, it is necessary to take into consideration of curvature of the earth. This type of surveys generally covers large areas.
- Plane Surveying: In plane surveying the curvature of the earth is not taken into consideration as the surveys extend covers small areas.

5.2 Representative Factor

It is the ratio of map distance to the corresponding ground distance. That is, if the distance between two points in the ground is 100 m and in the drawing the distance between the points representing the same is 10 cm then the representative factor (R.F.) is given by

$$RF = \frac{10\text{cm}}{10000\text{cm}} = \frac{1}{1000} = 1:1000$$

5.3 Distance Measuring Instruments

Pacing
Passometer
Pedometer
Odometer
Speedometer
Perambulator
Speedometer

5.4 Correction Factors in Line Measurement

5.4.1 Correction Factor Due to Incorrect Chain : It is often observed that the length of the measuring tape or chain is incorrect due to expansion or contraction and, hence the measured length with this incorrect chain should be corrected before plotting.

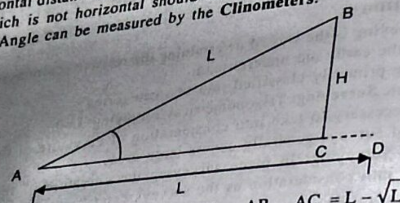
Correction factor due to incorect chain (C.F.) $= \frac{L \pm \Delta L}{L}$

Where, L = true length of the chain or tape
$(L \pm \Delta L)$ = incorrect length of the chain or tape

[**Note :** + sign, if the chain is longer than the indication i.e. the chain is too long and, – sign, if the chain is shorter than the indication i.e. the chain is too short]

$$\Rightarrow \text{True Length of Line} = \frac{L \pm \Delta L}{L} * \text{Measured Length of Line}$$

5.4.2 Slope Correction Factor : Since the map distance always indicates the horizontal distance between any two points. Thus the measurements along the plane which is not horizontal should be corrected to plot the exact horizontal distance. Angle can be measured by the **Clinometers.**



$$\text{Slope Correction Factor (S.C.F.)} = CD = AB - AC = L - \sqrt{L^2 - H^2}$$

$$= L - L\,[1 - H^2/2L^2 - H^4/4L^4 - \ldots\ldots\ldots\ldots\ldots]$$

$$= H^2/2L \qquad \text{[Neglecting the higher powers as } H/L \text{ is very small]}$$

If 'θ' is known then the correction factor is given by

$$\text{S.C.F.} = CD = L - L^* \cos\theta = L\,(1 - \cos\theta)$$

5.4.3 Correction for Absolute Length : The measured length of any line can be corrected by the factor given by

$$Ca = \frac{L^*c}{l}$$

Where, Ca = correction for absolute length
L = Measured length of a line
c = correction to a tape
l = nominal length of a tape

⇒ Corrected length of measured distance D can be give by

Actual length = L ± Ca

[The sign of the correction will be the same as Ca]

5.4.4 Correction for Temperature : Since the length of the tape is increased as the temperature is raised, and consequently, the measured distance is too small. It is given by

$$C_t = \alpha\,(T_m - T_0)L$$

Where, C_t = the correction for temperature, in m
a = the coefficient of thermal expansion
T_m = the mean temperature during measurement
T_o = the temperature at which the tape is standardised
L = the measured length in m.

[The sign of the correction will be the same as C_t]

Actual length = L ± C_t

5.4.5 Correction for Pull : When the pull used during measurement is different from that at which the tape is standardised, this correction should be apply to obtain the actual distance. It is given by

$$Cp = \frac{(P - P0)^*L}{A^*E}$$

Where, Cp = the correction for pull, in m
P = the pull applied during measurement, in N
Po = the pull under which the tape is standardised, in N
L = the measured length in m
A = the cross-sectional area of the tape, in sq. cm.
E = the modulus of elasticity of steel

[The sign of the correction is always +Ve]

Actual length = L + Cp

5.4.6 Correction for Sag : The correction for sag is the difference in length between the arc and the subtending chord and the length measured along the curve. It requires only when the tape is suspended during measurement and the correction is always negative. It can be given by

$$C_s = \frac{L^*(M^*g)^2}{24^*P^2}$$

Where, Cs = the sag correction for a single span, in m
L = the distance between supports, in m
M = the mass of the tape between the two consecutive span, in kg
P = the applied pull, in N

[The sign of the correction will always −Ve]

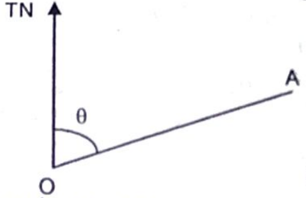
Actual length = L − Cs

5.5 Traversing

In traverse surveying the directions of the survey lines are set by angular measurements. There are two types of traverse namely closed traverse and open traverse.

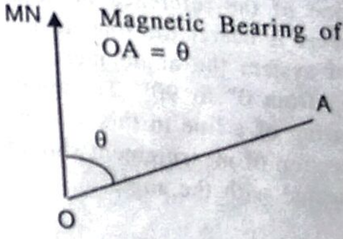
5.5.1 Bearing of Lines : It is the horizontal angle which the line makes with some reference directions or meridians. There are three types of meridian generally used for surveying like true meridian or geographical meridian, magnetic meridian and assumed meridian.

5.5.2 True Meridians or Geographical Meridians : The true or geographical meridian passing through a point on the earth's surface is the line in which the plane passing through the given point and the North Pole and South Pole intersects the surface of the earth. The point of intersection of the earth's axis and the surface of the earth are known as the north geographical pole and south geographical pole. The angle which a line makes with the true north is cad geographical bearing/ true bearing.



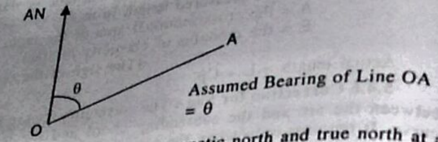
True Bearing of Line OA = θ

5.5.3 Magnetic Meridian : The direction indicated by a freely suspended magnetic needle at a place free from local attraction is called the magnetic meridian. The angle which a line makes with the magnetic north is called the magnetic bearing.

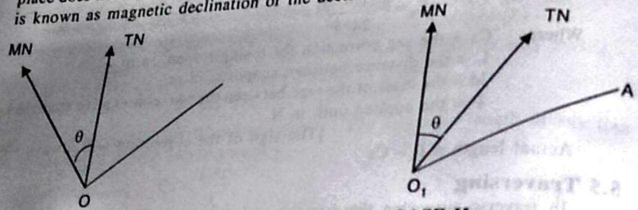


Magnetic Bearing of Line OA = θ

5.5.4 Arbitrary Meridian : The angle between the any assumed meridian and a line is known as assumed bearing of the line.



5.5.5 Magnetic Declination : Since the magnetic north and true north at a place does not coincide except in few places, the horizontal angle between the two is known as magnetic declination or the declination of the needle.



CASE I CASE II

CASE I : Magnetic north is situated in θ° W of true north

True Bearing of line OA = Magnetic Bearing of line OA - θ°

CASE II : Magnetic north is situated in θ° E of true north

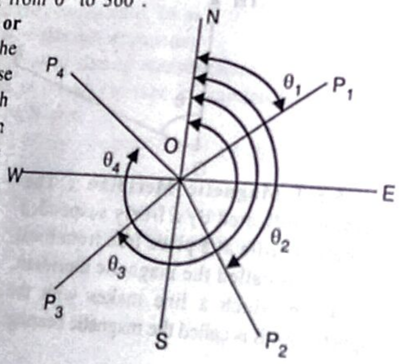
True Bearing of line OA = Magnetic Bearing of line OA + θ°

['θ' is called the magnetic declination]

5.5.6 Designation of Bearing : Bearing of a line can be represented by two types of system.

5.5.6.1 **Whole Circle Bearing :** *In this system, the bearing of a line is always measured clockwise from the north direction of the reference meridian. The angle thus measured varying from 0° to 360°.*

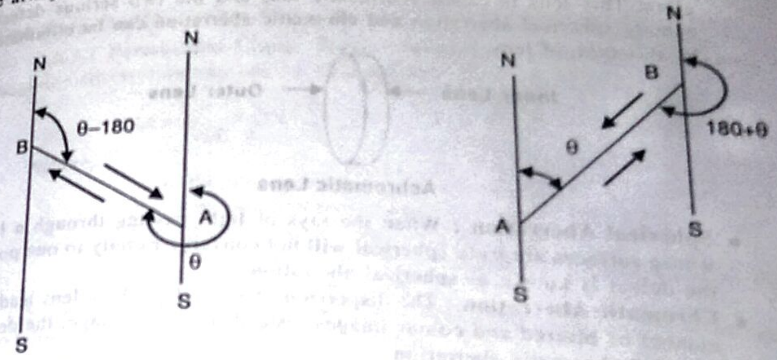
5.5.6.2 **Quadrantal Bearing or Reduced Bearing :** *In this system the bearing of a line is measured clockwise or anti-clockwise from north or south direction of the reference meridian whichever is nearer the line. In this type of system the angle measured varying from 0° to 90°. To denote the bearing of a line in this system, the direction of measurement should be indicated with the angle.*



Bearing of lines OP_1, OP_2, OP_3 and OP_4 can be represented by :

Line	Whole Circle Bearing (W.C.B.)	Qudrant Bearing or Reduced Bearing (R.B.)	Relation between W.C.B. and R.B. for different cases
OP_1	θ_1	$N\theta_1$ E	R.B = W.C.B [$0° < \theta < 90°$]
OP_2	θ_2	S(180° – θ_2)E	R.B = 180° – W.C.B [$90° < \theta < 180°$]
OP_3	θ_3	S(θ_3 – 180°)W	R.B = W.C.B – 180° [$180° < \theta < 270°$]
OP_3	θ_3	N(θ_3 – 180°)W	R.B = 360° – W.C.B [$270° < \theta < 360°$]

5.5.6.3 **Fore and Back Bearing :** The bearing of a line in the direction of the progress of survey is called the fore bearing (F.B) while the bearing of the same line in the reverse direction known as back bearing (B.B). Thus there will be always two bearings of any line on the basis of the direction of the line.



CASE I **CASE II**

Relation between F.B. and B.B :

CASE I : When F.B. > 180°

F.B. of line AB = θ

B.B. of line AB = F.B. of line BA = θ – 180°

Thus, F.B. = B.B. + 180°

CASE II : When F.B. < 180°

F.B. of line AB = θ

B.B. of line AB = F.B. of line BA = θ + 180°

Thus, F.B. = B.B. – 180°

5.6 Theodolite

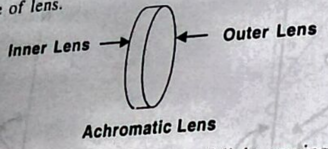
5.6.1 Introduction

Theodolite can be classified in two ways

➢ **Transit Theodolite :** When the telescope of the theodolite can be revolved through a complete revolution about its horizontal axis in a vertical plane.

Parameters of a Transit Theodolite :

- **Alidade :** The vernier plate completes with the standards and telescope is called the alidade of the theodolite.
- **Face Left :** When the vertical circle of the instrument is on the left of the observer when taking a reading, the position is called face left and the observation made called face left observation.
- **Face Right :** When the vertical circle of the instrument is on the right of the observer when taking a reading, the position is called face right and the observation made called face right observation.
- **Line of Collimation :** The line of collimation is the imaginary line joining the intersection of the cross-hairs of the diaphragm to the optical center of the object glass and its continuation.
- **Object Glass :** It is compound lens consisting of an outer double convex lens of hard crown glass and an inner concave-convex lens of dense flint glass. This lens is called achromatic lens and the two serious defects namely spherical aberration and chromatic aberration can be eliminated by this type of lens.

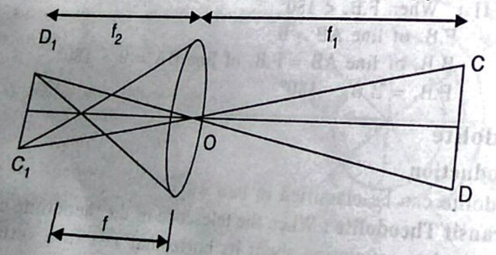


Achromatic Lens

- **Spherical Aberration :** When the rays of light passing through a lens whose surfaces are truly spherical will not converge exactly to one point, the defect is known as spherical aberration.
- **Chromatic Aberration :** The dispersion of white light by a lens leads to number of blurred and colour images instead of single image, the defect is called chromatic aberration.
- **Level Tube or Bubble Tube :** It consists of a sealed glass tube set in a brass tube with Plaster of Paris and filled with alcohol or ether, or a mixture of both.

➢ **Non-transit theodolite :** When the telescope of the theodolite cannot be revolved through a complete revolution about its horizontal axis in a vertical plane. This is of two types namely Y type and Everest type.

5.6.2 Magnification or Magnifying Power : Magnifying power of a telescope is the ratio of the angle subtended at the eye of the virtual image of the object and the real object. It can also be defined as the ratio of the focal length of the objective to the focal length of the eye piece.



$$\text{Magnifying Power (P)} = \frac{\text{Size of the object}}{\text{Size of the image}}$$

$$\frac{CD}{C_1D_1} = \frac{f_1}{f_2}$$

Where,

f_1 = the distance between the lens and object (CD)

f_2 = the distance between the lens and image (C_1D_1)

f = the focal length of the lens

Relation among f, f_1 and f_2 is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

5.6.3 Error in Theodolite Traverse

5.6.3.1 **Permissible Linear Error :** Permissible linear error of closure in theodolite traverse survey can be obtained by

$$Le = \frac{P}{1000} * \sqrt{1 + \frac{e^2N}{12}}$$

Where,

Le = the closing error of closure in m

P = the perimeter of a traverse in m

N = the number of sides of a traverse

e = permissible error per angle depends on the precision of the theodolite (commonly 1')

5.6.3.2 **Permissible Angular Error :** Permissible angular error of closure in theodolite traverse survey can be obtained by

$$Ae = L * \sqrt{N}$$

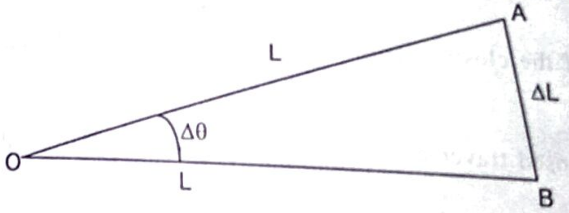
where,

Ae = the permissible angular error of closure

L = least count of the theodolite (generally 20'' or 1')

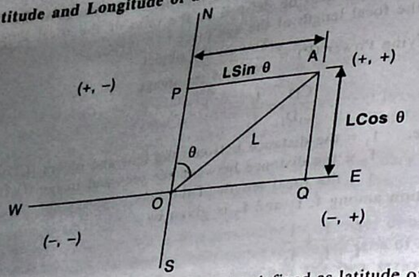
N = number of sides

5.6.4 Relation between Angular and Linear Measurement



$$\Delta L = L * \tan \Delta\theta$$

5.6.5 Latitude and Longitude of a Line :



The projection of any line in north-south line defined as latitude of the line. The sign is positive if the direction is in the north side and negative for south side.

Similarly, the projection of any line in east-west line is defined as longitude of the line. The sign is positive in eastward direction and negative for westward direction.

In the above figure, if 'θ' is the bearing of line OA

Latitude of OA $= L \cos\theta = OP$

Departure of OA $= L \sin\theta = OQ$

Co-ordinate of A is $(L \cos\theta, L \sin\theta)$

5.6.6 Close Traverse Survey

Sum of the latitudes of the lines $= \sum L\cos\theta = 0$

Sum of the latitudes of the lines $= \sum L\sin\theta = 0$

If the above two equation not holds, there is some error, called closing error and this is given by

Closing Error (E) $= \sqrt{\left(\sum L\cos\theta\right)^2 + \left(\sum L\sin\theta\right)}$

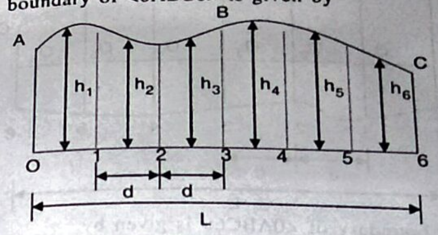
Direction of the closing error $= \beta = \tan^{-1}\left(\dfrac{\sum L\sin\theta}{\sum L.\cos\theta}\right) = \text{Reduced Bearing}$

Area of a closed traverse $= \frac{1}{2}[y_1(x_1 - x_2) + y_2(x_2 - x_3) + \ldots\ldots. y_n(x_1 - x_{n-1})]$

5.6.7 Computation of areas of Irregular Boundaries

5.6.7.1 Mid-Ordinate Rule

Area of the boundary of <0ABC6> is given by



Area of the boundary of <0ABC6> is given by

$$\Delta = \frac{(h_1 + h_2 + h_3 + h_4 + h_5 + h_6)}{n} * d * n = \frac{(h_1 + h_2 + h_3 + h_4 + h_5 + h_6)}{n} * L$$

Where,

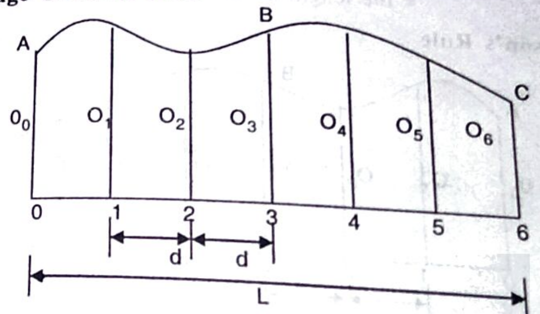
$h_1, h_2 \ldots. h_6$ = the ordinates at the mid-points of each division

n = the number of equal division of the base line

d = the common distance between the ordinates

L = the length of the base line

5.6.7.2 Average Ordinate Rule



Aarea of the boundary <OABC6> is given by

$$\Delta = \frac{(O_0' + O_1 + O_2 + O_3 + O_4 + O_5 + O_6)}{n} * d * n = \frac{(O_0 + O_1 + O_2 + O_3 + O_4 + O_5 + O_6)}{n} * L$$

Where,

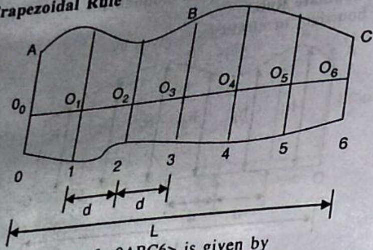
$O_1, O_2 \ldots. O_6$ = the ordinates at each of the points of division

n = the number of equal division of the base line

d = the common distance between the ordinates

L = the length of the base line

5.6.7.3 Trapezoidal Rule



Area of the boundary of <0ABC6> is given by

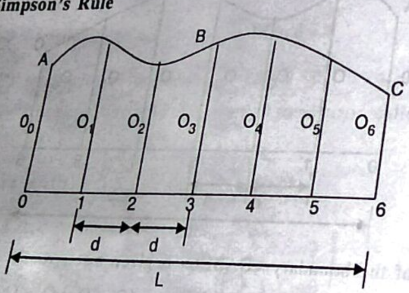
$$\Delta = \frac{(O_0 + 2O_1 + 2O_2 + 2O_3 + 2O_4 + 2O_5 + 2O_6)}{n} * d * n$$

$$= \frac{(O_0 + 2O_1 + 2O_2 + 2O_3 + 2O_4 + 2O_5 + 2O_6)}{n} * L$$

Where,

$O_1, O_2 \ldots. O_6$ = the ordinates at each of the points of division
n = the number of equal division of the base line
d = the common distance between the ordinates
L = the length of the base line

5.6.7.4 Simpson's Rule



Area of the boundary of <0ABC6> is given by

$$\text{Area } (\Delta) = \frac{(O_0 + 4O_1 + 2O_2 + 4O_3 + 2O_4 + 4O_5 + O_6)}{3} * d$$

Where,

$O_1, O_2 \ldots. O_6$ = the ordinates at each of the points of division
n = the number of equal division of the base line
d = the common distance between the ordinates
L = the length of the base line

5.7 Levelling

5.7.1 Introduction : Levelling is defined as the art of determining the relative heights or elevations of points or objects on the earth's surface. In India the relative heights are measured with respect to the datum of mean sea level. The measurement of elevation also is done with respect to the assumed bench marks. The measurement parameters of levelling are as follows:

- **Reduced Level (R.L.) :** The elevation of a point is its vertical distance above or below the datum fixed for the reference and it is known as reduced level (R.L.)
- **Back Sight (B.S.) :** It is staff reading on the point of known elevation and also the first reading after the level is set up and levelled.
- **Fore Sight (F.S.) :** It is staff reading on the point whose elevation is to be determined as on a change point.
- **Intermediate Sight (I.S.) :** All the staff readings taken between the back sight and foresight are called intermediate sight.
- **Height of the Instruments (H.I) :** It is the elevation (R.L) of the plane of collimation or line of sight when the instrument is correctly leveled.

5.7.2 Classification of Levelling

5.7.2.1 **Differential Levelling :** It is the operation of levelling to determine the elevations of points or to establish bench marks with respect to the known bench marks. It is also known as "taking flying levels"

5.7.2.2 **Check Levelling :** It is the operation of running levels for the purpose of checking a series of levels, which have been previously fixed.

5.7.2.3 **Profile Levelling :** It is the operation in which the object is to determine the elevations of points at known distance apart along a given line. It is also called longitudinal levelling or sectioning.

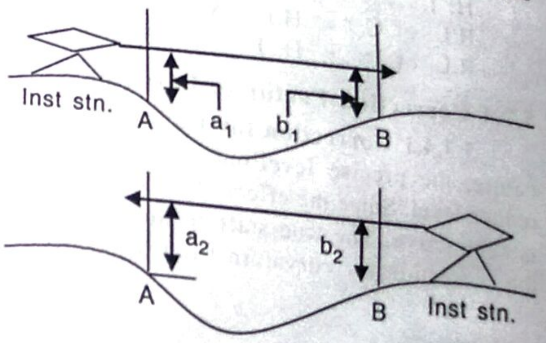
5.7.2.4 **Cross Sectioning :** It is the operation of levelling to determine the surface undulations to the give line.

5.7.2.5 **Reciprocal Levelling :** It is the method of levelling in which the difference between the two points situated at a large distance apart is accurately determined by two sets of reading taking from the two points. The advantage of this method is that it eliminates the errors due to curvature and refraction as well as due to the line of collimation not being exactly parallel to the bubble line.

If d and e are respectively the difference between the levels between two points and error, these can be obtained as

$$d = \frac{(b_1 - a_1) + (b_2 - a_2)}{2}$$

$$e = \frac{(b_1 - a_1) - (b_2 - a_2)}{2}$$



Where,

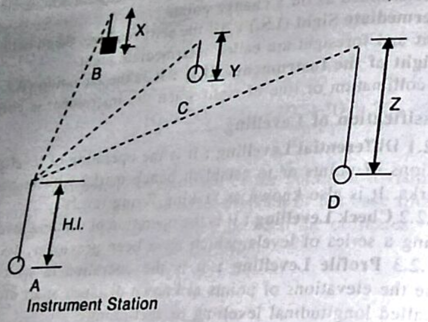
a_1 and a_2 are the staff reading at A measuring from two stations respectively
b_1 and b_2 are the staff reading at B measuring from two stations respectively

5.7.2.6 **Barometric Leveling :** It is the method of leveling in which the altitudes of points are determined by means of a barometric pressure.

5.7.2.7 **Hypsometry :** It is the method of leveling in which the altitudes of points in the hill are determined by means of observing the temperature.

5.7.2.8 **Trigonometric Leveling :** It is the process of leveling in which the elevations of points are determined from the vertical angles and horizontal distance measured in the field.

5.7.3 Determination of Reduced Level (R.L)



Assume :

A : Instrument station from where the reading of the staff height has been taken
B : Bench mark station (Reduced level of the B.M. = 100 m)
C : Intermediate station
D : Station of which the R.L has to be determined

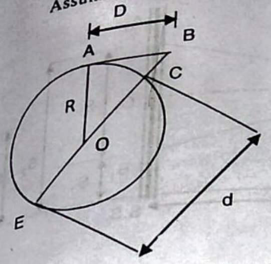
Instrument is set at 'A' and the staff reading of the points 'B', 'C' and 'D' has been taken and these are X ,Y and Z respectively.

H. I. = B.M. + X
R.L. of C = H.I. – Y = B.M. + X – Y
R.L. of D = H. I. – Z = B.M. + X – Z

5.7.4 Correction Factor in Levelling

5.7.4.1 **Correction for Curvature :** Since the earth surface is spherical in nature, the precise levelling needs some correction factor to obtain the exact reduce level. Since the effect of curvature increases the staff reading, the correction is subtractive. The true staff reading may, therefore, be obtained by subtracting the correction for curvature from the observed staff reading.

Assuming :



AC = level line through the instrument axis
O = centre of the earth
R = radius of the earth = OA
d = diameter of the earth = CE
BC = correction for curvature
AB = horizontal line (line of collimation)
D = distance between the inst. station and staff point

From tangent rule, we can get

$AB^2 = BC * BE = BC * (BC + CE) = BC^2 + BC * CE$

$\Rightarrow AB^2 = BC * CE$

[Since BC is very small as compared to diameter (CE) of the earth, the term BC^2 can be neglected]

$\Rightarrow D^2 = BC * d$

$\Rightarrow$ Correction for curvature of the earth = $BC = D^2/ d = D^2/ 2*R$

$\Rightarrow$ Correction for curvature of the earth = $BC = 0.0785\ D^2$

[Taking diameter of the earth = 12742 km]

True Staff reading = Observed staff reading – $0.0785 * D^2$

5.7.4.2 **Correction for Refraction :** The ray of light from the staff to the instrument is not straight rather it refracted or bent down due to the different air densities. Thus, the effect of refraction is to diminish the staff reading and hence the correction is additive. On an average it may be taken as one seventh of that of curvature and is of opposite effect.

Therefore,

Correction for refraction = $BC/ 7 = D^2/ 7*d = D^2/ 14*R$

True Staff reading = Observed staff reading + $0.0112 * D^2$

Combined Correction = Correction for curvature – Correction for refraction

$$= \left(BC - \frac{BC}{7}\right) = \frac{6}{7} * BC = \frac{6}{7} * \frac{D^2}{d} = \frac{6}{7} * \frac{D^2}{2R}$$

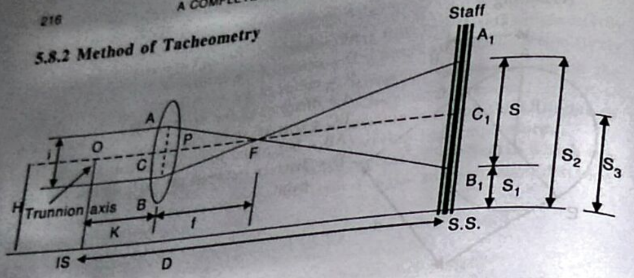
5.8 Tacheometric Survey

5.8.1 **Introduction :** This is a branch of angular surveying in which the horizontal and vertical positions of points are determined by instrumental observations without any linear measurements.

Essential features of a tacheometer

- multiplying constant (f/i) should have a value of 100
- telescope should be fitted with an anallatic lens
- the central horizontal wire must be exactly midway between the two lines

5.8.2 Method of Tacheometry



Let,

f = focal length of the object glass

i = distance between the upper and lower stadia hairs

$S = S_2 - S_1$ = staff intercept

D = horizontal distance from the instrument station to staff station

K = distance of trunnion axis from the object glass

S_3 = axial hair reading

From similar triangle ABF and A1B1F, we can obtain

$AB/A_1B_1 = CF/C_1F$

$\Rightarrow \quad i/S = f/D - f - K$

$\Rightarrow \quad D = f/i * S + (f + K)$

f/i = known as multiplying constant = 100

$(f + K)$ = known as additive constant = 0 for anallatic lens

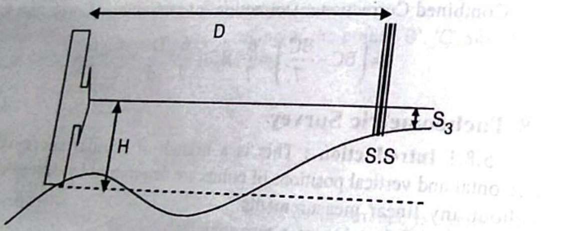
Therefore,

Horizontal distance $= D = f/i * S + (f + K) = f/i * S + C$ (assume)

R.L. of Staff Station= Height of instrument $- S_3$

5.8.3 Determination of Horizontal and Vertical Distance in Different Conditions

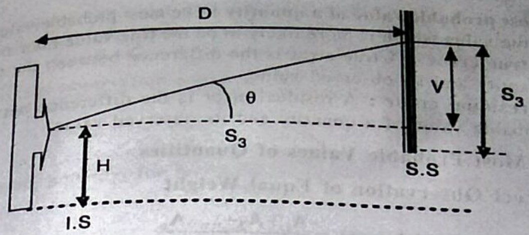
CASE I : when the line of sight is horizontal and the staff held vertical



$D = f/i * S + (f + K)$

R.L. of S.S. $= H - S_3$

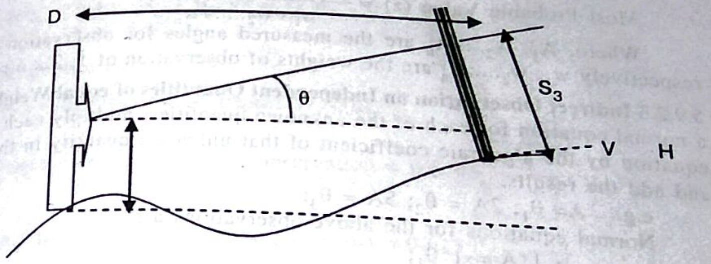
CASE II : When the line of sight is inclined to the horizontal and the staff is held vertically



$D = f/i * S \cos^2\theta + (f + K)\cos\theta$

R.L. of S.S. $= H + f/i * S \cos\theta * \sin\theta + (f+K)\sin\theta - S_3 = H + V - S_3$

CASE III : when the line of sight is inclined to the horizontal and staff is normal to the line of sight



$D = f/i * S \cos\theta + (f + K)\cos\theta + S3 * \sin\theta$

R. L. of S.S. $= H + f/i * S \cos\theta * \sin\theta + (f + K)\sin\theta - S_3 * \sin\theta$

$= H + V - S_3 \sin\theta$

5.9 Triangulation Adjustments

5.9.1 Definition of Various Observation and error Parameters

- **Direct Observation :** An observation is said to be direct when it is made directly upon a quantity whose value is desired. e.g., a single measurement of an angle.
- **Indirect Observation :** An observation is called indirect when made upon some function of quantities whose values are to be determined, e.g. measurement of a summation angle for total sum of the individual angles, measurement of an angle by repetition.
- **Observed Value of a Quantity :** The observed value of a quantity is the value obtained as a result of an observation, after applying the corrections for all known errors.
- **True Value of a Quantity :** The true value of a quantity is the value which is absolutely free from all errors.

- ***Most probable value of a quantity :*** *The most probable value of a quantity is the value which is more likely to be the true value than any other value.*
- ***A true error :*** *A true error is the difference between the true value of a quantity and its observed value.*
- ***A residual error :*** A residual error is the difference between the most probable value of a quantity and its observed value.

5.9.2 The Most Probable Values of Quantities

5.9.2.1 Direct Observation of Equal Weight

$$\text{Most Probable Value } (z) = \frac{A_1 + A_2 + \ldots\ldots A_n}{n}$$

Where, A_1, $A_2 \ldots\ldots A_n$ are the measured angles for observation 1, 2..... n *respectively.*

5.9.2.2 Direct Observation of Unequal Weight

$$\text{Most Probable Value } (z) = \frac{w_1A_1 + w_2A_2 + \ldots . w_nA_n}{w_1 + w_2 \ldots\ldots + w_n}$$

Where, A_1, $A_2 \ldots\ldots A_n$ are the measured angles for observation of 1, 2....n respectively w_1, $w_2 \ldots\ldots w_n$ are the weights of observation of 1, 2....n respectively.

5.9.2.3 Indirect Observation on Independent Quantities of equal Weight : To form a normal equation for each of the unknown quantities, multiply each observation equation by the algebraic coefficient of that unknown quantity in that equation, and add the results.

e.g. $A = \theta_1$; $2A = \theta_2$; $5A = \theta_3$;

Normal equations for the above observations are

$1{*}A = 1{*}\theta_1$;

$2{*}2A = 2{*}\theta_2$;

$5{*}5A = 5{*}\theta_3$;

$$\text{Most Probable Value } (A) = \frac{\theta_1 + 2\theta_2 + 5\theta_3}{1+4+25}$$

5.9.2.4 Indirect Observation on Independent Quantities of Unequal Weight : To form a normal equation for each of the unknown quantities, multiply each observation equation by the product of algebraic coefficient of that unknown quantity in that equation and the weight of that observation, and add the results.

e.g. $A = \theta_1$ weight 2; $2A = \theta_2$ weight 5

Normal equations for the above observations are

$1{*}2{*}A = 2\theta_1$;

$2{*}5{*}A = 10\theta_2$;

$$\text{Most Probable Value } (A) = \frac{2\theta_1 + 10\theta_2}{2+10}$$

5.9.2.5 Indirect Observation on Conditioned Quantities

In this case, there are one or more conditional equations in addition to the observation equations. The number of independent conditional equations is always less than the number of unknown quantities. In this method all the observation equations are written in terms of the independent quantities, for eliminating

conditional equations. The most probable values of the unknowns may then be found by the rules for independent quantities.

e.g. $A = \theta_1$, $B = \theta_2$, $C = \theta_3$ and $A + B = C$ (Conditional equation)

Therefore, the observation equations are

$A = \theta_1$ (1)

$B = \theta_2$ (2)

$A + B = \theta_3$ (3)

Thus, normal equation for A

$A = \theta_1$

$A + B = \theta_3$

$2A + B = \theta_1 + \theta_3$

Similarly, normal equation for B

$A + 2B = \theta_2 + \theta_3$

Solving the above two normal equations, we get

$A = [2(\theta_1 + \theta_3) - (\theta_2 + \theta_3)]/3 = (2\,\theta_1 - \theta_2 + \theta_3)/3$

$B = (2\,\theta_2 - \theta_1 + \theta_3)/3$

5.9.3 The Probable Error

5.9.3.1 Direct Observation of Equal Weight

$$\text{The probable error of a single observation} = 0.6745\sqrt{\frac{\Sigma v^2}{(n-1)}}$$

$$\text{The probable error of the arithmetic mean} = 0.675\sqrt{\frac{\Sigma v^2}{\Sigma_n(n-1)}}$$

5.9.3.2 Direct Observation of Unequal Weight

$$\text{The probable error of a single observation of unit weight} = 0.6745\sqrt{\frac{\Sigma wv^2}{(n-1)}}$$

$$\text{The probable error of the weighted arithmetic mean} = 0.6745\sqrt{\frac{\Sigma wv^2}{\Sigma w\,(n-1)}}$$

Where, v = the difference between the measured value and the arithmetic mean

n = the number of measurements.

w = the weight of measurement.

5.9.4 Laws of Weights : The following laws of weights are established by the method of least squares.

1. The weight of the arithmetic mean of observations of unit weight is equal to the number of observations.
2. The weight of the weighted arithmetic mean is equal to the sum of the individual weights.
3. If two or more quantities are added algebraically, the weight of the result is

equal to the reciprocal of the sum of the reciprocals of the individual weights.

e.g. $\alpha = \theta_1$ weight a_1

$\beta = \theta_2$ weight a_2

$$\text{Weight of } (\alpha + \beta) = \text{Weight of } (\alpha - \beta) = \frac{1}{\left(\frac{1}{a_1} + \frac{1}{a_2}\right)} = \frac{a_1 a_2}{a_1 + a_2}$$

4. If a quantity is multiplied by a factor, the weight of the resultant is equal to the weight of that quantity divided by the square of that factor.

   e.g. $\alpha = \theta_1$ weight a

   $3\alpha = 3\ \theta_1$ weight $a/3^2$

5. If a quantity is divided by a factor, the weight of the product is equal to the weight of that quantity multiplied by the square of that factor.

   e.g. $\alpha = \theta_1$ weight a

   $\alpha/3 = \theta_{1/3}$ weight $a*3^2$

6. Weights vary directly as the number of observations in case of angles, and inversely as the lengths of the various routes in the case of lines of levels.
7. Weights are inversely proportional to the squares of the corresponding probable errors, if the angles are measured a large number of times.

5.9.5 Computed Quantities

Case I : The computed quantity is the sum or difference of an observed quantity and a constant.

Let x = the computed quantity

ex = its probable error

a = the observed quantity

ea = its probable error

k = a constant

then, $x = \pm a \pm k$

and $ex = ea$

Case II : The computed quantity is obtained by the product of an observed quantity and a constant factor.

Then $x = k.a$

$ex = k.e_a$

Case III : The computed quantity is the algebraic sum of two or more independently observed quantities.

a_1, a_2, a_3 ...an are the independently observed quantities.

e_1, e_2, e_3...en are the probable errors of a_1, a_2, a_3 ...an respectively

Then, $x = \pm a_1 \pm a_2 \pm a_3 \pm \ldots a_n$

and $e_x = \sqrt{e_1^2 + e_2^2 + \ldots e_n^2}$

Case IV : The computed quantity is any function of a single observed quantity.

$x = \phi(a)$

Then, $e_x = e_a \dfrac{dx}{da}$

Case V : The computed quantity is any function of two or more independently observed quantities.

$x = \phi(a, b)$

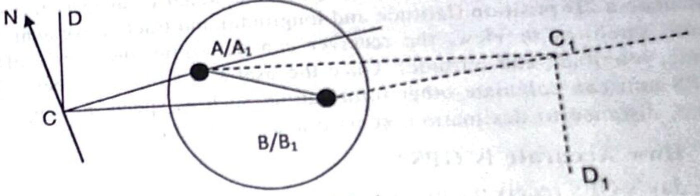
Then, $e_x = \sqrt{\left(e_a \dfrac{dx}{da}\right)^2 + \left(e_b \dfrac{dx}{db}\right)^2}$

5.10 Correlation

5.10.1 **Introduction** : Correlation is the type of survey, where the relative position of the underground base line is determined with reference to the surface base line. There are number of correlation methods such as

- Direct traversing through mines entry
- Precise magnetic methods
- Gyro theodolite method
- Optical transfer
- **Shaft plumbing** : There are a number of shaft plumbing methods :
  - two shafts plumbing → Co-planning or exact alignment
  - single shaft plumbing → Weissbach triangle method
    → Weiss quadrilateral method

5.10.2 Weisbach Triangle Method



CD = Surface base line whose bearing is known

C_1D_1 = Underground base line whose bearing has to be determined with respect to CD

ABC and $A_1B_1C_1$, known as weissbach triangle

Theodolite is set at the point C and the angles <DCA & <ACB has determined

Assuming, $< DCA = \alpha$ and $< ACB = \beta$

In triangle ABC,

AB/ Sin C = AC/ Sin B

$\Rightarrow$ Sin B = b*Sin C/ c

Since angles B and C are very small, Sin B = <B and Sin C = <C

Therefore, <B = (b* angle C)/ c = γ (Assume)

azimuth of AB = azimuth of CD + <DCA + <XAB

= azimuth of CD + <DCA + <ACB + <ABC

Now the theodolite is fixed in underground station C1 and the angles $B_1C_1D_1$ and $A_1C_1B_1$ has been measured

Again from sine rule, we can get

Angle $A_1 = (B_1C_1 / A_1B_1) * \text{angle } C_1 = a_1 * \text{angle } C_1 / c_1$

Azimuth of line B_1C_1 = azimuth of AB + angle A_1 + angle C_1

Azimuth of line C_1D_1 = azimuth of line B_1C_1 + angle $(180 - {<}B_1C_1D_1)$

5.11 GPS

5.11.1 **Introduction :** The Global Positioning System (GPS) is a satellite-based navigation system made up of a network of 24 satellites placed into orbit by the U.S. Department of Defence. GPS was originally intended for military applications, but in the 1980s, the government made the system available for civilian use. GPS works in any weather conditions, anywhere in the world, 24 hours a day. There are no subscription fees or setup charges to use GPS.

5.11.2 How it Works?

GPS satellites circle the earth twice a day in a very precise orbit and transmit signal information to earth. GPS receivers take this information and use triangulation to calculate the user's exact location. Essentially, the GPS receiver compares the time a signal was transmitted by a satellite with the time it was received. The time difference tells the GPS receiver how far away the satellite is. Now, with distance measurements from a few more satellites, the receiver can determine the user's position and display it on the unit's electronic map.

A GPS receiver must be locked on to the signal of at least three satellites to calculate a 2D position (latitude and longitude) and track movement. With four or more satellites in view, the receiver can determine the user's 3D position (latitude, longitude and altitude). Once the user's position has been determined, the GPS unit can calculate other information, such as speed, bearing, track, trip distance, distance to destination, sunrise and sunset time and more.

5.11.3 How Accurate is GPS?

Today's GPS receivers are extremely accurate, thanks to their parallel multi-channel design. Garmin's 12 parallel channel receivers are quick to lock onto satellites when first turned on and they maintain strong locks, even in dense foliage or urban settings with tall buildings. Certain atmospheric factors and other sources of error can affect the accuracy of GPS receivers. GPS receivers are accurate to within 15 meters on average.

5.11.4 GPS Applications in Coal Industry

GPS entered the Mining Industry as a fast and cost-effective instrument for survey. A shifting landscape is the very nature of mining operations; as shovels and dozers remove coal and ore, they reshape the mine's surface. Real-time GPS allows mining operations to keep on top of these constant changes and provide updated operating instructions to heavy equipment operators. In addition, GPS systems provide a fast and accurate solution for replacing and maintaining control points and calculating the volume of material moved.

Moving mining assets, including dozers, shovels, graders and draglines, are managed and guided using advanced GPS technology. Advanced GPS systems also track and monitor the status and location of dump trucks, providing reports to their heading and velocity as well as the size of the truck's load. Live GPS is becoming commonplace for monitoring and dispatching haul trucks or drills and for providing grade control on shovels. These data can also be tied to a GIS to monitor the location of all equipment, in real time.

GPS being an all-weather real time, continuously available, economic and very precise positioning technique, would have wide range of applications in Indian coal industry. The potential areas for usage of GPS in Indian coal industry includes –

- **Surveying**

Several modern surveying techniques like Satellite Remote Sensing, Photogrammetric, Field surveying procedures using digital theodolites, short and long range EDM instruments like Total Station are available today. However, the advantages of using the satellite based GPS techniques for surveying are :

- GPS measurements do not require inter-visibility between points whereas the conventional surveying tools require line of sight for measurements.
- GPS technique provides a three dimensional position for the point. That is in one go, we get the horizontal and vertical position of the point, unlike in conventional surveying where we need two operations viz., horizontal traverse for planimetric control and a level loop for height control.
- A very high accuracy measurement can be made in a relatively short time for baseline lengths of a few hundred meters to few hundred kilometers and can provide the same accuracy anywhere on earth, in almost any weather condition and at any time of the day.

GPS offers many advantages compared with conventional survey methods. Because there is no need for a rod person, each surveyor can work alone when necessary. GPS also requires much less setup time than did traditional surveying equipment, so the crew can use its time more efficiently. It can also keep a much more flexible schedule and move from one area to the next or one pit to another as needed.

- **Opencast Mine Operations**

The entire opencast mine earthmoving process can be monitored and enhanced by GPS equipped heavy earth mining machinery (HEMM). Computerized mine operations management system would include tracking of mining equipment, maintenance, monitoring and diagnostic systems, transmission of loading instructions to dumpers etc for better control.

5.11.5 The GPS Satellite System

The 24 satellites that make up the GPS space segment are orbiting the earth about 12,000 miles above us. They are constantly moving, making two complete orbits in less than 24 hours. These satellites are travelling at speeds of roughly 7,000 miles an hour.

GPS satellites are powered by solar energy. They have backup batteries onboard to keep them running in the event of a solar eclipse, when there's no solar power. Small rocket boosters on each satellite keep them flying in the correct path.

Here are some other interesting facts about the GPS satellites (also called NAVSTAR, the official U.S. Department of Defence name for GPS) :

- The first GPS satellite was launched in 1978.
- A full constellation of 24 satellites was achieved in 1994.
- Each satellite is built to last about 10 years. Replacements are constantly being built and launched into orbit.
- A GPS satellite weighs approximately 2,000 pounds and is about 17 feet across with the solar panels extended.
- Transmitter power is only 50 watts or less.

For better understanding of GPS, we normally consider three segments viz. space segment, control segment and user segment. Space segment deals with GPS satellite system, control segment describes ground based time and orbit control prediction and in user segment various types of existing GPS receiver and its application is dealt.

Space segment consists of 21 GPS satellite with an addition of 3 active spares. These satellites are placed in almost six circular orbits with an inclination of 55°. An orbital height of these satellites is about 20,200 km corresponding to about 26,600 km from the semi major axis. Final arrangement of 21 satellite constellation known as Primary Satellite Constellation. There are 6 orbital planes A to F with a separation of 60 at right ascension. GPS satellites are mainly divided into three blocks.

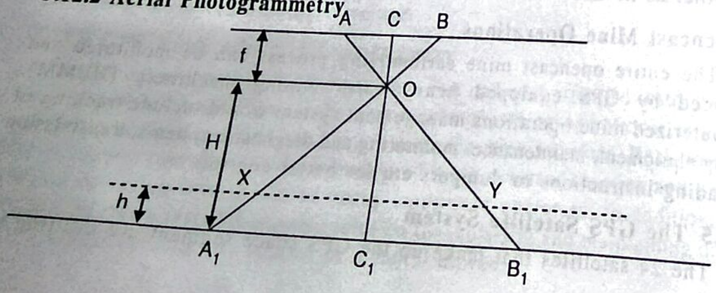
5.12 Photogrammetry

5.12.1 Introduction : This is a type of surveying in which plans or maps are prepared from photographs taken at suitable camera stations. In photo-theodolite, camera is a fixed focus type. The focal length of the lens should be 15 cm or more.

There are two types of photogrammetry survey

- Terrestrial or ground photogrammetry
- Aerial photogrammetry

5.12.2 Aerial Photogrammetry



The above figure represents the aerial photography methods where

A_1B_1 = ground surface covered by the camera

C_1OC = principal axis of the camera

CO = principal distance and should be equal to the focal length (f) of the lens

Scale of the photograph or R.F. along the ground level $A_1B_1 = f / H$

Scale of the photograph or R.F. along the line $XY = f / (H-h)$

Average photo scale can be obtained by

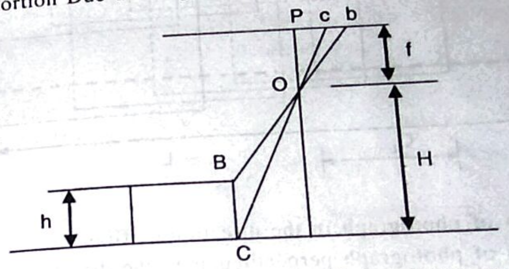
Average photo scale = $f / (H-h_{av})$

Average height of ground = $(h_1 + h_2 + \dots hn)/n$

Where, h_{av} = average height of ground

$h_1, h_2 \dots h_n$ = are the ground height at n number of points.

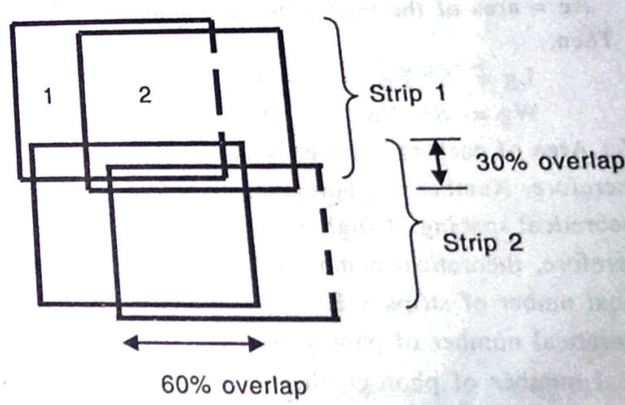
5.12.2.1 Distortion Due to Height or Relief :



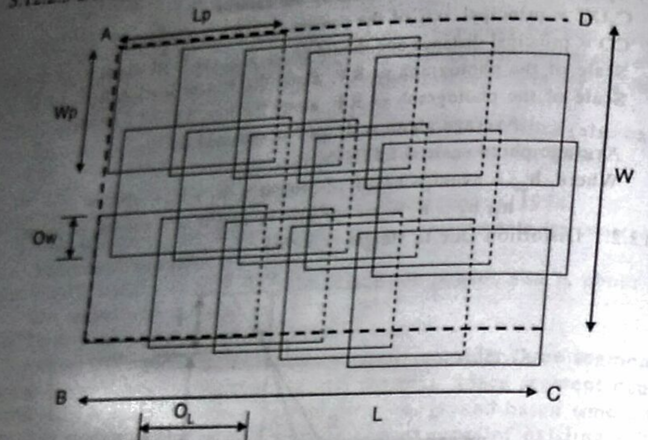
Though, B is exactly above the point C, it appears as cb in photo-print. bc is called the distortion due to the height BC.

$$\text{Distortion} = \frac{h}{H-h} * Pc = \frac{h}{H} * Pb$$

5.12.2.2 **Flight Planning :** In order to obtain stereo pairs, every parts of the ground to be surveyed must be photographed at least twice. To achieve this, aeroplane flies in strips and takes photograph with some portion overlapped (generally 60 %) with the adjacent photographs as shown in figure below.



5.12.2.3 Determination of Number of Photographs for a Particular Area :



Assuming,

Lp = length of photograph in the direction of flight.

Wp = width of photograph perpendicular to the direction of flight.

O_L = the percentage of the longitudinal overlap.

O_W = the percentage of the side overlap.

Lg = net ground distance corresponding to Lp.

Wg = net ground distance corresponding to Wp.

S = the scale of photograph (1cm = S m).

N = the number of photographs required.

Ap = net area of each photograph.

Ag = area of the tract to be photographed.

Then,

Lg = S* Lp * (1 – OL)

Wg = S* Wp* (1 – OW)

Net Area of each photograph = Ap = Lg * Wg

Therefore, Number of photographs required = Ag / Ap

Theoretical spacing of flight strips = net width of a single photograph = Wg

Therefore, theoretical number of strips = Width of the area / Wg = K

Actual nmber of strips = K + 1 [one strips required to cover the sides]

Theoretical number of photographs per strip = Length of the area/Lg = M

Actual number of photographs per strip = M + 1 [one photograph being added to cover the ends of the area]

Thus, in practice there would be (K + 1) strips with (M + 1) photographs in each.

Actual number of photographs for complete coverage of the area = (K+1) * (M+1)

5.12.2.4 Intervals between Exposures : The time interval between exposures depends upon the speed of the aeroplane and the distance it travels between exposures.

Time interval = 3600*Lg / V [Second]

Where, V is the speed of the aeroplane in km/ hr. and

Lg, the distance the aeroplane travels between exposures in km.

EXERCISE - 5

1. The ground distance between the mine managers office and pit top is 300 m and it is represented by 3 cm in the mine plan. What is the representative factor?

a. 1: 100
b. 1: 1000
c. 1: 10,000
d. 1: 300

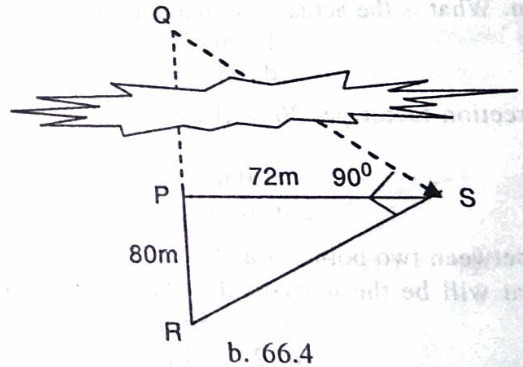
2. Match the following :

Instrument	Principal features	Application
P Tilting level	1 Micrometer	a Levelling
Q Microoptic theodolite	2 Magnetic needle	b Traversing
R Telescopic alidade	3 U-tube	c Azimuth (Bearing)
S Compass	4 Plane table surveying	d Contouring

a. P-1-b, Q-2-c, R-4-a, S-3-d
b. P-4-b, Q-3-a, R-1-c, S-2-d
c. P-2-c, Q-3-b, R-4-a, S-1-d
d. P-3-a, Q-1-b, R-4-d, S-2-c

[GATE 2008]

3. In the figure shown below, the distances RP and PS are measured to be 80 m, and 72 m respectively. The distance PQ in m is



a. 60.4
b. 66.4
c. 64.8
d. 68.4

[GATE 2008]

4. The length of a line measured with a 20 m chain was found to be 536 m. It was afterwards found that the chain was 0.05 m too long. What will be the true length of the line?

a. 568.5 b. 537.3
c. 587.5 d. 512.3

5. A 30 m tape has an error of ± 0.005. If a length of 1500 m is measured with this tape, the expected total error made in the measurement in m is
a. ± 0.025 b. ± 0.030
c. ± 0.035 d. ± 0.040 [GATE 2008]

6. The length of a line measured with a 20 m chain was found to be 536 m. It was afterwards found that the chain was 0.05 m too short. What will be the true length of the line?
a. 568.1 b. 534.7
c. 523.5 d. 512.3

7. The length of a line measured by means of a 20 m chain was found to be 610.2 m known to be 608.0. What was the actual length of the chain?
a. 20.066 m b. 20.012 m
c. 19.906 m d. 19.928 m

8. The length of a line measured by means of a 20 m chain was found to be 610.2 m known to be 612.0. What was the actual length of the chain?
a. 20.066 m b. 20.059 m
c. 19.059 m d. 19.012 m

9. A certain field was measured with a 30 m chain and found to contain 45 km^2. It was afterwards found that the chain was 0.1 m too short. What is the true area of the field?
a. 44.70 km^2 b. 45.76 km^2
c. 70.52 km^2 d. 65.20 km^2

10. If in a length of one 100 m chain along a slope, the ground rises 5 m, find the correction factor for 1 chain length.
a. 0.125 b. 0.325
c. 0.250 d. 0.050

11. The length measured along slope with a 20 m chain is 100 m and the ground rises is 15 m. What is the actual horizontal distance between the two points?
a. 96.857 b. 96.825
c. 100.85 d. 98.875

12. Find the correction factor per 20 m chain for a line measured along a slope of 15°.
a. 0.681 m b. 0.678 m
c. 0.568 m d. 0.786 m

13. The distance between two points A and B measured along a slope of 1 in 2.5 is 275 m. What will be the horizontal distance between two points?
a. 265.83 m b. 216.56 m
c. 287.67 m d. 255.33 m

14. A line was measured with a steel tape which was exactly 30 m long at 18° C and found to be 550 m. The temperature during measurement was 32°C. Find the true length of line, if the coefficient of expansion of the tape is 1.18*10-5/ °C.

a. 550.09 m b. 551.10 m
c. 549.01 m d. 549.09 m

15. A steel tape is exactly 30 m long at 20°C and coefficient of linear expansion is 11*10-6 per °C. The mean temperature during measurement is 15°C. The correction for temperature in mm is
a. +1.65 b. –5.50
c. –1.65 d. +7.50 [GATE 2002]

16. A 30 m tape is suspended between the ends under a pull of 150 N. The mass of the tape is 1.5 kg. What is the correct length of the tape?
a. 29.988 m b. 29.908 m
c. 28.980 m d. 30.008 m

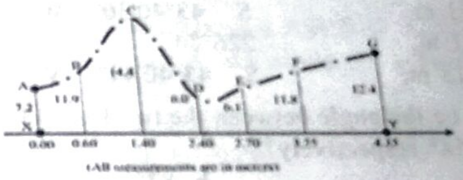
17. A 30 m steel tape having an area of cross-section of 5×10^{-6} m^2 is standardized at 20^0C, supported under a tension of 5.45 N. It is used to measure a horizontal distance of 81.15 m under an applied tension of 9.09 N. The error, due to incorrect pulling arrangement in this observation, in m is (E_{steel} = 200 GPa)
a. 0.148 b. 0.295
c. 1.820 d. 3.640 [GATE 2012]

18. A 30 m steel tape used in a mine survey is found to be 20 cm short when compared with a standard tape. If the measured volume of a dump using the tape is 4000 m^3, its actual volume in m^3 is
a. 3881 b. 3902
c. 3920 d. 4121 [GATE 2012]

19. The boundary of a mine is plotted on a scale of 1:2000. If a planimeter measures the plotted area as 58 cm2, the actual mine area in m^2 is
a. 5800 b. 11600
c. 23200 d. 29000 [GATE 2013]

20. An area of 100 m^2 is measured on a plan having a R.F. of 1/800. If the R.F. were to be 1/2000, the area in m^2 would be
a. 16 b. 40
c. 250 d. 625 [GATE 2009]

21. The figure shows the values of seven perpendicular offsets and the respective locations along the line XY as observed while carrying out a traversesurvey. The area of the plot XABCDEFGY in m^2 is



a. 26.10 b. 43.38
c. 44.92 d. 62.50 [GATE 2012]

22. Odometer is used for measurement of
a. Distance b. Pressure
c. Temperature d. Area

23. The angle 5°15'25" is expressed in hours, minutes, and seconds as
    a. 1h 20 min 1.67 s
    b. 1h 20 min 16 s
    c. 0h 21 min 1.67 s
    d. 0h 21 min 16 s

24. A circular curve has a radius of 200 m and deflection angle of 65°. The length of the curve in m is
    a. 221
    b. 227
    c. 235
    d. 262
    [GATE 2014]

25. What will be Quadrantal bearing of a line whose whole circle bearing is 236°25'?
    a. S 56°25' W
    b. N 56°25' W
    c. S 56°25' E
    d. S 43°45' E

26. The reduced bearing of a line is S 57°36' W. What will be the whole circle bearing of the line?
    a. 237°36'
    b. 327°36'
    c. 137°36'
    d. 212°24'

27. The fore bearing of a line AB is N 36° W. What will be the back bearing of the line AB?
    a. 144°
    b. 54°
    c. 324°
    d. 36°

28. The fore bearing of a line AB is N 36° W. What will be the back bearing of the line AB?
    a. N 36° W
    b. S 36° E
    c. N 54° W
    d. S 54° E

29. The fore bearing of a line AB is 136°. What will be the back bearing of the line AB?
    a. 144°
    b. 54°
    c. 316°'
    d. 44°

30. For two survey points the coordinates are given. The line CD has length and bearing

Point	Southing (m)	Westing (m)
C	280.4	310.8
D	620.9	635.9

    a. 1307.18 m, N 43°40'36" E
    b. 470.81 m, S 43°40'36" W
    c. 470.81 m, 226°24'
    d. 1307.18 m, S 43°40'44" W

31. What will be the angle between the two line OA and OB whose bearings are 56° and 154° respectively?
    a. 98°
    b. 82°
    c. 210°
    d. 18°

32. What will be the angle between the two lines OA and OB whose bearings are S56°E and N 54°E respectively?
    a. 98°
    b. 70°
    c. 2°
    d. 110°

33. The magnetic bearing of a line AB is 136°. Find its true bearing, if the magnetic declination is 8° W.
    a. 128°
    b. 144°
    c. 38°
    d. 54°

34. The magnetic bearing of a line AB is S 32° E and the magnetic declination is 4°E. What is the true bearing of the line AB?
    a. 126°
    b. 152°
    c. 36°
    d. 28°

35. The magnetic bearing of a line AB is S 32° E and the magnetic declination is 4°E. What is the true bearing of the line AB?
    a. S 36° E
    b. N 36° E
    c. S 28° E
    d. N 28° E

36. The true bearing of a line is 326° and the magnetic declination is 8° E. Find the magnetic bearing of the line.
    a. 318°
    b. 334°
    c. 138°
    d. 154°

37. The true bearing of a line is 349° and the magnetic declination is 15° W. Find the magnetic bearing of the line.
    a. 274°
    b. 4°
    c. 334°
    d. 154°

38. Find the magnetic declination, if the magnetic bearing of the sun at noon is 185°
    a. 5° E
    b. 5° W
    c. 10° E
    d. 10° W

39. Find the magnetic declination, if the magnetic bearing of the sun at noon is 358°
    a. 7° E
    b. 7° W
    c. 2° E
    d. 2° W

40. The magnetic bearing and declination of a line were recorded in the year 1906 as S 43°30'E and 2° E respectively. If the declination in the year 2006 is 3° W, the magnetic bearing of the line is
    a. S 48°30' E
    b. S 45°30' E
    c. S 41°30' E
    d. S 38°30' E
    [GATE 2007]

41. The bearing of side AB of a regular hexagon ABCDEF is S50°10' E. If the station C is easterly from the station B, the whole circle bearing of the side BC is
    a. 65°15'25''
    b. 69°50'25''
    c. 69°15'25''
    d. 69°50'0''
    [GATE 2014]

42. The whole circle bearing of the line AB is 116°20'20". If there exists an east declination of 20°, the true quadrantal bearing of line AB is
    a. S41°59'40"E
    b. S43°39'40"E
    c. S45°59'40"W
    d. S47°59'40"W
    [GATE 2011]

43. The results of theodolite survey are given below. The line AB has length

Point	Northing (m)	Easting (m)
A	400.5	620.2
B	750.5	320.5

a. 460.78 m
b. 349.70 m
c. 106.6 m
d. 50.30 m
[GATE 2007]

44. In the above question, the line AB has bearing
a. 23.17 NE
b. 23.17 NW
c. 40.57 NW
d. 40.57 NE
[GATE 2014]

45. Transit theodolite is a
a. micro-optic theodolite
b. theodolite with face left and face right reading facilities
c. theodolite with stadia hairs
d. theodolite with two vertical circles
[GATE 2013]

46. The following observations are taken during a closed traverse.

Side	Length (m)	WCB
AB	100	90^0
BC	173	180^0
CA	200	330^0

The closing error of the traverse in mm is
a. 205
b. 20.5
c. 2.05
d. 0.205
[GATE 2013]

47. In the above problem, the reduced bearing of the closing error in degrees is
a. 87.21
b. 64.03
c. 14.04
d. 0
[GATE 2013]

48. A closed transverse ABCDE of perimeter 425 m has a total error + 0.25 m in latitude and – 0.44 m in departure. The precision of transverse is
a. 1 in 556
b. 1 in 785
c. 1 in 840
d. 1 in 1024
[GATE 2010]

49. The adjusted values of departure and latitude for a traverse line AB obtained in a field survey of a mine are 225.520 m and 388.835 m respectively. The length in m and azimuth of line AB are
a. 449.50, 30.11°
b. 614.36, 30.11°
c. 614.36, 45.11°
d. 449.50, 45.11°
[GATE 2012]

50. The maximum permissible angular error in seconds in a closed underground traverse of 16 stations measured with a 20 seconds theodolite is
a. 200
b. 120
c. 80
d. 160

51. A tacheometer is set up at a station B. The R.L. of the station B is 150 m above the MSL. By holding a staff vertically at a station A, the following readings are taken :

Vertical Angle	Staff Readings (m)		
	Lower	Middle	Upper
26^{0}36'	0.80	3.08	5.36

The multiplying factor and additive constant of the instrument are 100 and 1.9 respectively. The horizontal distance between the stations A and B in m is
a. 364.6
b. 366.3
c. 409.4
d. 457.6
[GATE 2009]

52. In the above problem, if the height of the instrument is 1.2 m, the RL of the station A above the MSL in m is
a. 337.6
b. 334.5
c. 331.5
d. 330.3
[GATE 2009]

53. In tacheometric observations, the staff readings are found to be 2.4 m, 3.0 m and 3.6 m. If the angle of elevation is 45, the multiplying factor is 100 and the additive constant is 0.3 m then the horizontal distance between the staff and the instrument station, expressed in m is
a. 30.20
b. 59.76
c. 60.19
d. 73.21

54. Contours in a topographic map
a. are not closed upon themselves although the earth is a continuous surface
b. are not perpendicular to the direction of maximum slope
c. provide an indication of presence of valley or ridge in the area
d. are the lines joining the points of same declination at different elevations
[GATE 2012]

55. The following consecutive staff readings were taken on continuously sloping ground at a common interval of 30 m:

0.5 0.9 1.2 2.6 3.8
A ——————————— B

If the elevation of A was 500 m, what will be the elevation of point B?
a. 496.7 m
b. 500.3 m
c. 503.3 m
d. 503.8 m

56. The following consecutive staff readings were taken with a theodolite on continuously sloping ground at a common interval of 30 m :
0.5 on A, 1.3, 2.2, 1.6, 2.8 on B

If the elevation of A was 510 m, what will be the elevation of point B?
a. 496.7 m
b. 504.2 m
c. 507.7 m
d. 503.8 m

57. The following consecutive readings were taken with a theodolite on continuously sloping ground at a common interval of 30 m :
0.5 on A, 1.3, 2.2, 1.6, 2.8 on B

If the elevation of A was 500 m, what will be the gradient of line AB?
a. 5.5°
b. 6.5°
c. 1.1°
d. 2.5°

58. In levelling between two points A & B on opposite side of an opencast

mine, the level was set up near A and the staff readings on A and B were 2.5 m and 3.7 m respectively. The level was then moved and set up near B, and the respective staff readings on A and B were 2.0 m and 3.0 m. Find the true difference of level A and B.

a. 1.0 m
b. 1.1 m
c. 0.6 m
d. 1.3 m

59. Two points A and B were 1000 m apart across a river. The following readings were taken with one level :

Level near station	**Reading on**	
	A	B
A	1.5	2.8
B	1.4	2.6

The error in the collimation adjustment of the level was + 0.002 m per 100 m. Determine the true difference of level between A and B.

a. 1.25
b. 0.05
c. 2.5
d. 0.1

60. Two points A and B were 1000 m apart across a river. the following readings were taken with one level:

Level near station	Reading on	
	A	B
A	1.5	2.8
B	1.4	2.6

The error in the collimation adjustment of the level was + 0.002 m per 100 m. Determine the correction for refraction.

a. 0.0450
b. 0.0485
c. 0.4850
d. 0.1850

61. The backsight reading on a benchmark of RL 100.0 m is 1.45m. If the inverse staff reading on a foresight is 2.23m, the RL of the staff station in m is

a. 105.13
b. 103.68
c. 100.78
d. 98.55

[GATE 2010]

62. The effect of curvature in levelling is an error which is

a. additive
b. subtractive
c. multiplicative
d. divisive

63. The effect of refraction in levelling is an error which is

a. additive
b. subtractive
c. multiplicative
d. divisive

64. The combined effect of curvature and refraction in levelling is an error which is

a. additive
b. subtractive
c. multiplicative
d. divisive

65. What is the correction for curvature for a distance of 1200 m?

a. – 0.0113 m
b. + 0.0113 m
c. – 0.113 m
d. + 0.113 m

66. What will be the correction for refraction for a distance of 800 m if the diameter of the earth is 12742 km?

a. –0.007 m
b. + 0.007 m
c. – 0.7 m
d. + 0.7 m

67. What is the combined correction for curvature and refraction for a distance of 1200 m if the diameter of the earth is 12000 km?

a. – 0.103 m
b. + 0.103 m
c. – 1.030 m
d. + 1.030 m

68. A level is set up at O on a line AB 100 m from A and 1500 m from B. The back sight on A is 0.548 m and the fore sight on B is 5.263 m. What will be the true difference of level A and B?

a. 4.563 m
b. 4.356 m
c. 4.715 m
d. 5.801 m

69. A level is set up at O on a line AB 600 m from A and 800 m from B. The back sight on A is 0.348 m and the fore sight on B is 1.263 m. What will be the true difference of level A and B?

a. 0.896 m
b. 0.915 m
c. 1.544 m
d. 1.611 m

70. From the following page of a levelling field book, the missing values in F.S. and B.S respectively are

Station	B.S.	I.S.	F.S.	Rise	Fall	Remarks
1	4.550					Starting Point
2	2.125		?		0.750	Change point
3		2.225				
4	?		1.975			Change point
5		2.445		1.500		

a. 3.804, 0.945
b. 3.804, 3.945
c. 5.300, 0.945
d. 5.300, 3.945

[GATE 2007]

71. Identify the correct statement for the rise and fall method of RL computation

a. $\Sigma FS - \Sigma BS = \Sigma Rise - \Sigma Fall = \Sigma RL$ of first point – ΣRL of last point
b. $\Sigma BS - \Sigma FS = \Sigma Rise - \Sigma Fall = \Sigma RL$ of last point – ΣRL of first point
c. $\Sigma FS - \Sigma BS = \Sigma RL$ of last point – ΣRL of first point
d. $\Sigma BS - \Sigma FS = \Sigma Fall - \Sigma Rise$

72. In tacheometric observations, the staff readings are found to be 2.4 m, 3.0 m and 3.6 m with the angle of elevation is 45°. In a second set of observation, the readings are found to be 3.2 m, 3.8, and 5.0 m with the angle of elevation 60°. The horizontal distance between the instrument station and staff station are found to be 75.5 m and 60 m, respectively in first and second set of observation. If the additive constant of the instrument is 0.3 m, determine the multiplying constant.

a. 100.20
b. 102.54
c. 99.75
d. 105.21

73. In tacheometric observations, the staff readings are found to be 1.2 m, 2.0 m and 3.0 m. If the angle of elevation is 30°, the multiplying factor is 100 and the additive constant is 0, then the horizontal distance between the staff and the instrument station, expressed in m is
a. 110
b. 150
c. 135
d. 115

74. The most probable value of the angle A from the following observation equations.
A = 10°; 2A = 20°2'; 5A = 50°15'
a. 10°10'
b. 10°3.8'
c. 9°50'
d. 10°2.6'

75. The most probable value of the angle A from the following observation equations.
A = 15° weight 2; 2A = 30°10'; weight 4
a. 15°4.4'
b. 15°6.8'
c. 15°4'
d. 15°4.9'

76. The most probable value of A for the following data :
A = 20°; B = 50°; A + B = 70°10'
a. 20°5.5'
b. 20°3.3'
c. 15°4.3'
d. 20°5'

77. Given A = 30° ± 1.2; B = 40° ± 1.4;
The most probable error of A+B is
a. 70° ± 1.3
b. 70° ± 1.5
c. 70° ± 1.6
d. 70° ± 1.8

78. Given A = 30° ± 1.4; B = 40° ± 1.6;
The most probable error of A+B is
a. 1.45
b. 2.56
c. 2.13
d. 3.66

79. The most probable value of the area of a circle whose radius is 10 ± 0.04
a. 314 ± 2.5
b. 314 ± 2.6
c. 314 ± 2.7
d. 314 ± 2.8

80. The most probable value and the probable error of the area of a rectangle whose sides are 50 ± 0.04 m and 150 ± 0.06 m
a. 7500 ± 2.7
b. 7500 ± 4.7
c. 7500 ± 5.7
d. 7500 ± 6.7

81. The most probable value of the volume of a cylinder whose height and radius are 10 ± 0.02 and 5 ± 0.01
a. 785 ± 3.51 m^3
b. 785 ± 2.87 m^3
c. 785 ± 5.34 m^3
d. 785 ± 0.53 m^3

82. The lengths and standard errors of three sections AB, BC, and CD of a straight line AD are given below
AB = 125.85 ± 0.021 m; BC= 205.72 ± 0.029 m; CD = 246.21 ± 0.025 m
The standard error in total length AD in m is

a. ± 0.0436
b. ± 0.0350
c. ± 0.0250
d. ± 0.0019

83. In shaft plumbing, the plumb bobs are immersed in a bucket of water to prevent
a. oscillation of wires
b. effect of ventilation current
c. kinking of the wires
d. none of the above

84. In weissbach triangle method of correlation, number of plumb bobs requires for suspensions
a. one
b. two
c. three
d. four

85. In co-planing or exact alignment method of correlation, number of plumb bobs requires for suspensions
a. one
b. two
c. three
d. four

86. In weissbach quadrilateral method of correlation, number of plumb bobs requires for suspensions
a. one
b. two
c. three
d. four

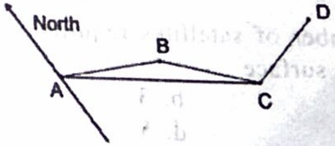
87. In precise magnetic method of correlation, number of plumb bobs requires for suspensions
a. one
b. two
c. three
d. four

88. In weiss quadrilateral method of correlation, number of shaft used for correlation
a. one
b. two
c. three
d. four

89. In precise magnetic method of correlation, number of shaft used for correlation
a. one
b. two
c. three
d. four

90. In weissbach triangle method of correlation, number of shaft used for correlation
a. one
b. two
c. three
d. four

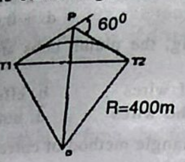
91. In an underground correlation survey by the Weisbach triangle (figure below) the following data are obtained.
AB = 3.50m, BC = 2.75m, CA = 6.20m, ∠ACD = 179° 14' 33'', ∠BCD = 179°10'17'' and bearing of AB = 115°23'49''. The bearing of raverse CD is



a. 102° 27'16"
b. 114°41'49"
c. 115° 27'16"
d. 179°14'16"

[GATE 2011]

92. Match the elements of a simple curve as given in the figure below :



The tangent length in m is
a. 215.5
b. 220.4
c. 228.4
d. 230.9
[GATE 2010]

93. In the above question, the length of the chord in m is
a. 375
b. 400
c. 415
d. 450
[GATE 2010]

94. Match the following

Instruments	Purpose/Measurement
1. Abneys level	a. Horizontal and vertical angles
2. Pantograph	b. Area of plotted figure
3. Planimeter	c. enlargement and reduction of plotted maps
4. Box sextant	d. Angle of inclination

a. 1-a, 2-c, 3-d, 4-b
b. 1-c, 2-b, 3-d, 4-a
c. 1-d, 2-a, 3-d, 4-c
d. 1-d, 2-c, 3-b, 4-a

95. The basic concept behind GPS is
a. Triangulation
b. Trilateration
c. Traverse
d. Tacheometry

96. The local mean time at longitude 75°30' is 8hr. 45 min. The corresponding standard time with reference to 82°30' meridian is
a. 8 hr. 13 min
b. 9 hr. 13 min
c. 9 hr. 17 min
d. 10 hr. 17 min
[GATE 2008]

97. The number of satellites and orbital planes in a GPS constellation (US system) are
a. 21 and 3
b. 24 and 4
c. 24 and 6
d. 25 and 5

98. The GPS satellites orbital planes are inclined to the equator and spaced in the longitude. The inclination and the spacing of the longitude in degrees are
a. 45° and 55°
b. 55° and 60°
c. 60° and 55°
d. 60° and 60°

99. The minimum number of satellites required to locate the position of any object on the earth surface
a. 2
b. 3
c. 4
d. 5

100. The minimum number of satellites required to locate the position of any object on the air

---

a. 2
b. 3
c. 4
d. 5

101. GPS gives height from
a. WGS83
b. WGS84
c. WGS85
d. None of the above

102. What is the name of navigation and positioning system proposed by India?
a. GLONASS
b. GSTB
c. IRNSS
d. EGNOS

103. C/A stands for
a. coarse acquisition
b. conditional access
c. code access
d. code acquisition

104. Satellite orbits are inclined at an angle of 55° from
a. Greenwich meridian
b. Equator
c. Specific latitude
d. Specific longitude

105. Orbital altitude of GPS satellite from earth surface
a. 15000 km
b. 20000 km
c. 25000 km
d. 30000 km

106. GPS is a satellite based radio navigation system developed and operated by
a. Department of Commerce
b. Department of Defence
c. Department of Mines
d. Geological Survey

107. The three segments, whose synchronous functioning is essential for GPS operations, are
a. space, control and user
b. signal, control and user
c. space, control and geo-registration
d. signal, control and geo-registration
[GATE 2012]

108. DGPS stands for
a. Different Global Positioning System
b. Direct Global Positioning System
c. Differential Global Positioning System
d. Dynamic Global Positioning System

109. GIS stands for
a. Global Information System
b. Geological Information System
c. Geographical Information System
d. Group Information System

110. Determine the airplane flying height to obtain the average scale of the photograph equal 1/5000. Ground surface elevations vary frum 150 to 300 m. Focal length of the camera lens is 150 mm.
a. 997
b. 975
c. 1275
d. 1359

111. The scale of the photograph is 1:1000. The photograph size is 20 cm X 20 cm. Determine the number of photographs required to cover an area of 50 km2 if the longitudinal overlap is 60 % and the side overlap is 30 %.
a. 4412
b. 4465
c. 4483
d. 4056

112. *Determine the number of photographs required to cover an area of 10 X 20 km², if the longitudinal overlap is 70% and the side overlap is 30%. The scale of the photograph is 1: 10000 and the photograph size is 25 cm X 25 cm.*
a. 145
b. 135
c. 153
d. 195

113. *An image of the top of a building is 90 mm from the principal point of the photograph. The elevation of the top of the building is 500 m and the flying height is 5000 m above the datum level. Determine the relief displacement.*
a. 9 mm
b. 11 mm
c. 10 mm
d. 7 mm

114. *A line 2500 m long lying at an elevation of 500 m measures 10 cm on a vertical photograph. The focal length of the camera is 20 cm. Determine the scale of the photograph for the area having an elevation of 1000 m.*
a. 1: 24550
b. 1: 22250
c. 1: 2250
d. 1: 21250

115. *A line measures 10 cm on a photograph taken with a camera having a focal length of 20 cm. The same line measures 3 cm on a map drawn to a scale of 1: 50000. Calculate the flying height of the aircraft if the average altitude of the ground is 300 m.*
a. 3020 m
b. 3000 m
c. 4300 m
d. 3300 m

116. A vertical photograph was taken from a flying height of 3000 m above mean sea level (MSL) of a terrain of average elevation of 750 m. If the focal length of the camera is 150 mm, then the scale of the photograph is
a. 1:16000
b. 1: 18000
c. 1:15000
d. 1: 14000

[GATE 2002]

Answer

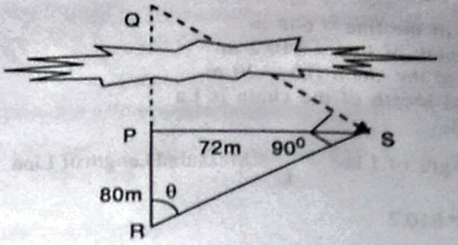
1. c	2. d	3. c	4. b	5. c	6. b	7. d
8. b	9. a	10. a	11. d	12. a	13. d	14. a
15. c	16. a	17. b	18. a	19. c	20. a	21. c
22. a	23. c	24. b	25. a	26. a	27. a	28. b
29. c	30. b	31. a	32. b	33. a	34. b	35. c
36. a	37. b	38. b	39. c	40. d	41. d	42. b
43. a	44. c	45. b	46. a	47. d	48. c	49. a
50. c	51. b	52. c	53. c	54. c	55. a	56. c
57. c	58. b	59. a	60. b	61. b	62. b	63. a
64. b	65. c	66. b	67. a	68. a	69. a	70. d
71. b	72. b	73. c	74. d	75. a	76. b	77. d
78. c	79. a	80. d	81. a	82. a	83. a	84. b
85. b	86. b	87. a	88. a	89. a	90. a	91. b
92. d	93. b	94. d	95. a	96. b	97. c	98. b
99. b	100. c	101. b	102. c	103. a	104. b	105. b
106. b	107. a	108. c	109. c	110. b	111. b	112. c
113. c	114. b	115. d	116. c			

Solution of Exercise 5

1. Given that,
Ground distance (x) = 300 m
Map distance (y) = 3 cm = 0.03 m

$$\text{Representative Factor (R.F.)}\ \frac{\text{Map distance (y)}}{\text{Ground distance (x)}} = \frac{0.03}{300} = 1{:}10{,}000$$

3.



From right angle triangle PRS

$$RS = \sqrt{80^2 + 72^2} = 107.62\text{m}$$

From similar triangle PRS and QRS, we get

$$\frac{PR}{RS} = \frac{RS}{QR}$$

$$\Rightarrow \frac{80}{107.62} = \frac{107.62}{PQ + 80}$$

$$\Rightarrow PQ + 80 = 144.78$$

$$\Rightarrow PQ = 64.78 \approx 64.8$$

4. True length of the chain (L) = 20 m
Incorrect length of the chain or tape = (L + ΔL) = (20 + 0.05) m
[Note : + sign, if the chain/tape is longer than the indication i.e. the chain is too long]
Measured length of the line = 536 m

$$\text{Threfore, true Length of Line} = \frac{L + \Delta L}{L} * \text{Measured Length of Line}$$

$$= \frac{20.05}{20} * 536 = 537.34\text{ m}$$

5. True length of the tape (L) = 30 m
Error in tape (ΔL) = ± 0.005
Measured length of the line = 1500 m
Number of measurements required to measure 1500 m with 30 m tape = 50

Expected total error made in the measurement = $\pm 0.005\sqrt{50} = \pm 0.035$

6. True length of the chain (L) = 20 m
Incorrect length of the chain or tape = (L - ΔL) = (20 – 0.05) m
[Note : –ve sign, if the chain/tape is shorter than the indication i.e. the chain is too short]

Measured length of the line = 536 m

$$\text{Therefore, true Length of Line} = \frac{L-\Delta L}{L} * \text{Measured Length Line}$$

$$= \frac{19.95}{20} * 536 = 534.66\text{m}$$

7. Given data,
True length of the line = 608 m
Measured length of line = 610.2 m
True length of the chain (L) = 20 m
Let the actual length of the chain is La
We have,

$$\text{True Length of Line} = \frac{L_a}{L} * \text{Measured Length of Line}$$

$$\Rightarrow 608 = \frac{L_a}{20} * 610.2$$

$$\Rightarrow L_a = \frac{608}{610.2} * 20 = 19.928\text{m}$$

8. Given data,
True length of the line = 612 m
Measured length of line = 610.2 m
True length of the chain (L) = 20 m
Let the actual length of the chain is La
We have,

$$\text{True Length of Line} = \frac{L_a}{L} * \text{Measured Length of Line}$$

$$\Rightarrow 612 = \frac{L_a}{20} * 610.2$$

$$\Rightarrow L_a = \frac{612}{610.2} * 20 = 20.059\text{m}$$

9. Given data,
Measured area of the plot (Am) = 45 km^2
True length of the chain (L) = 30 m
Actual length of the chain (La) = (L – ΔL) = (30 – 0.1) = 29.9 m
[Note : –ve sign, if the chain/tape is shorter than the indication i.e. the chain is too short]
Let the true or actual area of the plot is Aa
We have,

$$\text{True of Actual area of the plot} = \left(\frac{L_a}{L}\right)^2 * \text{Measured area of the plot}$$

$$\Rightarrow A_a = \left(\frac{29.9}{30}\right)^2 * 45$$

$$\Rightarrow A_a = 44.7\,\text{km}^2$$

---

10. Given data,
Length of chain (L) = 100 m
Ground rise per chain length (h) = 5 m

$$\text{Correction factor for 1 chain length} = \frac{h^2}{2L} = \frac{5^2}{2*100} = 0.125\text{m}$$

11. Given data,
Measured length along the slope (L) = 100 m
Ground rise (h) = 15 m

$$\text{Correction factor for 100m chain length } (L_c) = \frac{h^2}{2L} = \frac{15^2}{2*100} = 1.125\text{m}$$

Therefore, Horizontal distance between the two points = $L - L_c$
= 100 – 1.125 = 98.875m

12. Given data,
Length of chain (L) = 20 m
Slope angle (θ) = 15°
Correction factor per 20 m chain length = L (1–cosθ) = 20(1–cos15) = 0.681 m

13. Given data,
Measured length along slope (L) = 275 m
Slope (m) = tanθ = 1 in 2.5 [θ is slope angle]
⇒ cosθ = 0.93
Horizontal length between two points = L cosθ = 275*0.93 = 255.33 m

14. Given data,
Coefficient of thermal expansion = $\alpha = 1.18*10^{-5}/°\text{C}$
Mean temperature during measurement = T_m = 32°C
Temperature at which the tape is standardised = T_o = 18°C
Measured length of the line = L = 550 m.
Let C_t is the correction for temperature and Actual length of the line is L_a
We have,
$C_t = \alpha(T_m - T_0)L = 1.18 * 10^{-5} (32 - 18) * 550 = 0.09068\text{m}$
Therefore, actual length $(L_a) = L + C_t = 550 + 0.09068 = 550.09068\text{m}$

15. Given data,
Coefficient of thermal expansion = $\alpha = 11*10^{-6}/°\text{C}$
Mean temperature during measurement = T_m = 15°C
Temperature at which the tape is standardised = T_o = 20°C
True length of tape = L = 30m
Let C_t is the correction for temperature
We have,
$C_t = \alpha(T_m - T_0)L = 11 * 10^{-6} (15 - 20) * 30 = -0.0165\text{m} = -1.65\text{mm}$

16. Given data,
True length of tape = L = 30 m.
Mass of the tape between the two consecutive span = M = 1.5 kg
Applied Pull = P = 150 N
Let C_s is the sag correction factor for a single span and the actual length is L_a

We have,

$$C_s = \frac{L*(M*g)^2}{24*P^2} = \frac{30*(1.5*9.81)^2}{24*150^2} = 0.012m$$

Therefore, actual length of the line = L − C_s = 30 − 0.012 = 29.988
[Sag correction is always negative]

17. Given data,
True length of steel tape = L = 30 m.
Cross sectional area of the tape at standardised temperature =A = $5*10^{-6}$ m^2
Mass of the tape between the two consecutive span = M = 1.5 kg
Applied Pull during standardised = P_0 = 5.45 N
Pull during measurement = P = 9.09 N
Modulus of elasticity of steel = E = 200 GPa = 200*109 Pa
Let C_p is the pull correction for measuring the 81.15 m distance
We have,

$$C_P = \frac{L*(P-P_0)}{A*E} = \frac{81.15*(9.09-5.45)}{5*10^{-6}*200*10^9} = 0.000295m = 0.295\,mm$$

18. Given data,
Measured volume of the dump (Vm) = 4000 m^3
True length of the chain (L) = 20 m
Actual length of the chain (La) = (L − ΔL) = (30 − 0.20) = 29.8 m 19.8
[Note : –ve sign, if the chain/tape is shorter than the indication i.e. the chain is too short]
Let the actual volume of the dump is V_a
We have,

$$\text{Actual volume of the dump} = \left(\frac{L_a}{L}\right)^3 * \text{Measured volume of the dump}$$

$$\Rightarrow V_a = \left(\frac{29.8}{30}\right)^3 * 4000 = 3920m^3$$ ⇒ 388/m³

19. Given that,
Representative factor (RF) is 1:2000
Map area (Am) = 58 cm^2 = $58*10^{-4}$ m^2
Let the actual area of the mine plot is A_a
Actual area (A_a) = $(2000)^2 * 58*10^{-4}$ = 23200 m^2

20. Given data,
Condition 1 :
Map area (Am_1) = 100 m^2
Representative Factor (RF_1) = 1: 800
Condition 2 :
Let the map area is A_{m2}
Representative Factor (RF_2) = 1: 2000
Let the actual area of the plot is A_a

We have,

$$A_a = \frac{A_{m1}}{RF_1^2} = \frac{A_{m2}}{RF_2^2}$$

$$\Rightarrow \frac{100}{\left(\frac{1}{800}\right)^2} = \frac{A_{m2}}{\left(\frac{1}{2000}\right)^2}$$

$$\Rightarrow A_{m2} = 100*\left(\frac{800}{2000}\right)^2 = 16m^2$$

21. Area of XABCDEFGY $= \frac{1}{2}\Sigma_{i=1}^{6}(a_i + b_i)(h_i)$

$$= \frac{1}{2}[(7.2+11.9)(0.6)+(11.9+14.4)(0.8)+(14.4+6)(1)+(6+6.1)(1.3)+ (6.1+11.8)(1.05)+(11.8+12.4)(0.6)]$$

= 5.73 + 10.52 + 10.2 + 1.815 + 9.3975 + 7.26 = 44.9225 m^2

23. Given angle is 5°15'25"
Again, 5°15'25" = 5*60*60" +15*60" + 25"= 18925"
Since, 360° equivalent to 24 hours
⇒ 360*60*60" equivalent to 24*60*60 sec
⇒ 18925" equivalent to (24*60*60/360*60*60) *18925 sec
= (24*18925)/360 = 1221.667 sec
= 21 min 1.67 sec

24. Given data,
Radius of the curve (r) = 200 m
Deflection angle (θ) = 65°

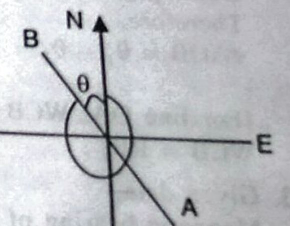
$$\text{Therefore, the length of the curve} = \frac{\theta}{360}*2\pi r$$

$$= \frac{65}{360}*2\times3.14*200 = 226.78m$$

25. WCB (θ) = 236°25'
For, 180° ≤ θ ≤ 270°
Quadrant bearing = S (WCB − 180°) W = S (236°25'−180°) W = S56°25'W

26. Reduced Bearing (RB) = S57°36'W
For S-W direction,
WCB = 180 + RB = 180 + 57°36' = 237°36'

27.



Fore bearing (FB) of line AB = N 36° W = 360° − 36° = 324°
Therefore, back bearing of line AB = FB − 180° = 324° − 180° = 144°

28. *In the above problem, we have determined the back bearing of line AB is 144°*
And, this is equivalent to S (180° − 144°) E = S 36° E

29. *Fore bearing (FB) of line AB = 136°*
Therefore, back bearing of line AB = FB + 180° = 136° + 180° = 316°

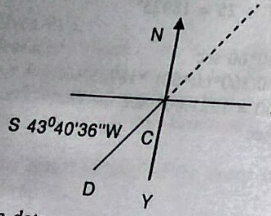
30. *Given data,*
Coordinate of point C is (x_1 = −280.4, y_1 = −310.8)
Coordinate of point D is (x_2 = −620.9, y_2 = −635.9)
[-ve signs indicate that points are located in fourth quadrant]
Therefore,

$$\text{Length of the line } CD = L\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-280.4+620.9)^2+(-310.8+635.9)^2} = \sqrt{(340.5)^2+(325.1)^2}$$

$$= 470.78\text{m}$$

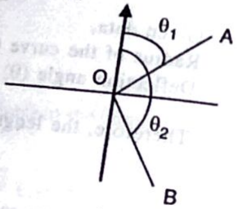
$$\text{We have, } <DCY = \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \tan^{-1}\left(\frac{-310.8+635.9}{-280.4+620.9}\right)$$



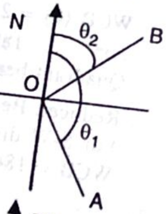
$$= \tan^{-1}\left(\frac{325.1}{340.5}\right) = S\,43.67^0\,W$$

$$= 226.32^0$$

Bearing of line CD = S 43°40'36"W

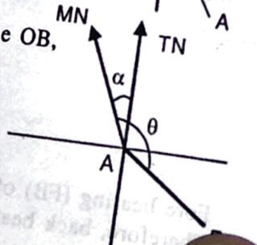


31. Given data,
Bearing of line OA = θ_1 = 56°
Bearing of line OB = θ_2 = 154°
Therefore,
<AOB = $\theta_2 - \theta_1$ = 154° −56° = 98°



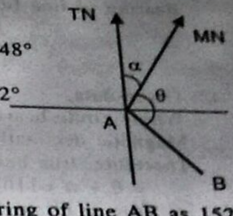
32. Given data,
Bearing of line OA = θ_1 = S56°E = 180° −56° = 124°
Bearing of line OB = θ_2 = N54°E = 54°
Therefore,
<AOB = $\theta_1 - \theta_2$ = 124° −54° = 70°

[For line OA, WCB = (180 − RB) and for line OB, WCB = RB]

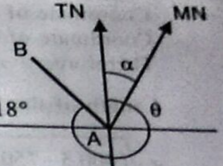


33. Given data,
Magnetic bearing of line AB = θ = 136°
Magnetic declination = α = 8°W
True bearing = θ − α = 136° − 8° = 128°

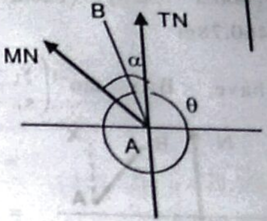
34. Given data,
Magnetic bearing of line AB = θ = S32°E
= 180°−32° = 148°
Magnetic declination = α = 4°E
True bearing of line AB = θ + α = 148°+ 4° = 152°



35. In the above solution, we determined the true bearing of line AB as 152°
This can be represented in reduced bearing as S (180°-152°) E = S 28° E



36. Given data,
True bearing of line AB = θ = 326°
Magnetic declination = α = 8°E
Magnetic bearing of line AB = θ − α = 326°- 8° = 318°



37. Given data,
True bearing of line AB = θ = 349°
Magnetic declination = α = 15°W
Magnetic bearing = α − (360 − θ)
= 15°− (360°−349°) = 4°

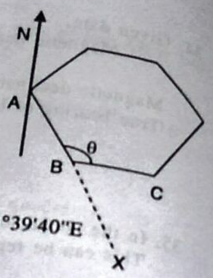
38. Magnetic bearing of the sun at noon = φ = 185°
True bearing of sun at noon = θ = 180°
Therefore, magnetic declination (α) = 5° W

39. Magnetic bearing of the sun at noon = φ = 358°
True bearing of sun at noon = θ = 360°
Therefore, magnetic declination (α) = 2° E

40. Given data,
Magnetic bearing of a line in the year 1906 (θ_1) = S43°30'E
= 180° − 43°30' = 136°30' [WCB = 180° − RB]
Magnetic declination at the point in the year 1906 = α_1 = 2°E
Magnetic declination at the same point in the year 2006 = α_2 = 3°W
True bearing of the line (f) in the year 1906 = $\theta_1 + \theta_1$ = 136°30'+2^0 = 138°30'
Since, the true north is always constant. Therefore, the value of φ remains same in the year 2006.
Magnetic bearing of line in 2006 = φ + α_2 = 138°30' + 3^0 = 141°30'
= S (180° - 141°30') E = S 38°30'E

41. Angle of a regular hexagon (θ) = 120°
Bearing of line AB = S50°10'E = 180° −50°10'
= 129°50'

Bearing of line BC = Bearing of line AB − <CBX
= 129°50′ − (180° − 120°)
= 69°50′



42. Given data,
Whole circle bearing of line AB = θ = 116°20′20″
Magnetic declination = α = 20°E
Therefore, true bearing of line AB
= $\theta + \alpha$ = 116°20′20″ + 20° = 136°20′20″
⇒ True quadrantal bearing = 180° − 136°20′20″ = S43°39′40″E

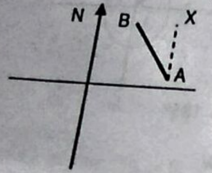
43. Given data,
Coordinate of point A is (x_1 = 400.5, y_1 = 620.2)
Coordinate of point D is (x_2 = 750.5, y_2 = 320.5)
Therefore,

$$\text{Length of the line } AB = L = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(400.5 - 750.5)^2 + (620.2 - 320.5)^2} = \sqrt{(-350)^2 + (299.7)^2}$$

= 460.78m

44. We have, $< BAx = \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right)$



$$= \tan^{-1}\left(\frac{620.2 - 320.5}{400.5 - 750.5}\right) = \tan^{-1}\left(-\frac{299.7}{350}\right)$$

= N 40.57⁰W [for negative sign, this is in W]
Bearing of line AB = N 40.57⁰ W

46. Given data,

Side	Length (m)	WCB
AB	100	90°
BC	173	180°
CA	200	330°

Therefore the latitude and departure of various lines are as follows :

Side	Length (m)	WCB	Latitude	Departure
AB	100	90°	100*cos90°=0	100*sin90°=100
BC	173	180°	173*cos180°=−173	173*sin 180° = 0
CA	200	330°	200*cos330° = 173.205	200*sin33° = − 100
			Σ Latitude = 0.205	Σ Departure = 0

We have,
Closing error (e) $= \sqrt{(\Sigma_i \text{ Latitude})^2 + (\Sigma_i \text{ Departure})^2} = \sqrt{0.205^2 + 0^2}$
= 0.205m = 205mm [i is the number of lines]

47. We have,
Reduced bearing of closing error line $= \tan^{-1}\left(\frac{\Sigma_i \text{ Departure}}{\Sigma_i \text{ Latitude}}\right) = \tan^{-1}\left(\frac{0}{0.205}\right)$
$= \tan^{-1} 0 = 0°$

48. Given data,
Perimeter of the traverse ABCDE = P = 425 m
Error in latitude = Σ Latitude = 0.25 m
Error in departure = Σ Departure = −0.44 m
Therefore, Closing error (e) $= \sqrt{(\Sigma \text{ Latitude})^2 + (\Sigma \text{ Departure})^2}$
$= \sqrt{(0.25)^2 + (-0.44)^2} = 0.506\text{m}$

Precision of traverse $= \frac{e}{P} = \frac{0.506}{425} = 1 \text{ in } 840$

49. Given data,
Departure of line AB = 225.52m
Latitude of line AB = 388.835m

Therefroe, length of line AB $= \sqrt{(\text{Latitude})^2 + (\text{Departure})^2}$
$= \sqrt{(388.835)^2 + (225.52)^2} = 449.50\text{m}$

Again, azimuth of line AB $= \tan^{-1}\left(\frac{\text{Departure}}{\text{Latitude}}\right) = \tan^{-1}\left(\frac{225.52}{388.835}\right)$
$= \tan^{-1}(0.58) = 30.11^0$

50. Given data,
Number of stations in traverse = n = 16
Least count of theodolite = L = 20″
Permissible angular error (A_e) $= L * \sqrt{n} = 20 * \sqrt{16} = 80''$

51. Given Data
Vertical angle (θ) = 26°36′
Staff readings = 0.80m, 3.08m, 5.36m
Multiplying factor (f/i) = 100
Additive constant (f + k) = 1.9
Let the horizontal distance between the stations A and B is D
Therefore, the horizontal distance between the instrument and staff station is given by $D = \frac{f}{i} * S * (\cos\theta)^2 + (f + k)\cos\theta$
[Applying the condition of vertical staff and inclined line of collimation]
= 100 * (5.36 − 0.80) (cos 26°36′)² 1.9*cos26°36′
[S = staff intercept = upper reading − lower reading]
= 100*4.56*0.8 + 1.9 × 0.894 = 366.27m

52. Given data,

Height of the instruent (h) = 1.2m

R.L. of station B is 150 m

Let the vertical distance between A and B is V

Therefore, we have

$$V=\frac{f}{i}*S*\cos\theta*\sin\theta+(f+k)\sin\theta$$

[Applying the condition of vertical staff and inclined line of collimation]

$V = 100 * (5.36 - 0.80) \cos 26°36'* \sin 26°36'\ 1.9*\text{sins}26°36'$

$= 182.56 + 0.85 = 183.41\text{m}$

Therefore

R.L. of A = R.L. of B + h + V– S_3 = 150 + 1.2+183.41–3.08 = 331.53

53. Given data,

Vertical angle (θ) = 45°

Staff readings = 2.40 m, 3.0m, 3.6m

Multiplying factor (f/i) = 100

Additive constant (f + k) = 0.3

Let the horizontal distance between the stations A and B is D

Therefore, the horizontal distance between the instrument and staff station is given by

$$D=\frac{f}{i}*S*(\cos\theta)^2+(f+k)\cos\theta$$

[Applying the condition of vertical staff and inclined line of collimation]

$= 100* (3.6 - 2.4) (\cos 45)^2 + 0.3 * \cos 45$

[S = staff intercept = upper reading – lower reading]

$= 100 * 1.2 * 0.50 + 0.3 * 0.707 = 60.2\text{m}$

55. Given data,

Elevation of A = 500 m

Staff reading at A = s_1 = 0.5 m

Staff reading at B = s_2 = 3.8 m

Therefore, Elevation of B = Elevation of A + $s_1 - s_2$

$= 500 + 0.5 - 3.8 = 496.7$ m

56. Given data,

Elevation of A = 510 m

Staff reading at A = s_1 = 0.5 m

Staff reading at B = s_2 = 2.8 m

Therefore, Elevation of B = Elevation of A + $s_1 - s_2$

$= 510 + 0.5 - 2.8 = 507.7$ m

57. Let the gradient of line AB is θ

There are three intermediate stations in between A and B, and the horizontal distance between each consecutive station is 30 m.

Thus, the horizontal distance between A and B = 4*30 =120 m

We have,

$$\text{Gradient of line } AB=\theta=\tan^{-1}\left(\frac{\text{R.L. of B}-\text{R.L. of A}}{\text{Horizontal dis tan ce between A and B}}\right)$$

$$=\tan^{-1}\left(\frac{507.7-510}{120}\right)=\tan^{-1}\left(-\frac{2.3}{120}\right)=-1.09^0$$

[–ve sign indicate that slope is in downward direction from A to B]

58. When the instrument is set near A :

Staff reading at A = a_1 = 2.5 m

Staff reading at B = b1 = 3.7 m

When the instrument is set near B :

Staff reading at A = a_2 = 2 m

Staff reading at B = b_2 = 3 m

Let the true difference of level A and B is d

$$\text{We have, } d=\frac{(b_1-a_1)+(b_2-a_2)}{2}=\frac{(3.7-2.5)+(3-2)}{2}=\frac{2.2}{2}=1.1\text{m}$$

59. When the instrument is set near A :

Staff reading at A = a_1 = 1.5 m

Staff reading at B = b_1 = 2.8 m

When the instrument is set near B :

Staff reading at A = a_2 = 1.4 m

Staff reading at B = b_2 = 2.6 m

Let the true difference of level A and B is da

$$\text{We have, } d=\frac{(b_1-a_1)+(b_2-a_2)}{2}=\frac{(2.8-1.5)+(2.6-1.4)}{2}=\frac{2.5}{2}=1.25\text{m}$$

60. Let the error in levelling due to curvature, refraction, and collimation are e_1, e_2, and e_3 respectively.

Given that, error due to collimation is 0.002 m per 100 m. and the distance between A and B is 1000 m.

Therefore, total error in collimation (e_3) = 0.002*1000/100 = 0.02 m

Again, error due to curvature of the surface (e_1) = $0.0785*d^2$

[Where, d is the distance in km between A and B]

$\Rightarrow e_1 = 0.0785*12 = 0.0785$

Total error in level measurement can be determined as

$$\text{Total error (e)}=\frac{(b_1-a_1)-(b_2-a_2)}{2}=\frac{(2.8-1.5)+(2.6-1.4)}{2}=\frac{0.1}{2}=0.05\text{m}$$

We have,

$$e = (e_1 - e_2) + e_3$$

[($e_1 - e_2$) is the combined correction of curvature and refraction. Since refraction error compensate the curvature, negative sign given]

Therefore, error due to refrection (e_2) = $-e + e_1 + e_3$ = –0.05 + 0.0785 + 0.02

refraction = 0.0485m

61. Given data,

RL of bench mark (B.M.) station = 100 m

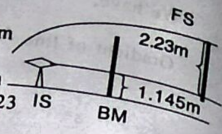
Back sight reading on B.M. station = BS = 1.45 m

Inverse staff reading on foresight = FS= 2.23 m

Therefore, R.L. of staff station or foresight station

= RL of B.M. + BS + FS = 100 + 1.45 + 2.23

= 103.68 m



65. Correction for curvature for a distance of 1200 m is determined as

$= 0.0785*D^2$ in m

[Where, D is distance in km]

$= 0.0785*(1.2)^2 = 0.11304$ m

Correction for curvature is subtractive in nature and hence this is -0.11304 m

66. Correction for refraction can be determined as

$$= \frac{\text{correctiion for curvature}}{7} = \frac{1}{7} * \frac{D^2}{2R}$$

Where, D is distance, and R is the radius of the earth

$$= \frac{(800)^2}{7*12742*1000} = 0.007\text{m}$$

Correction for refraction is additive in nature and hence the value is + ve

67. Given data,

Diameter of the earth (d) = 12000 km

Distance (D) = 1200m

We have,

Combine correction for curvature and refraction

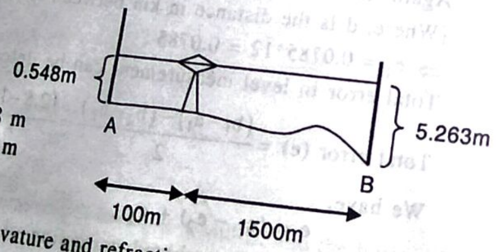
= – correction for curvature + correction of refraction

$$= -\text{correction for curvature} + \frac{\text{correction for curvature}}{7}$$

$$= -\frac{6*\text{correction for curvature}}{7} = \frac{6*D^2}{7*d} = -\frac{6*1200*1200}{7*12000*1000}$$

= –0.103m

68.



Given data,

BS reading at A = 0.548 m

FS reading at B = 5.263 m

OA = 100 m

OB = 1500 m

Combine correction for curvature and refraction at FS station B is given by

$$= -\frac{6*\text{correction for curvature}}{7} = -\frac{6*D^2}{7*d} = -\frac{6*1500*1500}{7*12742*1000} = -0.151\text{m}$$

Therefore, actual reading at B = 5.263 – 0.151 = 5.112 m

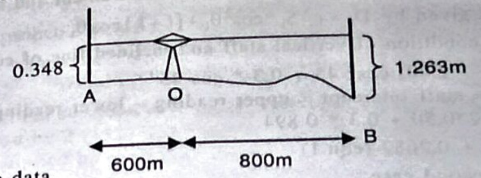
True difference of level at A and B

= Actual reading at B – Actual reading at A

= 5.112 – 0.548 = 4.564 m

[Since the distance of staff station A from instrument station O is very less, the combined error of curvature and refraction is neglected]

69.



Given data,

BS reading at A = 0.348 m

FS reading at B = 1.263 m

OA = 600 m

OB = 800 m

Combine correction for curvature and refraction at FS station B is given by

$$= -\frac{6*\text{correction for curvature}}{7} = -\frac{6*D^2}{7*d} = -\frac{6*800*800}{7*12742*1000} = -0.043\text{m}$$

Therefore, actual reading at B = 1.263 – 0.043 = 1.220m

Similarly, combine correction for curvature and refraction at BS station A is given by

$$= -\frac{6*\text{correction for curvature}}{7} = -\frac{6*D^2}{7*d} = -\frac{6*600*600}{7*12742*1000} = -0.024\text{m}$$

Therefore, actual reading at B = 0.348 – 0.024 = 0.324 m

True difference of level at A and B

= Actual reading at B – Actual reading at A

= 1.220 – 0.324 = 0.896 m

70. We have,

B.S. reading at station 1 – F.S. reading at station 2 = – 0.750

[Negative sign indicate fall]

⇒ F.S. reading at station 2 = 4.550 + 0.750

⇒ F.S. reading at station 2 = 5.30

Again, B.S. reading of station 4 – F.S. reading of station 5 = 1.50

⇒ B.S. reading at station 4 = 2.445 + 1.5 = 3.945

72. Given data,

Vertical angle in first set of observation $(\theta_1) = 45°$

Vertical angle in second set of observation $(\theta_2) = 60°$

Staff readings in first set of observation = 2.40 m, 3.00 m, 3.60 m

Staff readings in second set of observation = 3.20 m, 3.80, 5.00 m

Additive constant (f + k) = 0.3

Horizontal distance between the instrument stations and staff station in first set of observation = $D_1 = 75.5$ m

Horizontal distance between the instrument stations and staff station in second set of observation = $D_2 = 60$ m

Let the multiplying constant is c

Therefore, the horizontal distance between the instrument and staff station in first case is given by $D_1 = c * S_1 * \cos^2\theta_1 + (f+k)\cos\theta_1$

[Applying the condition of vertical staff and inclined line of collimation]

$75.5 = c*(3.60 - 2.40)\cos^2 45 + 0.3 * \cos 45^0$

[S_1 = staff intercept = upper reading – lower reading]

$\Rightarrow 75.5 = c*1.2*0.50 + 0.3 * 0.894$

$\Rightarrow 75.5 = 0.6c + 0.2682$ (eqn.1)

Similarly, for second case,

$D_2 = c*S_2*\cos^2\theta_2 + (f + k)\cos\theta_2$

$\Rightarrow 60 = c * (5-3.2) * \cos^2 60 + 0.3 * \cos 60$

$\Rightarrow 60 = 0.45c + 0.15$ (eqn. 2)

Solving above two equations, we get

$c = 102.54$

73. Given data,

Vertical angle $(\theta) = 30^\circ$

Staff readings = 1.20 m, 2.00 m, 3.00 m

Multiplying factor (f/i) = 100

Additive constant (f + k) = 0

Let the horizontal distance between the staff and instrument is D

Therefore, the horizontal distance between the instrument and staff station is given by $D = \frac{f}{i} * S * \cos^2\theta + (f+k)\cos\theta$

[Applying the condition of vertical staff and inclined line of collimation]

$= 100 * (3.0 - 1.2)\cos^2 30 + 0$

[S = staff intercept = upper reading – lower reading]

$= 100 * 1.8 * 0.750 = 135$m

74. Given that,

$A = 10^\circ$

$2A = 20^\circ 2'$

$5A = 50^\circ 15'$

Therefore, the normalised equation for A can be derived by multiplying the first, second, and third equation by 1, 2, and 5 respectively.

Therefore, the normal equations in A are as follows :

$A = 10^\circ$ (i)

$2*2A = 2*(20^\circ 2')$

$\Rightarrow 4A = 40^\circ 4'$ (ii)

$5*5A = 5*(50^\circ 15')$

$\Rightarrow 25A = 251^\circ 15'$ (iii)

Adding the above three equations, we get

$A + 4A + 25A = 10^\circ + 40^\circ 4' + 251^\circ 15'$

$\Rightarrow 30\ A = 301^\circ 19' = 300^\circ 79'$

$\Rightarrow A = 10^\circ 2.6'$

Most probable value of 'A' is10°2.6'

75. Given observation equations :

$A = 15^0$ Weight 2 and $2A = 30^0 10''$ Weight 4

The normal equation in A can be derived by multiplying the first and second equation by 2 (1*2) and 8 (2*4) respectively. That is each equation should be multiplied by multiplication of coefficient of A and weight of equation.

Therefore, the normal equations are as follows:

$2*A = 2*15^\circ$

$\Rightarrow 2A = 30^\circ$ (i)

$8*2A = 8*30^\circ 10'$

$\Rightarrow 16A = 241^\circ 20'$ (ii)

Adding the above two equations, we get

$18A = 271^\circ 20'$

$\Rightarrow A = 15^\circ 4.44'$

76. Given observations,

$A = 20^\circ$ (i)

$B = 50^\circ$ (ii)

$A + B = 70^\circ 10'$ (iii)

The normal equation in A can be derived by adding eqn. (i) and eqn. (iii)

$A + (A+B) = 20^\circ + 70^\circ 10'$

$\Rightarrow 2A + B = 90^\circ 10'$ (iv) normal equation in A

Similarly, the normal equation in A can be derived by adding eqn. (ii) and eqn. (iii)

$B + (A + B) = 50^\circ + 70^\circ 10'$

$\Rightarrow A + 2B = 120^\circ 10'$ (v) normal equation in B

Now solving eqn. (iv) and eqn. (v), we get

$2*(2A + B) - (A+2B) = 2*90^\circ 10' - 120^\circ 10'$

$\Rightarrow 3A = 180^\circ 20' - 120^\circ 10'$

$\Rightarrow A = 60^\circ 10'/3 = 20^\circ 3.33'$

Therefore, most probable value of A is 20°3.33'

77. Given data,

$A = 30^\circ \pm 1.2 = a \pm ea$ (assumed)

And, $B = 40^\circ \pm 1.4 = b \pm eb$ (assumed)

Let the most probable value of A + B is x and the error level is e_x

Therefore,

$x = (a + b) + e_x = (30° + 40°) + e_x = 70° + e_x$

The value of ex can be determined as

$$e_x = \sqrt{e_a^2 + e_b^2} = \sqrt{1.2^2 + 1.4^2} = \sqrt{1.44 + 1.96} = \sqrt{4.52} = 2.13$$

Therefore, the most probable value of (A+B) is

$x = 70° \pm 1.844$

78. Given data,

$A = 30° \pm 1.4 = a \pm ea$ (assumed)

and, $B = 40° \pm 1.6 = b \pm eb$ (assumed)

Let the probable error of A+B is ex

The value of e_x can be determined as

$$e_x = \sqrt{e_a^2 + e_b^2} = \sqrt{1.4^2 + 1.6^2} = \sqrt{1.96 + 2.56} = \sqrt{4.52} = 2.13$$

79. Given that,

Radius of the circle = $10 \pm 0.04 = r \pm er$ (assumed)

Let the area of the circle is A and the probable error in area is eA

We have,

$A = \pi r^2 = 3.14 * 10^2 = 314$

e_A can be determined as

$e_A = e_r * dA/dr = 0.04 * 2\pi r = 0.04 * 2 * 3.14 * 10 = 2.512$

Therefore, the most probable area of the circle = $A \pm e_A = 314 \pm 2.512$

80. Given that,

The probable values of sides of a rectangle are

$X = 50 \pm 0.04 = a \pm e_a$ (assumed)

And, $Y = 150 \pm 0.06 = b \pm e_b$ (assumed)

Area of the rectangle = $A = a*b = 50* \; 150 = 7500$

Let the error in area of rectangle is e_A

e_A can be determined as

$$e_A = \sqrt{\left(e_a \frac{\partial A}{\partial a}\right)^2 + \left(e_b \frac{\partial A}{\partial b}\right)^2} = \sqrt{(0.04 * b)^2 + (0.06 * a)^2}$$

$$= \sqrt{(0.04 * 150)^2 + (0.06 * 50)^2} = \sqrt{36 + 9} = 6.7$$

Therefore, the most probable area of the rectangle = $A \pm e_A = 7500 \pm 6.7$

81. Given that,

The probable value of radius of cylinder = $r \pm er = 5 \pm 0.01$

The probable value of height of cylinder = $h \pm eh = 10 \pm 0.02$

Therefore, volume of the cylinder = $V = \pi r^2 h = 3.14 * 52 * \; 10 = 785$

Let the error in volume of cylinder is e_v

e_v can be determined as

$$e_v = \sqrt{\left(e_r \frac{\partial V}{\partial r}\right)^2 + \left(e_h \frac{\partial V}{\partial h}\right)^2} = \sqrt{(0.01 * 2\pi rh)^2 + (0.02 * \pi r^2)^2}$$

$$= \sqrt{(0.01 * 2 * 3.14 * 5 * 10)^2 + (0.02 * 314 * 5^2)^2}$$

$$= \sqrt{9.86 + 2.46} = \sqrt{12.32} = 3.51$$

Therefore, the most probable area of the rectangle = $V \pm e_v = 785 \pm 3.51$

82. Given data,

$AB = 125.85 \pm 0.021$ m

$BC = 205.72 \pm 0.029$ m

$CD = 246.21 \pm 0.025$ m

Therefore, error in AB = $e_{ab} = 0.021$

Error in BC = $e_{bc} = 0.029$

Error in CD = $e_{cd} = 0.025$

Let the standard error in total length AD is e_{ad}

This can be determined as

$$e_{ad} = \pm\sqrt{(e_{ab})^2 + (e_{bc})^2 + (e_{cd})^2} = \pm\sqrt{(0.021)^2 + (0.029)^2 + (0.025)^2}$$

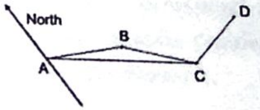
$$= \pm\sqrt{0.001907} = \pm 0.0436$$

91. Given data,

AB = 3.50 m, BC = 2.75 m, CA = 6.20 m, <ACD=179° 14' 33", <BCD=179°10' 17"

Bearing of AB = α =115°23'49".

Let the bearing of traverse line CD is θ



We have,

<NAB = Bearing of line AB = 115°23'49"

<ACB = <ACD - <BCD = 179°14' 33" −179°10' 17" = 4'16"

Again, applying triangle rule in ABC we get,

$$\frac{BC}{\sin < BAC} = \frac{AB}{\sin < ACB}$$ [For small angle, we can write sin θ = θ]

$$\Rightarrow \frac{2.75}{< BAC} = \frac{3.50}{\frac{256}{3600} \times \frac{3.14}{180}}$$

$$< BAC = \left(\frac{256}{3600} \times \frac{3.14}{180} \times \frac{2.75}{3.50}\right)^0 = \left(256 \times \frac{3.14}{180} \times \frac{2.75}{3.50}\right) = 3.50' = 3'30"$$

Again <DCP = 180 − <ACD = 180 − 179°14'33''=45'27''

Now,

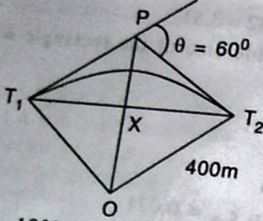
Bearing of line AC = Bearing of line AB + <BAC

= 115°23'49" + 3'30" =115°27'19"

Therefore, bearing of line CD = Bearing of line AC - <DCP

=115°27'19" – 45'27" = 114°41'52"

92. Given diagram,



We have,

$<T_1OT_2 + <T_1PT_2 = 180°$

$\Rightarrow <T1OT2 + (180 - \theta) = 180$

$\Rightarrow <T1OT_2 = \theta$

For symmetry in both the side, we get

$<PO\ T_1 = <POT_2 = \theta/2 = 30°$

In ΔPT_1O

$$\Rightarrow \tan 30 = \frac{PT_1}{400}$$

$$\Rightarrow PT_1 = 400*0.577 = 230.94m$$

Therefore, tangent length = $PT_1 = PT_2$ = 230.94 m

93. Again, from ΔXT_1O

$$\mathrm{Sin}\left(\frac{\theta}{2}\right) = \frac{XT_1}{OT_1}$$

$$\Rightarrow \sin 30 = \frac{(T_1T_2/2)}{400}$$

$$\Rightarrow T_1T_2 = 800*\frac{1}{2} = 400m$$

Therefore, the length of the chord = T_1T_2 = 400 m

96. Given that, the local mean time at longitude 75°30' is 8 hr. 45 min

Let the corresponding standard time with reference to 82°30' is t.

Differences in longitude = 82°30' – 75°30' = 7°

360° is equivalent to 24 hr.

Therefore, 7° is equivalent to (24*7)/360 hr.

= (24*7*3600)/360 sec

= 1680 sec = 1680/60 min

= 28 min

The corresponding standard time (t) = 8 hr. 45 min + 28 min

= 9 hr. 13 min

110. Given data,

Average scale of photograph (S) = 1/5000

Ground surface elevation is 150 to 300 m

Focal length of the camera lens (f) = 150 mm

Average surface elevation (h) = 150+ (300-150)/2 = 150 + 75 = 225 m

Let the air flying height is H

We have,

$$\text{Scale of Photograph (s)} = \frac{f}{H-h}$$

$$\Rightarrow \frac{1}{5000} = \frac{0.150}{H-225}$$

$$\Rightarrow H - 225 = 750$$

$$\Rightarrow H = 975m$$

111. Given data,

Length of photograph in the direction of flight = Lp = 20 cm

Width of photograph perpendicular to the direction of flight = Wp = 20 cm

Longitudinal overlap = OL = 60 % = 0.6

Side overlap = OW = 30 % = 0.3

The scale of photograph = S = 1/1000

Area of the tract to be photographed = Ag = 50 km^2

Let Lg is net ground distance corresponding to Lp.

Wg is net ground distance corresponding to Wp.

N is the number of photographs required.

Ap is net area of each photograph.

Then,

Lg = Lp*(1 – OL)/S = 1000*20*(1–0.6) = 8000 cm = 0.080 km

Wg = Wp* (1 – OW)/S = 1000*20*(1–0.3) = 14000 cm = 0.14 km

Net Area of each photograph = Ap = Lg * Wg = 0.080*0.14 = 0.0112 km^2

Therefore, Number of photographs required = Ag / Ap

= 50/ 0.0112 = 4464.28 = 4465 nos.

112. Given data,

Length of photograph in the direction of flight = Lp = 25 cm

Width of photograph perpendicular to the direction of flight = Wp = 25 cm

Longitudinal overlap = OL = 70 % = 0.7

Side overlap = OW = 30 % = 0.3

The scale of photograph = S = 1/10000

Area of the tract to be photographed = Ag = 200 km^2

Let Lg is net ground distance corresponding to Lp.

Wg is net ground distance corresponding to Wp.

N is the number of photographs required.

Ap is net area of each photograph.

Then,

$Lg = Lp*(1 - OL)/S = 10000*25*(1-0.7) = 75000$ cm = 0.75 km

$Wg = Wp*(1 - OW)/S = 10000*25*(1-0.3) = 175000$ cm = 1.75 km

Net Area of each photograph = $Ap = Lg * Wg = 0.75*1.75 = 1.3125$ km^2

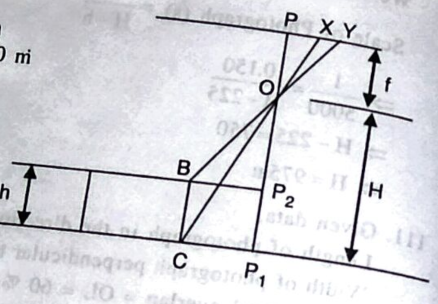
Therefore, Number of photographs required = Ag / Ap

= 200/1.3125 = 152.38 = 153 nos.

113. Given data,

Flying height = H = 5000 m

Height of building = h = 500 m



From similar ΔOBP_2 and ΔOPY

$$\frac{f}{H-h}=\frac{PY}{BP_2} \quad \text{(i)}$$

Again, from ΔOCP_1 and ΔOPX

$$\frac{f}{H}=\frac{PX}{CP_1}=\frac{PX}{BP_2} \quad \text{(ii)}$$

Dividing eqn. (i) by eqn. (ii), we get

$$\frac{H}{H-h}=\frac{PY}{PX}=\frac{PX+XY}{PX}=1+\frac{XY}{PX}$$

$$\Rightarrow XY=\frac{h}{H-h}*PX$$

Relief displacement $= XY=\frac{h}{H-h}*PX=\frac{500}{5000-500}*90=10mm$

114. Given data,

Focal length of the camera = f = 20 cm = 0.2 m

Let the flying height is H m

Scale of the photograph at 500m elevation $=\frac{0.1}{2500}=\frac{f}{H-h}$

$$\Rightarrow \frac{0.1}{2500}=\frac{0.2}{H-450}$$

$$\Rightarrow H=5450m$$

Scale of photograph for the area having elevation 1000m is given by

$$\frac{f}{H-h'}=\frac{0.20}{5450-1000}=\frac{1}{22250}$$

115. Given data,

Focal length of the camera = f = 20 cm = 0.2 m

Average altitude of the ground = h = 300 m

Let the flying height is H and the length of the line is L

Therefore,

Scale of the photograph at an average elevation of 300m $=\frac{0.1}{L}=\frac{f}{H-h}$

$$\Rightarrow \frac{0.1}{L}=\frac{0.2}{H-300} \quad \text{(i)}$$

Again, we have

$$\text{Scale of map}=\frac{1}{50000}=\frac{0.03}{L}$$

$$\Rightarrow \frac{0.03}{L}=\frac{1}{50000} \quad \text{(ii)}$$

On diving eqn. (i) by eqn. (ii) we get,

$$\frac{\frac{0.1}{L}}{\frac{0.03}{L}}=\frac{\frac{0.2}{H-300}}{\frac{1}{50000}}$$

$$\Rightarrow \frac{10}{3}=\frac{0.2*50000}{H-300}$$

$$\Rightarrow H=3300m$$

116. Given data,

Focal length of the camera = f = 150 mm = 0.15 m

Average elevation of the terrain = h = 750 m

Flying height = H = 3000 m

Let the scale of the photograph is S

Therefore,

Scale of the photograph at an average elevation of 750m $= S=\frac{f}{H-h}$

$$\Rightarrow S=\frac{0.15}{3000-750}=\frac{0.15}{2250}=\frac{1}{15000}$$

■

6.0 Mining Machinery

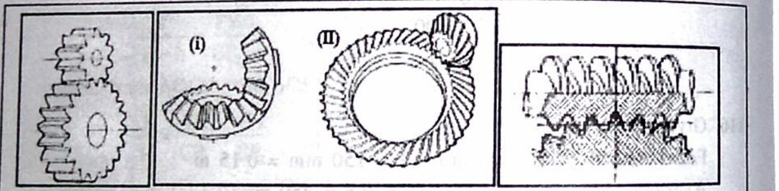
6.1 Types of Clutches

- Friction Clutch: Friction clutches can be engaged or disengaged without shock when the driving shaft is revolving at full speed, and the load on the driving engine or motor can be increased gradually from zero to full load. It may also be so arrange that the clutch slips if the load becomes excessive avoiding breakage of some important part of the plant. It is used on endless rope hauling engines.
- Claw or Dog or Jaw Clutch: This type of clutches can be engaged or disengaged at very low speed or nearly stationary. This type of clutches provides positive drive that cannot slip. Winding drum and large direct haulages require positive drive that cannot slip and hence use this type of clutches.

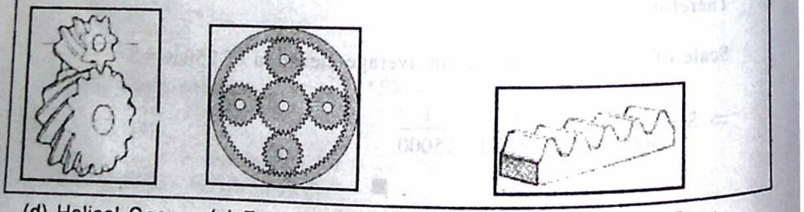
6.2 Types of Gears

Various types of gears as shown in Figure 6.1 are explained below.

- **Spur Gears :** Spur gears are the most common type used. Tooth contact is primarily rolling, with sliding occurring during engagement and disengagement.



(a) Spur Gear (b) Bevel Gear (I) Straight (II) Spiral (c) Worm Gear



(d) Helical Gear (e) Epicyclic Gear (f) Rack and Pinion Gear

Fig. 6.1 Types of Gear

- **Bevel Gears :** There are two types of Bevel gears. These are Straight Bevel Gear and Spiral Bevel Gear. These are used for transmission of power between two shafts whose axes, if extended, would meet at a point.

Straight bevel gears have tapered conical teeth which intersect the same tooth geometry. Bevel gears are used to transmit motion between shafts with intersecting center lines. The intersecting angle is normally 90^0 but may be as high as 180^0. Spiral bevel gears have a helical angle spiral teeth.

- **Worm Gears :** Operation of worm gears is analogous to a screw. The relative motion between these gears is sliding rather than rolling. This form of gearing is convenient to large speed reduction.
- **Helical Gear :** Helical gear is a cylindrical shaped gear with helicoid teeth. Helical gears operate with less noise and vibration than spur gears. At any time, the load on helical gears is distributed over several teeth, resulting in reduced wear. These are preferred to heavy drives.
- **Epicyclic Gear or Sun and Planetary Gear :** It is used for compactness and high reduction ratio between the speeds of the driver and driven shafts.
- **Rack and Pinion :** Rack and pinion gears are essentially a linear shaped variation of spur gears. The spur rack is a portion of a spur gear with an infinite radius.

6.2.1 Gear Calculation

Diametral Pitch (DP)

The Diametral Pitch (DP) describes the gear tooth size. Pitch is defined as the number of teeth per inch of pitch diameter. That is, it is the ratio of number of teeth to pitch diameter.

Pitch (P) = N/D

Pitch Diameter (D)

The Pitch Diameter refers to the diameter of the pitch circle. The diameter of the Pitch Circle from which the gear is designed is an imaginary circle, which will contact the pitch circle of another gear when in mesh.

Pitch diameter (D) = P/N

Where, D = Pitch Diameter

N = Number of teeth on the gear

P = Diametral Pitch (Gear Size)

The Pitch Circle

The pitch circle is the geometrical starting point for designing gears and gear trains. Gear trains refer to systems of two or more meshing gears. The pitch circle is an imaginary circle that contacts the pitch circle of any other gear with which it is in mesh.

Centre-to-Centre Distance

The centre to centre distance of 2 spur gears is the distance from the centre shaft of one spur gear to the centre shaft of the other. Centre to centre distance for two gears in mesh can be calculated with this formula.

$$\text{Centre} - \text{to} - \text{Centre Distance} = \frac{PDG_1 + PDG_2}{2}$$

This is also calculated as

$$\text{Centre - to - Centre Distance} = \frac{(N_1 + N_2)m}{2}$$

Where, PDG1 and PDG2 are the pitch diameter of gear 1 and gear 2 respectively.
N_1 and N_2 are the teeth number of gear 1 and gear 2 respectively.
m is the module of the gear.

Gear Ratio

Gears of the same pitch, but differing numbers of teeth can be paired to obtain a wide range of Gear Ratios. Gear Ratios are used to increase mechanical advantage (torque) or increase rotational speed or velocity. The ratio of a given pair of spur gears is calculated by dividing the number of teeth on the driven gear to the number of teeth on the drive gear.

$$\text{Gear Ratio} = \frac{\text{Number of teeth on driven gear}}{\text{Number of teeth on drive gear}}$$

Velocity

Velocity refers to the rotational speed of a gear and can be expressed using a variety of units. Velocity is expressed as the distance a point along the circumference of the pitch circle will travel over a given unit of time. Velocity can be calculated using this formula

Velocity = Pitch Circle Circumference x RPM

6.3 Wire Ropes

6.3.1 Introduction : Construction and composition of wire ropes is very important in order to maintain the safety in the mines. The wire is subjected to various tests like tensile, torsion, bending, wrapping, looping before use in the mines. The ropes used in the wet area should be galvanized i.e. coated with molten zinc.

6.3.2 Classification of Wire Ropes

6.3.2.1 On the Basis of Operation

- **Standing Ropes :** The ropes are required more or less stationary to carry the load, e.g. guide ropes in shafts, bucket supporting ropes in bi-cable ropeways
- **Running Ropes :** The ropes are have to undergo frequent movement to carry the load e.g. ropes used in cranes, coal cutting machines, excavators etc.

6.3.2.2 On the Basis of Construction

- **Stranded Ropes :** A stranded wire is built up of strands and each strand consists of number of concentrically twisted wires. The central wire is called the king core. It may be of fibre or steel. Wire ropes are designated by stating the number of strand followed by the number of wires in each strand. The flexibility of a strand depends upon the following factors:
  - **Type of Core :** Flexibility of the core is increases; flexibility of the strand also increases.

  - **Thickness of Individual Wires :** thinner the wires, more is the flexibility
  - **Number of Wires :** larger the number of wires more is the flexibility.

  Stranded ropes are again of two types according to their laying :

  - **Lang's Lay :** If the wires in the strand are laid in the same direction as the strands are laid in the rope is called Lang's lay. This type of construction causes the rope to spin. These are favoured for winding and haulage purposes.
  - **Ordinary Lay or Regular Lay :** If the wires in the strand are laid in the opposite direction as the strands are laid in the rope is called ordinary lay.
- **Non-Stranded ropes :** These types of ropes are constructed with number of wires placed in layer wise. Locked coil rope is of non-stranded type.

6.3.3 Application of Wire Ropes in Mines:

- **Winding Ropes :** Flattened stranded rope of Lang's lay type
- Winding Ropes for Shaft Sinking : Non-rotating type (stranded regular lay) or locked coil type
- **Haulage Ropes :** Stranded type of Lang's lay or regular lay but preferable Lang's lay
- **Guide Rope :** Half locked coil type
- **Coal Cutting Machines (CCM) :** Regular lay- type

6.3.4 Space Factor : The space factor of a rope is the ratio of aggregate cross sectional area of the wires to the area of circle drawn around the rope. Strength of the ropes depends on the space factor. Higher is the space factor higher is the strength.

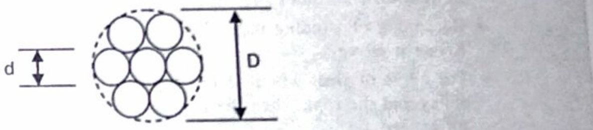


Fig. 6.2 Space Factor in Rope

For the rope shown in the above Figure 6.2,
Space Factor (SF) = $[7 * \pi * (d/2)^2] / [\pi * (D/2)^2] = 7 * d^2 / D^2$
Where, d = diameter of the individual wire
D = diameter of the rope

6.3.5 Mass of Rope : The mass of rope depends upon the construction materials and type i.e. space factor. It is given by

Mass of rope (m) = $k * D^2$ [kg/m]

Where, k = constant depending on the design of the rope and type of material
D = diameter of the rope in cm

6.3.6 Strength of Rope : Strength of rope is also depends on the type of construction and the material used. It is given by

Breaking strength of rope = $s * D^2$ [kN]

Where, s = constant depending on the design of rope and type of material
D = diameter of the rope in cm

6.3.7 Rope Splicing : Joining of two wire ropes permanently without special attachments called rope splicing.

6.3.8 Capping of Rope : It is a method to make the rope-end suitable for attachment of load. These are mainly of three types:

- Split capel with rivets
- Coned-socket type capel
- Interlocking type capel (Reliance capel)

6.4 Winding System

6.4.1 Types of Winding System in U/G Mines

6.4.1.1 Drum Winding

➢ Standards of Drum Winding System

- The diameter of the headgear pulley should be at least 100 times the rope diameter
- No spliced rope shall be in use for winding
- Rope is made of cold drawn steel wire
- Breaking load of the winding rope should be at least 10 times the maximum static load in case of a drum winding but it can be 7 times in case of Koepe winding
- Life of the winding rope should not be exceed 3.5 years
- The length of the tapered portion of the socket in rope capping should not be less than 12 times the rope diameter.
- Recapping of winding rope should be done once in 6 months except in Koepe winding
- The angle of fleet which is the angle between the vertical plane of the pulley and the rope when the cage is at the pit-top or pit-bottom, should not exceed 1.5°.

➢ Various Fittings in Drum Winding Arrangement

- **Arrangement of cage attachment**
  Rope => **Cappel** => D-Link => **Detaching Hook** => D-Link => **Bull Chain** => D-Link => **Triangular Plate** => Cage Chain => Cage
- **Detaching Hook :** This is a safety device which acts when overwind takes place and used only in drum winding not in koepe winding. Two types are King and Ormerod detaching safety hook.
- ***Safety Catches :*** This is a safety device which helps to prevent the cage falling back down the shaft during overwind and in case of failure of detaching plate.
- **Keps :** Keps are retractable support used in pit top for landing of the cage only in case of drum winding arrangement not in koepe winding system

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- **Tilted or Hinged Platform :** In koepe winding system, it is used at the pit top for landing of the cage.

➢ Torque -Time diagram at Various Stages of the Drum Winding Operation : The torque-time diagrams in drum winding are shown in Figure 6.3 and Figure 6.4.

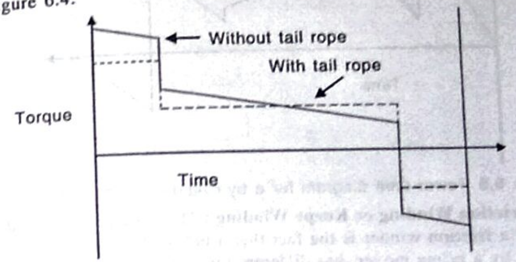


Fig. 6.3 Torque-time diagram for balanced cylindrical drum

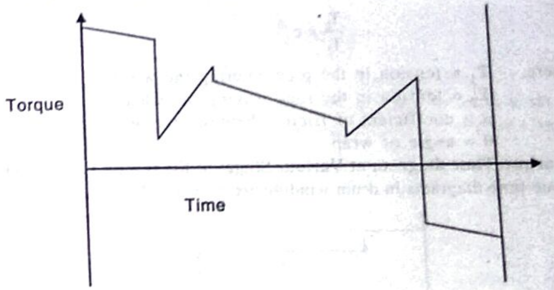


Fig. 6.4 Torque-time diagram for a by-cylindro-conical drum

➢ Power-Time diagram at Various Stages of the Drum Winding Operation : The power-time diagrams in drum winding are shown in Figure 6.5 and Figure 6.6.

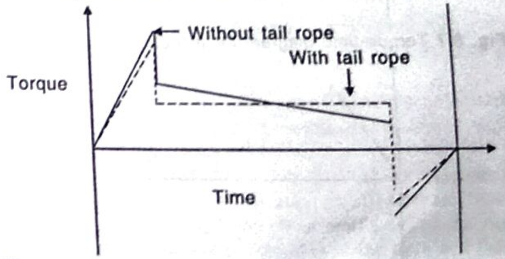


Fig. 6.5 Power-time diagram for balanced cylindrical drum

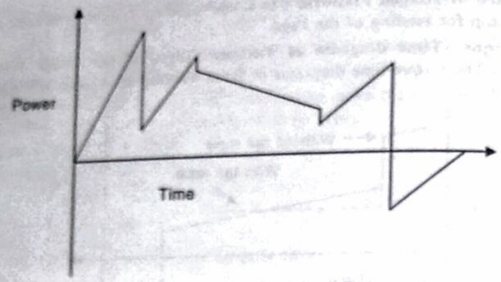


Fig. 6.6 Power-time diagram for a by-cylindro-conical drum

6.4.1.2 Friction Winding or Koepe Winding : The basic principle underlying the working of a friction winder is the fact that a belt or hoist rope passing over a wheel driven by a prime mover, has different tensions in the rope on two sides of the wheel. The relationship of these tensions is given by

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Where, T_1 = tension in the rope entering the wheel
T_2 = tension in the rope leaving the wheel
μ = coefficient of friction between the wheel and rope
θ = angle of wrap

➢ **Torque-Time diagram at Various Stages of the Koepe Winding Operation :** The torque-time diagrams in drum winding are shown in Figure 6.7 and Figure 6.8.

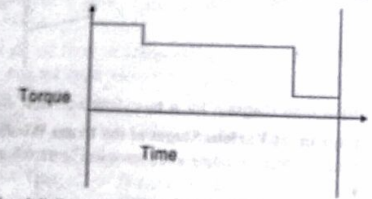


Fig. 6.7 Torque-time diagram for a friction/koepe winding

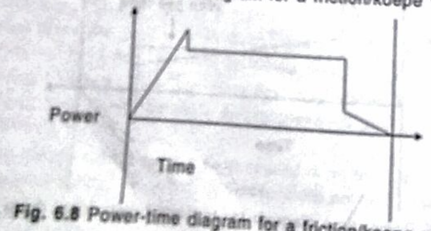


Fig. 6.8 Power-time diagram for a friction/koepe winding

6.4.2 Pit Top/Bottom Track Arrangement in Winding System

- Lofco system
- Run round arrangement
- Shunt back system
- Turn table circuit
- Traverse circuit

6.5 Underground Haulage System

6.5.1 Types of Haulage System

6.5.1.1 Direct Rope Haulage

- Minimum gradient for applicable is 12°
- The rope speed is generally 8 – 12 km/ hr.
- Slip-ring motor with drum controller is used
- Works in single track

6.5.1.2 Endless Haulage

- Suitable gradient for applicable is 1 in 6
- The rope speed is generally 3 -7 km/ hr.
- Rope moves uni-directionally
- Clifton pulley is used as a driving pulley
- A squirrel cage motor generally used
- This may have over rope or under rope arrangement
- Tubs can be attached with the rope individually or as in a set with the help of three types of clips (Screw clip, Smallman clip or Cam clip)
- It requires double track

6.5.1.3 Main and Tail Rope Haulage

- Suitable for undulating roadways
- It requires single track
- Tail rope is double the main rope

6.5.1.4 Tail Rope Haulage

- Haulage is situated at the bottom of the incline and the pull applied through a deflection pulley situated at the top of the incline

6.5.1.5 Gravity Haulage

- Minimum gradient for gravity haulage operation is 14°
- This is a haulage without any motor or external source of power
- Jig pulley is used in gravity haulage
- Loaded tubs transported from the top of the incline to the bottom with the help of gravity and simultaneously empty tubs moves from bottom to top of the incline.

6.5.2 Super-Elevation : Super elevation is tilting the roadway to help offset centripetal forces developed as the vehicle goes around a curve. That is the raising of outer rail or outer edge above the inner one, being called the super elevation. It depends on the radius of the curve, the speed of the train and the gauge of the track. as shown in Figure 6.9 and can be calculated by the following formula

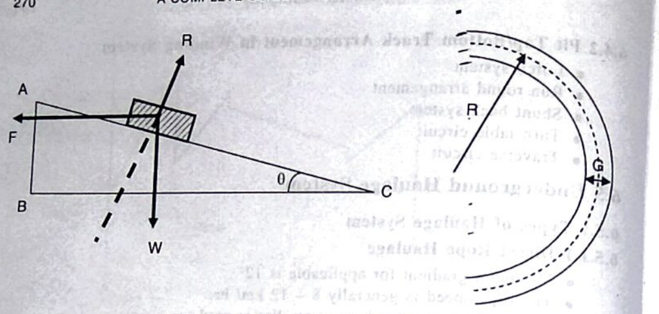


Fig. 6.9 Track gauage and radius

Assuming, G = the distance between the centres of the rails or gauge of track
V = velocity of train
R = radius of the curve in meter
g = acceleration due to gravity
W = weight of the train
h = super elevation
BC = d = the width of the road
AB = h = super elevation
F = centrifugal force
θ = inclination of the road or rail surface.

Then for equilibrium, the resultant of the weight of the train and the centrifugal force must be equal and opposite to the resultant action perpendicular to the road or rail surface.

$$\text{Thus, } F = \frac{W * V^2}{g * R}$$

$$\text{Again, } F \cos\theta = W \sin\theta$$

$$\Rightarrow \tan\theta = \frac{F}{W} = \frac{V^2}{gR}$$

Hence, the amount of super elevation (h) = BC tan θ = d tan θ

$$= \frac{V^2}{gR} \text{ [on road]}$$

$$= \frac{G.V^2}{gR} \text{ [on railways]}$$

6.5.3 Safety Devices Used in Haulage Arrangement

- **Monkey or Back Catch :** It is used in the endless haulage track to prevent the backward movement of tubs in case of rope breaks
- **Stop-block :** It is wooden block placed across the track to stop the rail in case of emergency

- **Backstay :** It is used behind the ascending set of tubs on direct haulage or endless haulage
- **Drop Warrick :** This is used to prevent the forward movement of tubs and is placed below the brow of the incline and also near the bottom.
- **A combined stop-block and runaway switch :** This is used at the brow of the direct haulage and is so constructed that that at one time only one of them is active. The minimum distance between the two is length of the set of the tubs used (L) plus 4.5 m.

6.5.4 Power Requirement of the Motor to Run a Haulage : The pull or force required to overcome the resistance to motion is known as drawbar pull. The resistance of the motion is equivalent to the frictional resistance plus or minus gradient resistance.

6.5.4.1 Direct rope haulage system : The force diagram of an ascending tub in a haulage system is shown in Figure 6.10.

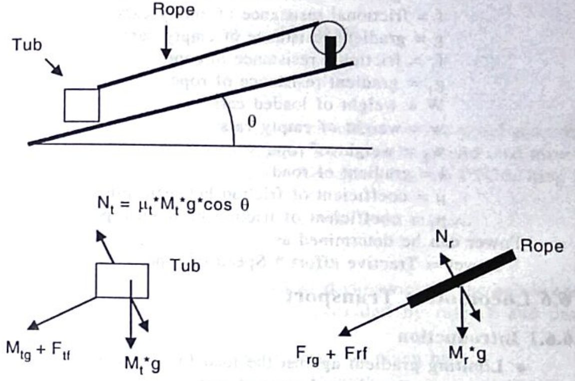


Fig. 6.10 Force diagram of the ascending tube

Where, F_{tg} = force due to gravity of the loaded tubs along the incline plane
F_{tf} = force due to friction of the loaded tubs along the incline plane
F_{rg} = force due to gravity of the rope along the incline plane
F_{rf} = force due to friction of the rope along the incline plane
θ = angle of inclination
M_t = total mass of the tubs
M_r = total mass of the rope
μ_t = coefficient of friction between the tub and incline surface
μ_r = coefficient of friction between the rope and incline surface

Total force acting along the incline plane (F) = $F_{tg} + F_{tf} + F_{rg} + F_{rf}$

$$\Rightarrow F = M_t*g*\sin\theta + \mu_t* M_t*g*\cos\theta + M_r*g*\sin\theta + \mu_r* M_r*g*\cos\theta$$

Power requirement to run the haulage (P) = F * V

$$= (M_t*g*\sin\theta + \mu_t* M_t*g*\cos\theta + M_r*g*\sin\theta + \mu_r* M_r*g*\cos\theta) * V$$

Where, V = velocity of rope

If the gradient (θ) is negative then the power requirement is

$P = (M_t*g*\sin\theta - \mu_t* M_t*g*\cos\theta + M_r*g*\sin\theta - \mu_r* M_r*g*\cos\theta) * V$

If the gradient (θ) is 0 then the power requirement is

$P = (\mu_t* M_t*g + \mu_r* M_r*g*\cos\theta) * V$

6.5.4.2 Endless Rope Haulage

Tractive effort = drawbar pull = Pull required to draw loaded mine cars + pull required to draw empty cars + pull required to draw rope

$= (F + G) + (f - g) + [(f_1 + g_1) + (f_1 - g_1)]$

$= F + f + 2f_1 + G - g$

$= \mu W + \mu w + 2\,\mu_1 w_1 + W - w_1$

Where, F = frictional resistance of loaded cars
G = gradient resistance of loaded cars
f = frictional resistance of empty cars
g = gradient resistance of empty cars
f_1 = frictional resistance of rope
g_1 = gradient resistance of rope
W = weight of loaded cars
w = weight of empty cars
w_1 = weight of rope
i = gradient of road
μ = coefficient of friction between mine car's wheel and track
μ_1 = coefficient of friction between rope and rollers

Power can be determined as

Power = Tractive effort * Speed of rope

6.6 Locomotive Transport

6.6.1 Introduction

- Limiting gradient against the load for the loco transportation is 1 in 15 but generally adopted on gradients milder than 1 in 25
- **Tractive Effort :** This is the force generated by the loco to move and accelerate the loco as well as to provide draw bar pull to move the mine cars. It depends on the weight of the loco and the coefficient of adhesion between the track and the tread of the wheel.

Tractive effort = μ W

Where, μ = coefficient of adhesion
W = weight of the Loco

- **Draw-bar pull :** This is the part of the tractive effort which remains available for pulling the connected load of mine cars after deducting the force required to move the loco itself from the total tractive force of the loco.

Draw-bar pull = Tractive Effort – Force required to move the loco

6.6.2 Types of Locomotive : There are three types of locomotives as follows

➢ **Diesel Locomotives**

- Locos used is u/g coal mines should have the power unit in flame proof enclosure
- Weight ranges from 3-15 te
- Power ranges from 15 -75 kW
- Maximum permissible CO in exhaust gases is 0.2 %
- Minimum flash point of diesel oil should be 65°C

➢ **Electric Battery Locomotive**

- Less powerful than diesel and trolley wire locomotives
- Power ranges from 4 -70 kW
- A fully charged battery gives service for nearly 8 hours

➢ **Overhead Wire Locomotive/ Trolley Wire Locomotive**

- D.C current is supplied through the overhead wire to supply power
- D.C. supply is at 250V
- Maximum efficiency comparison to the others

6.7 Conveyor Transport

6.7.1 Belt Conveyor

- Belts covered by a layer of PVC instead of rubber are fire proof and only such PVC coated belts is permissible to use in underground coal mines.
- The maximum angle of inclination for conveying coal on PVC belting is 16°
- The belt speed generally preferred 45 m/min to 60 m/min.
- Belt width varies from 0.6 to 1 m.
- Thrustor type brake is used to stop the belt
- Holdbacks are used to prevent the backward movement of the belt in case of a breakdown. This arrangement is provided by ratchet and pawl arrangement.

Carrying capacity of a troughed belt conveyor is given by

$T = a * b* V$ (te/sec)

Where, T = carrying capacity
a = average cross sectional area of material in $m^2 = \frac{w^2}{10}$ to $\frac{w^2}{12}$
b = bulk density of material in te/m^3
V = speed of conveyor belt in m/s
w = belt width

6.7.2 Cable Belt Conveyor

- In this type belt only carries the material and rest on the carrier rope situated on both side of the belt
- Suitable for long lengths and in inclination

6.7.3 Disc Conveyors

- These are designed for steep gradient

6.7.4 High Angle Conveyor

- These are designed for steep gradient up to 60°

6.8 Mine Pumps

6.8.1 Introduction

Before a pump can be selected or a prototype designed, the application must be clearly defined. The common requirement of all applications is to move liquid from one point to another. As pump requirements must match system characteristics, analysis of the overall system is necessary to establish pump conditions. The followings will ultimately determine pump selection.

- Capacity range of liquid to be moved
- Differential head required
- Net positive suction head available (NPSHA)
- Shape of head-capacity curve
- Pump speed
- Liquid characteristics
- Construction

The following parameters or terms are frequently used in pump :

❖ **Capacity :** Capacity means the flow rate with which liquid is moved or pushed by the pump to the desired point in the process.

The capacity depends on a number of factors like:

- Process liquid characteristics i.e. density, viscosity
- Size of the pump and its inlet and outlet sections
- Impeller size
- Impeller rotational speed RPM
- Size and shape of cavities between the vanes
- Pump suction and discharge temperature and pressure conditions

As liquids are essentially incompressible, the capacity is directly related with the velocity of flow in the suction pipe. This relationship is as follows :

$Q = A * V$

Where, Q = capacity in m^3/s

A = area of cross section in m^2

V = velocity in m/s

❖ **Head :** The head term is used to measure the kinetic energy created by the pump. In other words, head is a measurement of the height of a liquid column that the pump could create from the kinetic energy imparted to the liquid. Imagine a pipe shooting a jet of water straight up into the air, the height the water goes up would be the head. The main reason for using head instead of pressure to measure a centrifugal pump's energy is that the pressure from a pump will change if the specific gravity (weight) of the liquid changes, but the head will not change.

6.8.2 Pressure to Head Conversion Formula

The static head corresponding to any specific pressure is dependent upon the weight of the liquid according to the following formula :

$$\text{Head} = \frac{\text{Pressure (PSi)} * 231}{\text{Specific Gravity}}$$

6.8.3 Types of Head

- ***Static Suction Head*** **(h_S) :** Head resulting from elevation of the liquid relative to the pump centre line. If the liquid level is above pump centreline, hS is positive. If the liquid level is below pump centreline, hS is negative. Negative hS condition is commonly denoted as a "suction lift" condition.
- ***Static Discharge Head*** **(h_d) :** It is the vertical distance in feet between the pump centreline and the point of free discharge or the surface of the liquid in the discharge tank.
- ***Friction Head*** **(h_f) :** The head required to overcome the resistance to flow in the pipe and fittings. It is dependent upon the size, condition and type of pipe, number and type of pipe fittings, flow rate, and nature of the liquid.
- ***Vapour Pressure Head*** **(h_{vp}) :** Vapour pressure is the pressure at which a liquid and its vapour co-exist in equilibrium at a given temperature. When the vapour pressure is converted to head, it is referred to as vapour pressure head, hvp. The value of hvp of a liquid increases with the rising temperature and in effect, opposes the pressure on the liquid surface, the positive force that tends to cause liquid flow into the pump suction i.e. it reduces the suction pressure head.
- ***Pressure Head*** **(h_p) :** Pressure Head must be considered when a pumping system either begins or terminates in a tank which is under some pressure other than atmospheric. The pressure in such a tank must first be converted to feet of liquid. Denoted as hp, pressure head refers to absolute pressure on the surface of the liquid reservoir supplying the pump suction, converted to feet of head. If the system is open, hp equals atmospheric pressure head.
- ***Velocity Head*** **(h_v) :** Refers to the energy of a liquid as a result of its motion at some velocity 'v'. It is the equivalent head in feet through which the water would have to fall to acquire the same velocity, or in other words, the head necessary to accelerate the water. The velocity head is usually insignificant and can be ignored in most high head systems. However, it can be a large factor and must be considered in low head systems.
- ***Total Suction Head*** **(H_S) :** The suction reservoir pressure head **(hpS)** plus the static suction head (hS) plus the velocity head at the pump suction flange (hVS) minus the friction head in the suction line (hfS).

  $$H_S = h_{pS} + h_S + h_{vS} - h_{fS}$$

  The total suction head is the reading of the gauge on the suction flange, converted to feet of liquid.
- ***Total Discharge Head*** **(H_d) :** The discharge reservoir pressure head (hpd) plus static discharge head (h_d) plus the velocity head at the pump discharge flange (h_{vd}) plus the total friction head in the discharge line (hfd).

  $$H_d = hp_d + h_d + hv_d + hf_d$$

The total discharge head is the reading of a gauge at the discharge flange, converted to feet of liquid.

- ***Total Differential Head*** **(H_T) :** It is the total discharge head minus the total suction head or

$$H_T = H_d + H_S \text{ (with a suction lift)}$$
$$H_T = H_d - H_S \text{ (with a suction head)}$$

6.8.4 Power and Efficiency : The work performed by a pump is a function of the total head and the weight of the liquid pumped in a given time period.

Pump input or brake horsepower (BHP) is the actual horsepower delivered to the pump shaft.

This can be determined by the following formulas.

$$\text{B.H.P} = \frac{Q * H_T * \text{Sp.Gr.}}{3960 * \text{Efficiency}}$$

Where, Q = capacity in gallons per minute (gpm)
H_r = total differential head, ft.
Sp. Gr. = Specific gravity of the liquid
Efficiency = Pump efficiency in %

Pump output or hydraulic or water horsepower (WHP) is the liquid horsepower delivered by the pump. This can be determined by the following formulas.

$$\text{W.H.P.} = \frac{Q * H_r * \text{Sp.Gr}}{3960}$$

Where, Q = capacity in gallons per minute (gpm)
H_r = total differential head, ft.
Sp. Gr. = Specific gravity of the liquid
Efficiency = Pump efficiency in %

The constant 3960 is obtained by dividing the number of foot-pounds for one horsepower (33,000) by the weight of one gallon of water (8.33 pounds).

6.8.5 Specific Speed : Specific speed (Ns) is a non-dimensional design index that identifies the geometric similarity of pumps. It is used to classify pump impellers as to their type and proportions. Pumps of the same Ns but of different size are considered to be geometrically similar, one pump being a size-factor of the other.

$$Ns = \frac{N * Q^{0.5}}{H^{0.75}}$$

The following formula is used to determine specific speed :

Where, Q = capacity at best efficiency point at maximum impeller diameter, gpm.
H = head per stage at BEP at maximum impeller diameter, ft.
N = pump speed, rpm.

6.8.6 The Affinity Laws : The Affinity Laws are mathematical expressions that define changes in pump capacity, head, and BHP when a change is made to pump speed, impeller diameter, or both.

According to *Affinity Laws* :

- **Capacity, Q** changes in direct proportion to impeller diameter **D** ratio, or to speed **N** ratio :

$$\frac{Q_2}{Q_1} = \frac{D_2}{D_1}$$
$$\frac{Q_2}{Q_1} = \frac{N_2}{N_1}$$

- **Head, H** changes in direct proportion to the square of impeller diameter **D** ratio, or the square of speed N ratio :

$$\frac{H_2}{H_1} = \frac{D_2^2}{D_1^2}$$
$$\frac{H_2}{H_1} = \frac{N_2^2}{N_1^2}$$

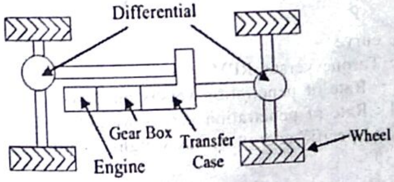
- **BHP** changes in direct proportion to the cube of impeller diameter ratio, or the cube of speed ratio :

$$\frac{BHP_2}{BHP_1} = \frac{D_2^3}{D_1^3}$$
$$\frac{BHP_2}{BHP_1} = \frac{N_2^3}{N_1^3}$$

Where the subscript : 1 refers to initial condition, 2 refer to new condition

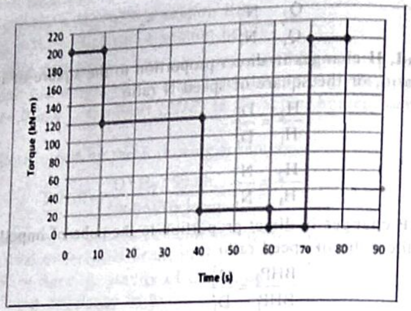
EXERCISE - 6

1. A spur gear system has two gears with 24 and 33 teeth of module 4. The centre to centre distance in mm between the driving and the driven shafts of the spur gear is
   a. 28.5 b. 57.0
   c. 114.0 d. 228.0
2. Two meshing spur gear wheels of Module 6 have 24 and 42 teeth. The distance in mm between the centres of the gear wheels is
   a. 1000 b. 198
   c. 126 d. 72 [GATE 2013]
3. The gear ratios of the first gear, transfer case and differential of a four wheel drive vehicle are 3.81:1, 2.72:1 and 4.11:1 respectively. If the engine is rotating at 1000 rpm and the wheel diameter is 1.2 m, the speed of the vehicle in first gear in km/h is



a. 5.31 b. 3.68
c. 2.42 d. 1.68 [GATE 2012]

4. The 3-period torque-time diagram of a statically balanced hoist is shown in the figure.

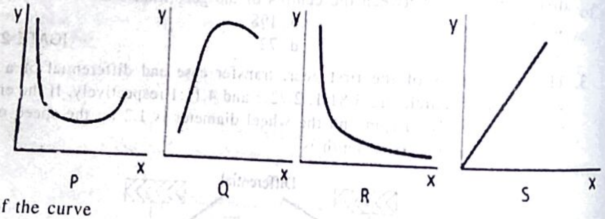


The rms torque of the motor is
a. 122.5 b. 124.5
c. 128.5 d. 130.5 [GATE 2015]

5. When the cage is at pit bottm, the minimum turns of the rope on the drum is
a. 2 b. 3
c. 4 d. 5

6. Given S is the setting load and Y is the yield load of a hydraulic prop, the correct relationship is
a. S < Y b. S > Y
c. S = Y d. S = Y^2 [GATE 2014]

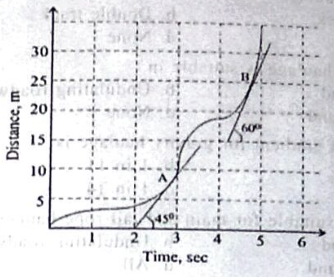
7. The following characteristic curves (P, Q, R, S) pertain to rotary drilling in rock.



Title of the curve
I : Torque versus RPM
II : Rate of penetration versus uniaxial compressive strength of rock
III : Rate of penetration versus weight on bit
IV : Specific energy versus weight on bit

Match the curves with their titles
a. P-III, Q-IV, R-II, S-I b. P-II, Q-IV, R-I, S-III
c. P-IV, Q-III, R-II, S-I d. P-I, Q-III, R-II, S-IV [GATE 2014]

8. The figure shows the distance vs time graph of a moving particle. The tangents to the curve at A and B make angles of 45° and 60° respectively with the time axis.



The ratio of the speeds of the particle at B and at A is
a. 0.72 b. 1.38
c. 1.58 d. 1.75 [GATE 2012]

9. The correct sequence of increasing the ductility of the material is
a. Au- Ag- Fe- Ni-Al-Cu b. Au- Fe-Ag- Al-Ni- Cu
c. Al- Cu- Ni- Fe- Ag- Au d. Au- Ni- Ag-Cu - Fe- Al

10. The tool used for borehole deviation is
a. String shot b. Kelly
c. Whipstock d. Ratchet [GATE 2008]

11. Rope used for winding purposes is
a. Lang's lay type b. Ordinary lay type
c. Full locked coil rope d. Half locked coil rope

12. The lowest ratio of metallic cross-sectional area to rope cross sectional area occurs in
a. Round strand rope b. Flattened strand rope
c. Half locked coil rope d. Full locked coil rope

13. Rope used for haulage transport in underground mine purposes is
a. Lang's lay type b. Ordinary lay type
c. Full locked coil rope d. Half locked coil rope

14. A wire rope, round stranded with fibre core, has a diameter of 2.54 cm. What is the mass of the rope per m length if the value of the constant k is 0.36?
a. 2.33 b. 2.36
c. 2.87 d. 3.21

15. A 6×19 wire rope would be
  a. 6 inches in diameter and 19 fathoms long
  b. 6 inches in circumference with 19 strands
  c. 6 strands with 19 wires in each strand
  d. 19 strands with 6 wires in each strand

16. Direct rope haulage works in
  a. Single track b. Double track
  c. Both d. None

17. Endless rope haulage is suitable in
  a. Flat ground b. Undulating roadway
  c. Steep ground d. None

18. The minimum gradient for gravity haulage is
  a. 1 in 10 b. 1 in 11
  c. 1 in 12 d. 1 in 14

19. The gradient suitable for main and tail rope haulage is
  a. Flat ground b. Undulating roadway
  c. Steep ground d. All

20. The type of rope haulage suitable for undulating roadways is
  a. direct rope haulage b. endless rope haulage
  c. gravity haulage d. main and tail rope haulage

21. Clifton pulley is used in
  a. direct rope haulage b. endless rope haulage
  c. gravity is haulage d. main and tail rope haulage

22. Jig pulley is used in
  a. direct rope haulage b. endless rope haulage
  c. gravity haulage d. main and tail rope haulage

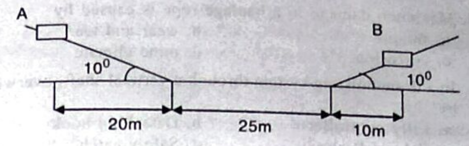
23. Slip ring motor is used in
  a. direct rope haulage b. endless rope haulage
  c. gravity haulage d. main and tail rope haulage

24. Squirrel cage motor is used in
  a. direct rope haulage b. endless rope haulage
  c. gravity haulage d. main and tail rope haulage

25. The type of rope suitable for winding during shaft sinking operation is
  a. half locked coil b. full locked coil
  c. lang's lay d. ordinary lay

26. In direct rope haulage, the total breaking force of all wires of the rope is 342500 N and the maximum resistive force to motion from all causes is 28406 N. Then the factor of safety against rope breakage will be
  a. 12.06 b. 1.21
  c. 0.08 d. 13.06

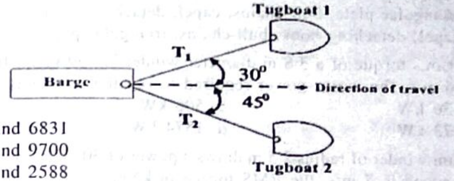
27. A mine car is released from rest from the point A. What is the velocity in m/s when the car is at the point B? Neglect all friction losses.



  a. 1.76
  b. 5.88
  c. 1.88
  d. 10.88

28. A 1 tonne mine car travelling at a constant speed of 10 km/h collides with a stationary buffer and comes to rest. If the buffer spring stiffness is 200 kN/m, the maximum compression in the spring in mm is
  a. 49 b. 98
  c. 196 d. 247 [GATE 2011]

29. In an iron ore handling port, a barge is pulled by ropes using two tugboats as shown in the figure. In equilibrium, the resultant of the forces T1 and T2 along the axis of the barge in the direction of its travel is 5000 N. The tensions T1 and T2 in N respectively are



  a. 9700 and 6831
  b. 6831 and 9700
  c. 3660 and 2588
  d. 2588 and 3660 [GATE 2011]

30. Neglecting the frictional forces, the force F required to prevent the mine car moving down (as shown in the figure) is
  a. $mg\left(\frac{1}{1+x^2}\right)$ b. $mg\left(\frac{1}{\sqrt{1+x^2}}\right)$
  c. $mg\left(\frac{x}{\sqrt{1+x^2}}\right)$ d. $mg\left(\frac{x^2}{1+x^2}\right)$

31. For a direct rope haulage installation in a mine drift if the total rope tension is 115 kN, the moment of inertia of the 3 m diameter drum is 9000 kg/m², and the speed of the rope is 12 m/s, the total torque of the drum in kNm is
  a. 172.5 b. 175.2
  c. 176.1 d. 179.1

32. Recapping of winding rope is done to
  a. Increase the flexural strength of the rope
  b. Increase the flexibility of the rope
  c. Remove a portion of the rope subjected to deterioration
  d. Prevent the rope from excessive rusting [GATE 2010]

33. Maximum damage to a haulage rope is caused by
    a. fatigue b. wear and tear
    c. corrosion d. mine climate

34. In a drum hoisting system through a vertical shaft, overwinding is prevented by
    a. Lilly controller b. Detaching hook
    c. Caliper Brake d. Safety catch

35. Steel wire rope of 25mm diameter weighing 37N/m has 6 strands of 7 wires each. The diameter and tensile strength of each wire are 2.5mm and 1800 MPa, respectively. The factor of safety for raising a cage of weight 60 kN from a depth of 200m is
    a. 5.60 b. 4.50
    c. 4.25 d. 4.15 [GATE 2010]

36. The sequence of attachments between the winding rope and the cage is
    a. bull chains, triangular plate, capel, detaching hook
    b. detaching hook, bull chains, triangular plate, capel
    c. triangular plate, bull chains, capel, detaching hook
    d. capel, detaching hook, bull chains, triangular plate [GATE 2000]

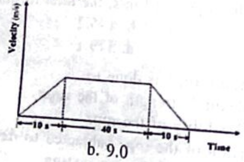
37. The r.m.s torque of a 3.8 m diameter winder is 109 kNm. If the rope speed is 7.6 m/s, the motor power required to run the winder is
    a. 436 kW b. 506 kW
    c. 872 kW d. 1574 kW

38. A drum winder of radius 2.5 m draws a power of 308 kW. When the maximum rope speed is 7 m/s, the RMS torque in kNm is
    a. 55 b. 76
    c. 110 d. 144 [GATE 2008]

39. According to mine regulations, the value of the fleet angle a, in degree of a drum winder installation lies in the range of
    a. 1.5 < a = 2.0 b. 0 < a =1.5
    c. 2.0 < a = 2.5 d. 2.5 < a = 3.0 [GATE 2011]

40. During over-winding, a cage is safely suspended in the headgear due to
    a. Bull chain b. Rope capel
    c. D-Link d. Detaching hook [GATE 2008]

41. A mine winder cage travelling 450 m from pit bottom to pit top is following a three period duty cycle as shown in the figure below. The maximum velocity attained by the cage in m/s is



    a. 7.5 b. 9.0
    c. 11.0 d. 12.0 [GATE 2011]

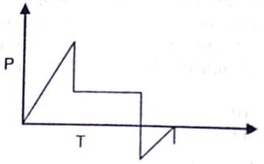
42. A cage weighing 12000 kg is raised by four chains each making an angle of 30° with the vertical. The tension in each chain in kN is
    a. 41 b. 34
    c. 25 d. 20 [GATE 2011]

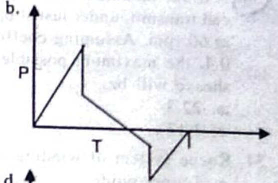
43. A cage of floor area 5 m^2 suspended in a shaft has a drag coefficient 2.5. If the velocity of air in the shaft is 6 m/s, the drag force (N) experienced by the cage is
    a. 120 b. 170
    c. 200 d. 270 [GATE 2008]

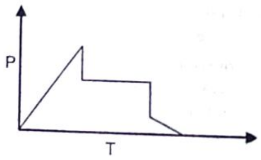
44. A balanced winder raises 3000 tonnes per day from a depth of 500 m. The payload of the winding cage is 7 tonnes. The energy consumed per day in kWh at 70% winder efficiency is
    a. 6030 b. 5840
    c. 5750 d. 5630 [GATE 2010]

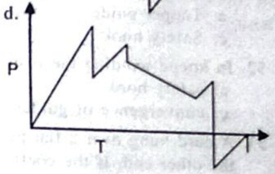
45. The centre lines of winding drum and winding pulley are parallel and 0.6 m apart. When one cage is at pit bottom, its winding rope is positioned at 1.8 m from the drum centre. The plan distance between the winding drum and the winding pulley axes is 60 m. the fleet angle of installation is
    a. 1.15° b. 1.35°
    c. 1.42° d. 1.82°

46. Which one of the followings is the power-time diagram for a koepe winder?

a.


b.


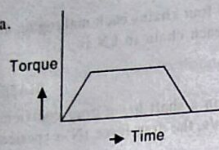
c.


d.


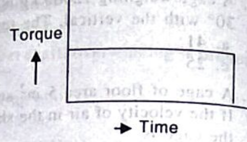
47. For tower mounted friction hoist having a coefficient of friction (μ) between the rope and the sheave of 0.30, the wrap factor $C^{\mu\theta}$ is
    a. 1.60 b. 2.57
    c. 4.11 d. None of the above

48. For a standard 3-period torque – time diagram of a mine hoist with constant acceleration and deceleration is

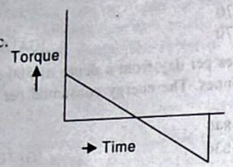
a.



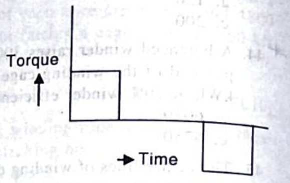
b.



c.



d.



49. The above figure shows the duty cycle of hoist system. The system is
   a. cylindrical drum with tail rope
   b. by-cylindro conical drum
   c. ground mounted friction sheave
   d. cylindrical drum without tail rope

* (50) A tower mounted friction hoist has a driving sheave of 4 m diameter, which can transmit, under just slipping condition, a power of 200 kW while rotating at 60 rpm. Assuming coefficient of friction between rope and rope tread is 0.4, the maximum possible rope tension in kN on the heavier side of the sheave will be

a. 22.3	b. 10.5
c. 1.17	d. 44.5

51. Koepe system of winding does not include

a. Tapper guide	b. Limit switches
c. Safety hook	d. Brake

[GATE 2009]

52. In koepe winding the over wind is prevented by

a. safety hook	b. breakage of rope
c. convergence of guides	d. thickening of guides

53. A cord hung over a flat pulley supports a weight W_1 at one end and W_2 at the other end. If the coefficient of friction is 0.3, find the greatest ratio of W_1 to W_2 possible without the cord slipping. Angle of contact is 180°.

a. 2.56	b. 2.85
c. 2.36	d. 3.25

54. If the r.m.s torque for a winder is 109 kNm, the cylindrical winding drum diameter is 4.4 m and the maximum rope speed is 7 m/s. What will be the minimum power requirement of the motor?

a. 347 kW	b. 375 kW
c. 365 kW	d. 247 kW

* (55) A round stranded fibre core winding rope of diameter 2.54 cm has a space factor of 0.5. The tensile strength of the rope steel is 1500 MPa. At a safety factor of 10, what is the maximum load the rope can take?

a. 38 kN	b. 76 kN
c. 3.8 MPa	d. 7.6 kg/ mm^2

* 56. A direct rope haulage pulls 8 tubs loaded with coal through an incline of length 500 m having an inclination of 1 in 6. Consider the following additional data.

Capacity of tub = 1.0 tonne
Tare weight of tub = 500 kg
Hauling speed = 9 km per hour
Coefficient of friction between wheel and rail = 1/60
Coefficient of friction between rope and drum = 1/10
Mass of rope per meter = 1.5 kg

The minimum power required to haul the tubs in kW is

a. 345.50	b. 348.60
c. 350.10	d. 365.50
e. 58.86	

[GATE 2014]

57. The maximum permissible gradient for locomotive is

a. 1 in 12	b. 1 in 14
c. 1 in 16	d. 1 in 15

58. Which one of the following is not a pit top or pit bottom layout?

a. Lofco system	b. Shunt circuit
c. Turntable circuit	d. Merry goround

59. An 8t locomotive has a coefficient of adhesion of 0.18 and a coefficient of rolling resistance of 100 N/te mass. The maximum tractive force available to haul the trailing load in kN along a straight track is

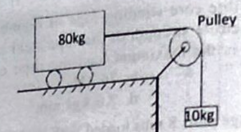
a. 0.64	b. 13.33
c. 14.13	d. 14.93

60. A locomotive haulage train transports 50 te of coal in its out by journey. In the return journey, it takes back 50 te of material in by. The ideal gradient for this locomotive haulage system is
   a. 1 in 50 in favour of coal movement
   b. 1 in 50 in favour of material movement
   c. zero
   d. none of the above

61. A 10,000 kg locomotive runs at a speed of 3 m/s along a level track. The coefficient of adhesion at the track is 0.23, and the rolling resistance of the locomotive is 90 N per 1000 kg. The maximum power available for hauling the train in kW is

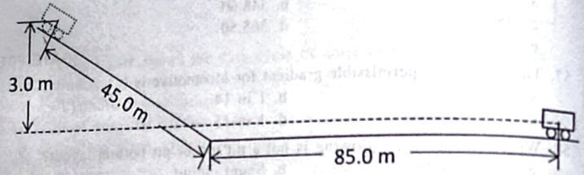
a. 40.7	b. 65.0
c. 67.1	d. 67.7

62. In the following figure, the coefficient of kinetic friction between the trolley and the surface is 0.04. When the block is released from rest, the acceleration of the trolley in m/s^2 becomes



a. 9.65
b. 1.23
c. 1.09
d. 0.74
[GATE 2013]

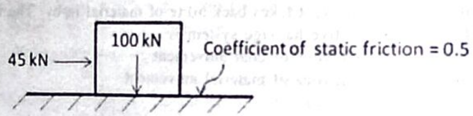
63. A two tonne mine car is released from the top of an incline at a height of 3 m as shown in the figure. The mine car travels 45 m along the inclined track and another 85 m along the horizontal track before coming to rest.



The specific rolling resistance of the car in N/tonne is

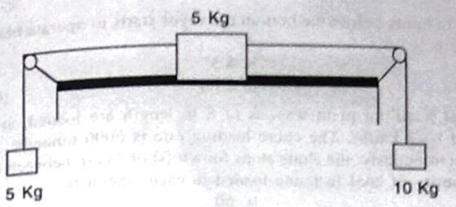
a. 0.226
b. 0.326
c. 0.526
d. 0.126
[GATE 2013]

64. A block of weight 100 kN rests on a floor as shown in the figure. The coefficient of static friction between the block and the floor is 0.5. A force of 45 kN is applied horizontally on the block. The static frictional force in kN is



a. 22.5
b. 50.0
c. 55.0
d. 100.0
[GATE 2014]

65. Two blocks of mass 5 kg and 10 kg are connected with cords and frictionless pulleys as shown. Friction coefficient between the 5 kg block and table is 0.2.



The acceleration of the system when the blocks are released from rest ('g' acceleration due to gravity), tension in the cord connected to 10 kg block and 5 kg block are

a. g/5, 8g, 6g
b. g/6, 6g, 8g
c. g, 4g, 6g
d. g/2, 2g, 4g
[GATE 2008]

66. Maximum permissible gradient for PVC belt conveyor transportation is

a. 1 in 14
b. 1 in 15
c. 1 in 16
d. 1 in 17

67. Maximum permissible gradient for high angle conveyor transportation is

a. 50°
b. 60°
c. 70°
d. 65°

68. An AFC of length 114 m runs at a speed of 0.535 m/s. The mass of chains and flights is 54 kg/m, and the coefficient of friction between the chain and trough 0.33. What is the power needed to drive the empty conveyor?

a. 6.94 kW
b. 10.68 kW
c. 21.36 kW
d. 25.74 kW

69. A conveyor transported the coal at the rate of 220 te/hr is 600 m long moving up a gradient of 1 in 60. If the belt width (W) is 0.75 m, and cross-sectional area of the material is 0.1 W^2, determine the belt speed. The bulk density of coal 0.8 te/m^3.

a. 0.04 m/s
b. 0.96 m/s
c. 1.36 m/s
d. 1.80 m/s

70. A belt conveyor conveys material of average cross-sectional area of 0.09 m2, of bulk density 1.5 tonnes/m^3, at a speed 2 m/s. The carrying capacity of the belt in tonne/hr is

a. 972
b. 864
c. 732
d. 643
[GATE 2008]

71. A conveyor of rated power 100 kW hauls coal up-dip at 30 kg/s along an inclination of 15° and distance 300 m. Heat added by the conveyor to the air in kW is

a. 56.4
b. 65.9
c. 77.2
d. 82.3
[GATE 2008]

72. Two belt conveyors load a ground bunker, each at a rate of 400 tph, which is initially filled with 10000 t of coal. Coal is discharged from the bottom of the ground bunker onto a belt conveyor at a rate of 1200 tph. The time

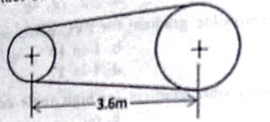
elapsed in hours before the bottom conveyor starts to operate below its rated capacity is

a. 6.5 b. 8.5
c. 12.5 d. 25.0 [GATE 2009]

73. In a coal handling plant wagons of 8 m length are loaded, at rake travel speed of 0.48 km/hr. The chute loading rate is 6000 tonne/hr. As the rake moves continuously, the chute stops for a total of 24s in between two wagons. The quantity of coal in tonne loaded in each wagon is

a. 52 b. 60
c. 76 d. 94 [GATE 2009]

74. An open belt drive connects two pulleys on parallel shafts that are 3.6 m apart as shown in the figure. The diameters of the pulleys are 2.4 m and 1.6 m. The angle of contact on the smaller pulley in degrees is



a. 167.38 b. 157.48
c. 166.48 d. 154.38 [GATE 2013]

75. A cable belt conveyor is required to carry coal of bulk density 0.8 t/m^3 at a rate 600 t/hr up an inclined length of 1200 m. The total vertical lift is 300 m. The power required to raise the material against gravity in kW will be

a. 490.5 b. 392.4
c. 120.0 d. 1765.8

76. A locomotive is negotiating a level curved path with radius of curvature 30m. The gauge of the rails is 0.75 m. If the maximum elevation of the track is 1/8 of the track gauge, the maximum turning speed in m/s of the locomotive is

a. 5.2 b. 5.8
c. 6.1 d. 6.5

77. A 12 tonne diesel locomotive of 60 kW is plying in an underground haulage roadway. The coefficient of adhesion is 0.25 and the maximum gear efficiency is 80 %. The speed in m/s at which it will haul a train at its full power is

a. 2.548 b. 2.448
c. 2.038 d. 1.630

78. A 24 t locomotive pulls 300 t of trailing load from a level siding. If antifriction bearings having specific rolling resistance of 100 N/t is fitted on all equipment and the coefficient of adhesion is 0.3, the maximum possible acceleration of the train in m/s^2 will be

a. 0.12 b. 1.30
c. 0.22 d. 1.20

79. A loco of mass 10000 kg has a coefficient of adhesion to the tracks as 0.25. The loco offers a running resistance equal to 10 % of its weight. The draw-bar-pull generated by the loco on a level ground in kN is

a. 11.3 b. 14.7
c. 15.8 d. 17.2 [GATE 2008]

80. In the above questions, the draw-bar-pull generated by the loco when the upward gradient of the track is 5° in kN is

a. 6.16 b. 7.9
c. 9.5 d. 11.5 [GATE 2008]

81. In a belt conveyor, the cross-sectional area of material in m^2 can be expresses as $B^2/11$, where B is the width of the belt in m. The belt transport coal of bulk density 1200 kg/m^3 at a rate of 188 kg/s. The speed of the belt is 1.5 m/s and the fill factor is 0.8. The required width of the belt in m is

a. 1.07 b. 1.12
c. 1.20 d. 1.44

82. For a 150 m long chain conveyor, the component of power required to move coal against gravity along an inclination 15° is 5 kW. Hourly capacity of the chain conveyor will be

a. 47.2 te b. 37.8 te
c. 13.1 te d. 3.4 te

83. A flat steel plate conveyor of width 0.72 m runs at a speed of 0.9 m/s. If the dynamic angle of repose of the material conveyed is 20°, the conveyor theoretical capacity in m^3/ hr is

a. 152 b. 212.3
c. 394 d. 1217

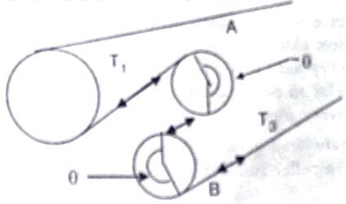
84. For AFC open-bottom pans are preferable to closed-bottom pans because

a. these don't sink into soft floor being of lighter weight
b. the return chain cannot get jammed due to accumulation of coal particles
c. these can be easily inspected and maintained
d. these make less noise

85. In a conveyor belt drive the tension on the tight side is double that on the slack side. If the value of μ is 0.3, find the angle of lap required if the belt is not to slip on its driving drum.

a. 132° b. 135°
c. 136° d. 150°

86. A conveyor belt has two drive drums A and B as shown in the figure. If each angle of wrap 'θ', in both the drive drums is 230° and coefficient of friction between the belt and the drum is 0.35, the ratio of the belt tensions T_1 and T_3 is



a. 4.1 b. 12.6
c. 8.2 d. 16.6

87. A flat belt conveyor is carrying coal of bulk density 1 tonne/m^3 at a rate of 400 tonne/h. The belt speed is 3 m/s. Coal is spread over the belt covering 80% of the belt width in a shape of a triangle. If the pile height is 1/4 of the belt width, the width of the belt in mm is
a. 1109 b. 909
c. 709 d. 609 [GATE 2011]

88. In a cable belt conveyor, the function of the cable is to
a. increase the lateral stiffness of the belt
b. increase the tensile strength of the belt
c. support and provide motion to the belt
d. minimize elongation of the belt under tension

89. Function of snub pulley in the belt conveyor system is
a. cleaning device b. safety device
c. braking device d. increasing the angle of wrap

90. Function of rotary brush in the belt conveyor system is
a. cleaning device b. safety device
c. braking device d. increasing the angle of wrap

91. Function of pull cord in the belt conveyor system is
a. cleaning device b. safety stopping device
c. material discharging on the side of the belt
d. increasing the angle of wrap

92. In a belt conveyor system, function of the snub pulley is to
a. clean the inner surface of the belt
b. clean the outer surface of the belt
c. increase the angle of contact of belt with drive drum
d. decrease the belt tension [GATE 2013]

93. Match the following

Mine car type	Mode of unloading
P. Granby	1. Bottom opening
Q. Gable bottom	2. Both side tilting
R. Drop bottom	3. Single side opening
S. Rocker dump	4. Both side opening

a. P-2, Q-4, R-3, S-1 b. P-4, Q-1, R-3, S-2
c. P-3, Q-1, R-4, S-2 d. P-3, Q-4, R-1, S-2 [GATE 2011]

94. Axial thrust in a turbine pump acts
a. from suction side to delivery side
b. from delivery side to suction side
c. perpendicular to pump shaft
d. in the delivery pipe

95. The approximate head generated by a single-stage centrifugal pump of 250 mm diameter impeller running at 1440 rev/min with manometric efficiency of 0.7 is

a. 25 m b. 36 m
c. 72 m d. 144 m [GATE 1998]

96. If a pump develops 10 bars of pressure and has a flow rate of 1500 l/min, what will be the pump motor power at 90 % efficiency?
a. 25 kW b. 28 kW
c. 30 kW d. 45 kW

97. Under standard temperature and pressure conditions the theoretical maximum height in m to which water can be lifted using an air-lift pump is
a. 10.33 b. 9.61
c. 7.45 d. 6.05 [GATE 2013]

98. Water will not be delivered by a centrifugal pump due to
a. lack of priming b. too low discharge head
c. wrong direction of rotation d. partial obstruction at discharge outlet [GATE 2011]

99. A turbine pump of efficiency 70% discharges water at the rate of 2100 L/min at a total head of 100 m. If the pump is run by a motor of efficiency 90 %, the input power required for the motor in kW is
a. 22.49 b. 34.31
c. 44.11 d. 54.50 [GATE 2009]

100. In the above question, If the velocity of water in suction and delivery pipes of the pump are 1.8 m/s and 2.5 m/s respectively, the diameter of suction and delivery pipes in cm are
a. 15.73 and 13.35 b. 7.86 and 6.67
c. 5.78 and 6.02 d. 4.97 and 4.22 [GATE 2009]

101. A centrifugal pump has a discharge rate of 2000 L of water per min against a total head of 200 m. If the pump efficiency is 75%, the input power to the pump in kW is
a. 87.20 b. 49.05
c. 13.33 d. 7.50 [GATE 2014]

102. Match the following

Hydraulic system components		Symbols
P Fixed displacement unidirectional flow pump	1	
Q Fixed displacement unidirectional flow motor	2	
R Accumulator	3	
S Filter	4	

a. P-4, Q-2, R-3, S-1 b. P-2, Q-4, R-3, S-1
c. P-3, Q-2, R-1, S-4 d. P-2, Q-3, R-1, S-4 [GATE 2011]

103. The inflation pressure in truck tyre as recommended by the tyre manufacturer is checked

a. when the truck has worked for 4 hours
b. when it fully loaded by the shovel
c. when it returns to the garage at the end of the shift
d. when it leaves the garage at the start of the shift

104. Lemniscate bar is the component of
a. Road header b. Shield support
c. DERDS d. AFC

105. Cowl is the component of
a. Road header b. Shield support
c. DERDS d. AFC

106. Flight bar is the component of
a. Road header b. Shield support
c. DERDS d. AFC

107. A twin cable ropeway consists of
a. two track ropes and one traction rope
b. one track rope and two traction ropes
c. two track ropes and two traction rope
d. three track ropes and one traction rope

108. The resistance to motion of a dumper on a horizontal track is $(35 + 0.1\ v^2)$ N per tonne, where v is the velocity in m/s. Its tractive effort is 325 N/te. The acceleration in m/s^2 at a velocity of 10 m/s is
a. 0.14 b. 7.14
c. 0.28 d. 28.00

109. The production rate of a bucket wheel excavator is 400 m^3/hr. in a material of digging resistance 25 N/ mm. Assuming digging resistance inversely varies with square root of production capacity of the excavator, the production rate in m^3/hr in another material of digging resistance of 45 N/mm will be
a. 1296.0 b. 298.1
c. 222.2 d. 123.5

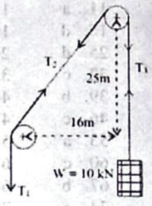
110. Match the following :

Equipment	Component
P Scraper	1 Dribble belt
Q Dragline	2 Dipper stick
R Bucket wheel excavator	3 Fair lead
S Rope shovel	4 Bowl

a. P-2, Q-4, R-3, S-1 b. P-4, Q-2, R-1, S-3
c. P-4, Q-3, R-1, S-2 d. P-2, Q-4, R-1, S-3 [GATE 2012]

111. The torque in N-m of a winder motor is described by the relationship T = 1450- 3.2 ω, where ω is the angular speed of the motor in rad/s. If the shaft is rotating at a speed of 1450 rpm, the power of the motor in kW is
a. 112.4 b. 146.4
c. 184.4 d. 212.4 [GATE 2012]

112. Figure shows a two pulley system for hoisting a loadof 10 kN. The coefficient of friction between each pulley and the rope is 0.2. The vertical and horizontal distances between the centers of the pulleys are 25 m and 16 m respectively.



The tensions T_1 and T_2 respectively in kN are
a. 6.00, 5.38 b. 12.37, 11.06
c. 18.74, 16.73 d. 25.11, 22.41 [GATE 2012]

113. A dump truck powered by 350 kW engine is running at a speed of 35 km/h. Considering the transmission efficiency of the truck as 85%, the rim pull of the truck in kN is
a. 21 b. 31
c. 41 d. 51 [GATE 2012]

114. It is proposed to connect two straights of a road by a simple circular curve. If the maximum speed of the vehicle is 60 km/h and the centrifugal ratio for the road is 1/4, the minimum radius of the curve in m is
a. 113.26 b. 98.18
c. 25.46 d. 15.50 [GATE 2011]

115. A drive shaft of an engine develops torque of 500 N-m. It rotates at a constant speed of 50 rpm. The power transmitted by the shaft in kW is
a. 1.46 b. 2.05
c. 2.62 d. 4.32 [GATE 2011]

116. A 1100 V, 3θ power supply system of a mine draws a load of 185 kW. The ammeter reading shows 115 A. The power factor of the system is
a. 0.84 b. 0.73
c. 0.64 d. 0.48 [GATE 2009]

117. A 400 V, 3 phase, star connected induction motor takes a line current of 10 A with 0.86 p.f. lagging. Total stator losses are 5 % of the input. Rotor copper losses are 4 % of the input to the rotor, and mechanical losses are 3 % of the input to the rotor. The input power to the rotor in Watts is
a. 5958 b. 5788
c. 5660 d. 5532 [GATE 2011]

118. In the above question, he shaft output power in Watts is
a. 5562 b. 5490
c. 5434 d. 5264 [GATE 2011]

Answer

1. c	2. b	3. a	4. b	5. a	6. a	7. c
8. b	9. c	10. c	11. a	12. d	13. a	14. a
15. c	16. a	17. a	18. d	19. b	20. d	21. b
22. c	23. a	24. b	25. d	26. a	27. b	28. c
29. d	30. c	31. a	32. c	33. b	34. b	35. a
36. d	37. a	38. c	39. b	40. d	41. b	42. b
43. d	44. b	45. a	46. c	47. b	48. d	49. d
50. a	51. c	52. c	53. a	54. a	55. b	56. e
57. d	58. d	59. d	60. c	61. b	62. d	63. a
64. b	65. a	66. c	67. b	68. b	69. c	70. a
71. c	72. d	73. b	74. a	75. a	76. a	77. d
78. a	79. b	80. a	81. c	82. a	83. a	84. d
85. a	86. d	87. d	88. c	89. d	90. a	91. b
92. c	93. d	94. b	95. a	96. b	97. a	98. a
99. d	100. a	101. a	102. b	103. d	104. b	105. c
106. d	107. a	108. c	109. d	110. c	111. b	112. c
113. b	114. a	115. c	116. a	117. a	118. d	

Solution of Exercise 6

1. We have,

Centre – to – Centre Distance (d) $= \dfrac{(N_1+N_2)m}{2}$

Where, N_1 and N_2 are the teeth number of gear 1 and gear 2 respectively. m is module of the gear.

$$\Rightarrow \quad d = \frac{(24+33)*4}{2} = 114\text{mm}$$

2. We have,

Centre – to – Centre Distance (d) $= \dfrac{(N_1+N_2)m}{2}$

Where, N_1 and N_2 are the teeth number of gear 1 and gear 2 respectively. m is module of the gear.

$$\Rightarrow \quad d = \frac{(24+42)*6}{2} = 198\text{mm}$$

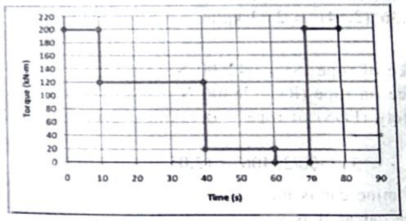
3. Given data,

Gear ratio of the first gear = R_1 = 3.81: 1
Gear ratio of transfer case = R_2 = 2.72: 1
Gear ratio of the differential = R_3 = 4.11: 1
Engine rotating speed = ω = 1000 rpm
Wheel diameter = D = 1.2 m
Let the speed of the vehicle is V
We have,

$$V = \frac{\pi D\omega/60}{R_1R_2R_3} \text{m/s}$$

$$\Rightarrow V = \frac{3.1491.2*1000/60}{3.81*2.72*4.11} = 1.47\text{ m/s} = 1.47*\frac{3600}{1000}\text{ km/hr} = 5.31\text{ km/hr}$$

4. Given condition :



The rms torque can be determined as

$$\tau = \sqrt{\frac{\Sigma_{i=1\text{ to }n}\tau_i^2 t_i}{\Sigma_{i=1\text{ to }n}t_i}}$$

Where, τ_i is the torque

T_i = time duration

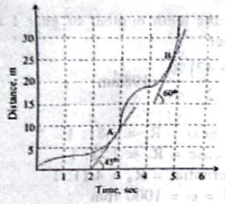
n = number of variation

$$\Rightarrow \tau_{rms} = \sqrt{\frac{200^2*10+120^2*30+20^2+0^2*10+200^2*10}{10+30+20+10+10}}$$

$$= \sqrt{\frac{400000+43000+8000+0+400000}{80}}$$

$$\Rightarrow \tau_{rms} = \sqrt{\frac{1240000}{80}} = \sqrt{15500} = 124.5\ \text{kN} - \text{m}$$

8.



$\frac{\tan\theta_B}{\tan\theta_A} = \frac{\Delta S_b/\Delta t_b}{\Delta S_A/\Delta t_A}$

$$\text{We have, } \frac{v_b}{v_a} = \frac{\Delta S_b/\Delta t_b}{\Delta S_a/\Delta t_a} = \frac{\frac{(24-15)}{(4.7-4.2)}}{\frac{(7-0)}{(2.8-2)}} = \frac{9*0.8}{6.5*0.8} = 1.38\ \text{m/s}$$

14. The mass of rope can be determined as
    Mass of rope (M) = k* D^2 [kg/m]
    Where, k = constant depending on the design of the rope and type of material
    D = diameter of the rope in cm
    ⇒ M = 0.36*$(2.54)^2$ = 2.33 kg/m

26. Given data,
    Breaking force of rope (S) = 342500 N
    Resistive force on rope (R) = 28406 N
    Factor of Safety (FOS) of rope = Breaking force of rope (S)/ Resistive force on rope (R)
    = 342500/28406 = 12.05

27. Let the mass of mine car is m,
    Velocity of mine car at B is v
    Height of mine car from ground level at A and B are h_1 and h_2 respectively.
    Appling the law of energy conservation, we have
    Potential energy at B = Potential energy at A + kinetic energy at A
    ⇒ $mgh_1 = mgh_2 + 1/2mv^2$

$g*20*\tan10 = g*10*\tan10 + 1/2*v^2$

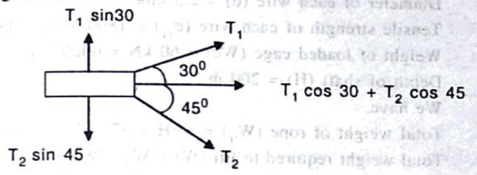
⇒ $v^2 = 2*g*10*\tan10 = 24.6$

⇒ v = 5.88 m/s

⇒

28. Given data,
    Weight of mine car (M) = 1 te = 1000 kg
    Speed of car (v) = 10 km/h = 10*5/18 m/s = 2.78 m/s
    Buffer spring stiffness of the buffer (k) = 200 kN/m
    Let the compression in the spring is x
    Applying the law of energy conservation, we have
    Loss in kinetic energy = energy stored in spring due to compression
    $1/2mv^2 = 1/2kx^2$
    ⇒ $x^2 = mv^2/k = 1000*(2.78)^2/200*10^3 = 0.038642$
    ⇒ x = 0.1966 m = 196.6 mm
    ⇒

29. Free body diagram of the Barge



In equilibrium condition :

The resultant forces along the direction of travel is 5000N

Therefore,

$T_1\cos30 + T_2\cos45 = 5000$ (i)

Again, $T_1\sin30 = T_2\sin45$

$\Rightarrow T_1 = \sqrt{2}\ T_2$

Putting the value of T_1 in eqn. (i), we get

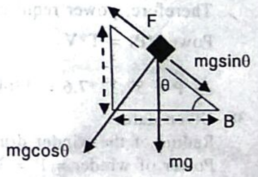
$\sqrt{2}\ T_2*\cos30 + T_2\cos45 = 5000$

$$\Rightarrow T_2 = \frac{5000}{\sqrt{3/2}+\sqrt{1/2}} = 2588\ \text{N}$$

Therefore, $T_1 = \sqrt{2}\ T_2 = 1.414*2588 = 3660$ N

30. Gravity force acting on mine car along the sloping line is

$$= mg\sin\theta = mg\frac{x}{\sqrt{1+x^2}}$$



Therefore, counter force require to prevent the mine car moving down is

$$F = mg\frac{x}{\sqrt{1+x^2}}$$

31. We have,

Total torque of the drum (τ) = Total rope tension (T) * radius of the drum (r)

$$\Rightarrow \tau = 115*\frac{3}{2} = 172.5\,\text{kNm}$$

35. Give data:

Diameter of rope (D) = 25 mm
Weight of rope (w) = 37 N/m
Number of strands (N_s) = 6
Number of wires in each strand (Nw) = 7
Diameter of each wire (d) = 2.5 mm
Tensile strength of each wire (σ_{tl}) = 1800 MPa = $18*10^8$ Pa
Weight of loaded cage (Wc) = 60 kN = 60000 N
Depth of shaft (H) = 200 m

We have,

Total weight of rope (W_{tr}) = w*H = 37 * 200 = 7400 N
Total weight required to lift (W) = W_{tr} + W_c = 7400 + 60000 = 67400 N
Total number of wires in the rope (Nwr) = N_s* N_w = 6*7 = 42
Load bearing capacity of 1 wire = σ_{tl}* Area of cross section of wire
= $18*10^8$*[3.14*(2.5*10–3)2/4] = 8831.25 N

Load bearing capacity of rope (T) = Load bearing capacity of 1 wire *Number of wires

= 490.625*42 = 370912.5 N

Therefore, factor of safety of the rope can be determined as

FOS = T/ W = 370912.5/67400 = 5.5

37. We have,

Total torque of the drum (τ) = Total rope tension (T) * radius of the drum (r)

$$\Rightarrow 109 = T*\left(\frac{3.8}{2}\right)$$

$$\Rightarrow T = \frac{109*2}{3.8} = 57.37\,\text{kN}$$

Therefore, power required to run the winder is given by

Power (P) = T*V

$\Rightarrow$ P = 57.37*7.6 = 436kW [Where, v is rope speed]

38. Given data,

Radius of the winder drum = r = 2.5 m
Power of winder = P = 308 kW = $308*10^3$ W

Rope speed = V = 7 m/s
Let the RMS torque is τ and the tension in the rope is T
Therefore, we have

$$P = T*V = \left(\frac{\tau}{r}\right)*V$$

$$\Rightarrow \tau = \frac{P*r}{V} = \frac{308*10^3*2.5}{7} = 110*10^3\,\text{Nm} = 110\,\text{kNm}$$

41. Let the maximum velocity attained by the cage is V.

There are three time cycles.
1^{st} Cycle : t_1 = 10 s
Initial velocity = u_1 = 0
Final velocity = v_1 = V
Acceleration = a_1 = a
2^{nd} Cycle : t_2 = 40 s
Initial velocity = u_2 = V
Final velocity = v_2 = V
Acceleration = a_2 = 0
3^{rd} Cycle : t_3 = 10 s
Initial velocity = u_3 = V
Final velocity = v_3 = 0
Acceleration = a_3 = – a

Applying Newton's first law, we get

Distance covered during 1st cycle = $S_1 = u_1*t_1 + ½ *a_1* t_1^2$

$\Rightarrow S_1 = 0 + 1/2*a*10^2$

$\Rightarrow S_1 = 50a$ (i)

Similarly, distance covered during 2nd cycle = $S_2 = u_2*t_2 + ½ *a_2* t_2^2$

$\Rightarrow S_2 = V*40 + 0$

$\Rightarrow S_2 = 40V$ (ii)

Again, distance covered during 3^{rd} cycle = $S_3 = u_3*3 + 1/2*a_3*t_3^2$

$\Rightarrow S_3 = V*10 + 1/2*(-a)*10^2$

$\Rightarrow S_3 = 10V - 50a$ (iii)

Adding the above three equations, we get

$S_1 + S_2 + S_3 = 10V - 50a + 40V + 50a$

$\Rightarrow 450 = 50V$

[total distance covered in three cycle is 450m]

$\Rightarrow V = 9\,m/s$

42. Let the tension in each rope is T.

Each chain makes an angle of 30° with the vertical. Therefore, vertical component of force acting on each chain is Tcos30°

Total force acting on the cage by four chains in upward direction = 4*T cos30°

This force is balanced by weight of cage.

Therefore, 4*T cos30° = M*g

$\Rightarrow T = \frac{12000*9.81}{4*0.866} = 33983.8N = 33.98kN$

43. Given data,

Floor area of cage = A = 5 m^2

Drag coefficient = C_d = 2.5

Velocity of air in the shaft = V = 6 m/s

Drag force eperienced by cage $= F_d = C_d * \frac{1}{2}\rho V^2 * A$

Where, ρ is the density of air and this is equal to 1.2 kg/m^3

$\Rightarrow F_d = 2.5*\frac{1}{2}*1.2*6^2*5 = 270N$

44. Given data,

Weight of Material raised per day = M = 3000 te = $3*10^6$ kg

Depth of shaft = h = 500 m

Winder efficiency = η = 70 % = 0.70

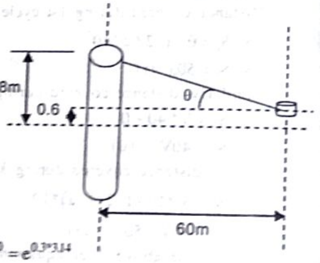
Therefore, energy consumed per day for raising the material $= \frac{M*g*h}{\eta}$

$= \frac{3*10^6*9.81*500}{0.7} = 210.21*10^8 W = 210.21*10^5 kW$

$= \frac{210.21*10^5}{3600} kWh = 5840 kWh$

45. Fleet angle $(\theta) = \tan^{-1}\left(\frac{1.8-0.6}{60}\right)$

$\Rightarrow \theta = 1.15^0$



47. Wrap factor $\left(C^{\mu\theta}\right) = e^{\mu\theta} = e^{0.3*180*\pi/180} = e^{0.3*3.14}$

μ = coefficient of friction between the wheel and rope

θ = angle of wrap = 180° (for tower mounted friction winding)

Wrap factor = 2.57

50. We have, $\frac{T_1}{T_2} = e^{\mu\theta}$

Where, T_1 = tension in the rope entering the wheel

T_2 = tension in the rope leaving the wheel

μ = coefficient of friction between the wheel and rope

θ = angle of wrap = 180^0 (for tower mounted friction winder)

$\Rightarrow \frac{T_1}{T_2} = e^{0.4*180*\frac{3.14}{180}} = e^{1.256} = 2.53$ → 3.51

Power $= (T_1 - T_2)*r*\varpi$

$\Rightarrow 200*1000 = (2.53*T_2 - T_2)*\left(\frac{4}{2}\right)*(60*2*3.14/60)$

$\Rightarrow 200000 = (1.53*T_2)*2*6.28$

$\Rightarrow T_2 = 10.4*10^3 N = 10.4kN$ → 6.369

53. We have, $\frac{W_1}{W_2} = e^{\mu\theta}$

Where, W_1 = weight in the rope entering the wheel

W_2 = weight in the rope leaving the wheel

μ = coefficient of friction between the wheel and rope

θ = angle of wrap

$\Rightarrow \frac{W_1}{W_2} = e^{0.3*180*\frac{3.14}{180}} = e^{0.942} = 2.56$

54. Given data,

r.m.s torque (τ) = 109 kNm = $109*10^3$ Nm

Maximum rope speed (v) = 7 m/s

Winding drum radius (r) = drum diameter/2 = 4.4/2 = 2.2 m

We have,

$\text{Power} = \tau.\varpi = \tau*\frac{v}{r} = 109*10^3*\frac{7}{2.2} = 348*10^3 W = 348 kW$

55. Given data,

Space factor (S) = 0.5

Diameter of rope (d) = 2.54cm = $2.54*10^{-2}$m

Tensile strength of rope (TS) = 1500MPa = $1500*10^6$ Pa

Factor of Safety (FoS) = 10

We have,

$\text{Space Factor (SF)} = \frac{\text{Aggregate rope diameter}(d_a)}{\text{Overall rope diameer}(d)}$

$\Rightarrow 0.5 = \frac{d_a}{2.54}$

$\Rightarrow d_a = 1.27$

Again,

$$FoS = \frac{\text{Strength of rope}}{\text{Strain on rope}}$$

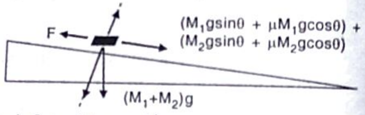
$$\Rightarrow 10 = \frac{1500*10^6}{\text{Maximum load the rope can take (P) / aggregate area of cross section (A)}}$$

$$\Rightarrow 10 = \frac{1500*10^6}{P/\pi(d_a/2)^2} = \frac{1500*10^6}{P}*3.14*(0.0254/2)^2 = \frac{0.759*10^6}{P}$$

$\Rightarrow P = 0.0759*10^6 N = 75.9 kN$

56. Capacity of tub (q) = 1 te = 1000 kg
Tare weight of tub (w) = 500 kg
Number of tub (n) = 8
Total mass of material including tare weight (M_1) = 8*(1000+500) =12000 kg
Hauling speed (v) = 9 km/hr = (9*5/18) m/s = 2.5 m/s
Coefficient of friction between wheel and rail (μ_1) = 1/60
Coefficient of friction between rope and drum (μ_2) = 1/10
Mass of rope = 1.5 kg/m
Length of incline plane (L) = 500 m
Therefore, total mass of rope (M_2) = 1.5*500 = 750 kg
Inclination (m) = 1/6
For small inclination,

We have, $\sin\theta = \tan\theta$ and $\cos\theta$


F
$(M_1 g\sin\theta + \mu M_1 g\cos\theta) +$
$(M_2 g\sin\theta + \mu M_2 g\cos\theta)$
$(M_1+M_2)g$

$F = (M_1 g\sin\theta + \mu_1 M_1 g\cos\theta) + (M_2 g\sin\theta + \mu_2 M_2 g\cos\theta)$

$\Rightarrow F = 12000*9.81(1/6+1/60) + 9.81(750*1/6+750/10)$

$\Rightarrow F = 12000*9.81(1/6 + 1/60) + 9.81\ (750*1/6 + 750/10)$

$\Rightarrow F = 21582 + 1962 = 23544N$

[For small θ, and $\sin\theta = \tan\theta = \frac{1}{6}$ $\cos\theta = 1$]

Therefore, power required to run the motor can be determined as
Power = F*v = 23544*2.5 = 58860 W = 58.860kW

59. Given data,
Mass of locomotive (m) = 8 te = 8000 kg
Rolling resistance (F_r) = 100 N/te = 100/1000 N/kg = 0.1 N/kg
Coefficient of adhesion (μ) = 0.18
We have,
Tractive effort = μ*m*g + F_r * m
= 0.18*8000*9.81 + 0.1*8000
= 14926.4 kg = 14.9264 te

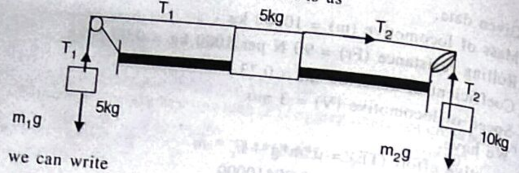
61. Given data,
Mass of locomotive (m) = 10000 kg
Rolling resistance (Fr) = 90 N per.1000 kg = 0.09 N/kg
Coefficient of adhesion (μ) = 0.23
Speed of locomotive (V) = 3 m/s
We have,
Tractive effort (TE) = μ*m*g – F_r * m
= 0.23*10000*9.81 – 0.09*10000
= 22563 – 900 = 21663 N
Therefore, power available for hauling = TE*V = 21663*3 = 64989 N = 64.989 kN

62. Mass of trolley (m_t) = 80 kg
Mass of the block (m_b) = 10 kg
Kinetic friction between the trolley and the surface (μ) = 0.04
Let the acceleration of the trolley is a and the tension in the rope is T
We have,
T – μ* m_t * g = m_t*a
⇒ T – 0.04*80*9.81 = 80a
⇒ T – 31.4 = 80 a (i)
Again,
m_b*g – T = m_b*a
⇒ 10*9.81 – T = 10a
⇒ 98.1 – T = 10a (ii)
Adding eqn.(i) and eqn.(ii), we get
T – 31.4 + 98.1 – T = 80 a +10a
⇒ 90a = 66.7
⇒ a = 0.74 m/s^2

63. Applying the law of energy conservation, we get
Loss in potential energy = Work done to cover a distance of 130 m
⇒ m*g*h = F*s
⇒ F = 2*9.81*3/130 = 0.452 N
Resistance force due to 2 te mine car is 0.452 N
Therefore, rolling resistance per te of load is
= 0.452/2 = 0.226 N/te

64. Given data,
Weight of block (w) = 100 kN
Coefficient of static friction (μ) = 0.5
Static frictional force (F) = μ*w = 0.5*100 = 50 kN

65. Free body force diagram of the situation is as



Thus, we can write

$T_1 - m_1g = m_1a \quad \Rightarrow \quad T_1 - 5g = 5a$

$m_2g - T_2 = m_2a \quad \Rightarrow \quad 10g - T_2 = 10a$

$T_2 - T_1 - \mu m_3g = m_3a \quad \Rightarrow \quad T_2 - T_1 - 0.2*5g = 5a$

Solving the above equations, we get

$T_1 = 6g$

$T_2 = 8g$

$a = g/5$

68. Given data,

Length of AFC (L) = 114 m

Speed of AFC (v) = 0.535 m/s

Mass of chains and flights (m) = 54 kg/m

Coefficient of friction between the chain and trough (μ) = 0.33

Total mass of chain and flights = M = L*m = 114*54 = 6156 kg

Total frictional force for moving the AFC (Fr) = μ*M*g = 0.33*6156*9.81

= 19928.8 N

Therefore, Power needed to drive the empty conveyor is given by

P = Fr*v = 19928.8*0.535 = 10661.9 W = 10.66 kW

69. Given data,

Rate of coal transportation (T) = 220 te/hr = (220/3600) te/sec

Length of conveyor (L) = 600 m

Gradient (m) = 1 in 60

Width of belt (W) = 0.75 m

Cross-sectional area of the material (a) = 0.1 $W^2 = 0.1*(0.75)^2 = 0.05625\ m^2$

Bulk density of coal (b) = 0.8 te/m^3

Let the speed of conveyor belt is v

Rate of coal transportation of a troughed belt conveyor is given by

T = a * b* v (te/sec)

$$\Rightarrow \frac{220}{3600} = 0.05625*0.8*V$$

$$\Rightarrow V = 1.36\ m/s$$

---

70. Given data,

Width of belt (W) = 0.75 m

Average cross-sectional area of the material (a) = 0.09 m^2

Bulk density of coal (b) = 1.5 te/m^3

Speed of conveyor belt (v) = 2 m/s

Let the carrying capacity of belt is T

Carrying capacity of belt conveyor is given by

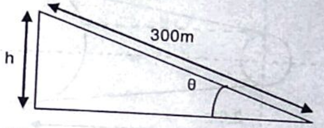
T = a * b* v (te/sec)

= 0.09*1.5*2 = 0.27 te/sec = 0.27*3600 te/hr = 972 te/hr

71. Given data,

Rated Power (P) = 100 kW = 100000 W

Rate of coal hauling (m) = 30 kg/s



Inclination (θ) = 15°

Heat added by the conveyor to the air = Rated Power – Work done

= Rated power – gain in potential energy

= 100000 – m*g*h

= 100000 – 30*9.8*sin15°*300 = 100000 – 22820

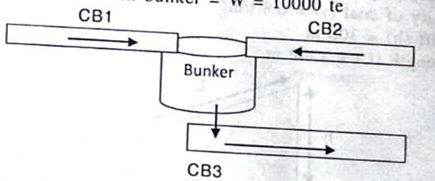
= 77180 W = 77.18 kW

72. Give data,

Capacity of CB 1 and CB 2 = q = 400 tph

Capacity of CB 3 = Q = 1200 tph

Initial weight of coal in bunker = W = 10000 te



Let the time elapsed before the bottom conveyor starts to operate below its rated capacity is T hr.

Therefore we have,

W + q*T +q*T = Q*T

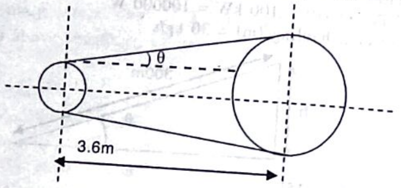
⇒ 10000 +2*400*T = 1200*T

⇒ 400T = 10000

⇒ T = 25 hr.

73. Given data :
Length of wagons (L) = 8 m
Chute loading rate (q) = 6000 te/hr = 6000/3600 te/s = 1.67 te/s
Rake travel speed (V) = 0.48 km/hr = 0.133 m/s
Chute's stopping time in between two wagons (t) = 24 sec
Loading time of each wagons (T) = (L/V – t) = 8/0.133 – 24 = 60 –24 = 36 sec
Therefore, quanity of coal loaded in each wagon (Q) = q*T = 1.67*36 = 60 te

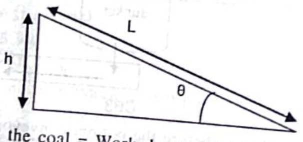
74. Given data,
Diameter of smaller pulley (d_1) =1.6 m
Diameter of bigger pulley (d_2) =2.4 m
Distance between the centre of the two pulley (D) = 3.6 m



Angle of rope with the horizontal (θ) $= \tan^{-1}\left[\dfrac{\frac{(2.4-1.6)}{2}}{3.6}\right] = \tan^{-1}(1/9) = 6.34^0$

Angle of contact on the smaller pulley = 180 –2*θ = 180 – 12.68^0 = 167.38^0

75. Given data,
Rate of coal transportation (Q) = 600 t/hr = (600000/3600) kg/s = (1000/6) kg/s
Bulk density of coal (γ) = 0.8 t/m^3
Vertical lift (h) = 300 m
Inclined length (L) = 1200 m



Power required to lift the coal = Work done = Gain in potential energy
= Q*g*h
= (1000/6)*9.8*300 = 490000W = 490kW

76. Given data,
Radius of curvature (R) = 30 m
Gauge width (w) = 0.75 m
Maximum elevation of the track (h) = w/8 = 0.75/8 m
Let the maximum turning speed of locomotive is V

We have,
Maximum elevation of track (h) $= \dfrac{V^2}{gR}$
g is acceleration due to gravity

$$\Rightarrow \frac{0.75}{8} = \frac{V^2}{9.81*30}$$

$$\Rightarrow V = \sqrt{\frac{9.81*30*0.75}{8}} = \sqrt{27.6} = 5.25\,m/s$$

77. Available Power for driving the locomotive (P)
= Power of Locomotive * Gear efficiency
= 60*103*0.80 = 48*10^3 W

Let the speed of locomotive is V
We have,
Available Power = Adhesion force * Velocity
⇒ 48*10^3 = (μ*M*g)*V
Where, μ is coefficient of adhesion
M is mass of locomotive
g is acceleration due to gravity
⇒ 48*10^3 = (0.25*12000*9.81)*V

$$\Rightarrow V = \frac{48*10^3}{0.25*12000*9.81} = 1.63\,m/s$$

78. Given data,
Weight of loco = m = 24 te = 24000 kg
Weight of trailing load = M = 300 te = 300000 kg
Rolling resistance = F= 100 N/te
Coefficient of adhesion = μ = 0.3
Let the acceleration of loco is a
We have,
Draw bar pull = Tractive effort – Rolling resistance
⇒ (M+m)a = μ*m*g – (M + m)*F
⇒ 324000a = 0.3*24000*9.81 – 324*100
⇒ 324a = 70.632 – 32.4
⇒ a = 38.232/324 = 0.118 m/s^2

79. Given data,
Mass of loco (M) = 10000 kg
Coefficient of adhesion (μ) = 0.25
Running resistance (F_r) = 0.1* weight of loco
Draw-bar-pull = Tractive effort – Running resistance
= μ*M*g – 0.1*M*g
= 0.25*10000*9.8 – 0.1*10000*9.8 = 14700 N = 14.7 kN

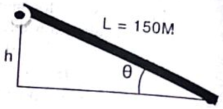
80. If the gradient of the track is 5°, the draw bar pull can be determined as
Draw-bar-pull = Tractive effort – Running resistance
= 0.25*10000*9.8 - 10000*9.8*10/100 - 10000*9.8*sin5° = 6.16 kN

81. Given data,
Average cross-sectional area of the material (a) = $B^2/11$
[B is width of the belt]
Bulk density of coal (b) = 1200 kg/m^3
Speed of conveyor belt (v) = 1.5 m/s
Fill factor (f) = 0.8
Rate of transportation (T) = 188 kg/s
Rate of transportation of belt conveyor is given by
T = a * b* v*f (kg/s)

$$\Rightarrow 0.188=\left(\frac{B^2}{11}\right)*1200*1.5*0.8$$

$$\Rightarrow B=\sqrt{\frac{11*188}{1200*1.5*0.8}}=\sqrt{1.44}=1.2\text{m}$$

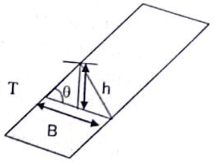
82. Given data,



Length of the chain conveyor (L) = 150 m
Inclination of the chain (θ) = 15°
Power required to move the coal (P) = 5 kW
= 5000 W
Therefore, Vertical lift of coal (h) = L sinθ
Let the mass of coal transportation per hour is M kg
We have,
Energy required to lift M kg of coal is given by
E = M*g*h = M*g*L*sinθ
This is equal to the energy consumed by the conveyor in 1 hour.
And, the energy consumed by conveyor belt in 1 hour = P*3600 = 5000*3600
Therefore,
5000*3600 = M*9.81*150*sin15°

$$\Rightarrow M=\frac{5000*3600}{9.81*150*0.26}=47262.42\text{kg}=47.26\text{ te}$$

83. Given data,
Width of plate conveyor (B) = 0.72 m
Speed of conveyor (V) = 0.9 m/s
Dynamic angle of repose of material (θ) = 20°
Let the theoretical capacity of plate conveyor is T
We have,
Capacity of conveyor (T) = a*V m^3/s
= a*V*3600 m^3/hr



Where, a = average cross-sectional area of the material
Therefore

$$T=\left(\frac{1}{2}*B*h\right)*V*3600=\frac{1}{2}*B*\left(\frac{B}{2}\tan\theta\right)*V*3600$$

$$=\frac{1}{4}*0.72*0.72*\tan 20^0*0.9*3600=152.82\text{m}^3/\text{hr}$$

85. Let the tension on slack side of the belt = T_1
Tension on tight side of the belt = T_2
Given that,
Coefficient of friction between the belt and drive drum (μ) = 0.3
Again, $T_2 = 2T_1$

We have, $\frac{T_2}{T_1}=e^{\mu\theta}$

Where, θ is angle of wrap

$$\Rightarrow \frac{2*T_1}{T_1}=e^{0.3*\theta}$$

$$\Rightarrow 2=e^{0.3*\theta}$$

$$\Rightarrow \theta=\frac{0.693}{0.3}=2.31\text{rad}=2.31*\frac{180}{\pi}=132.45^0$$

86. Given data,
Coefficient of friction between the belt and drum (μ) = 0.35
Angle of wrap in both the drum (θ) = 230°

We have, $\frac{T_1}{T_2}=e^{\mu\theta}$ ...... (i)

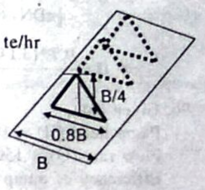
Again, $\frac{T_2}{T_3}=e^{\mu\theta}$ ...... (ii)

Multiplying the above two equations, we get

$$\frac{T_1}{T_2}*\frac{T_2}{T_3}=e^{\mu\theta}*e^{\mu\theta}$$

$$\Rightarrow \frac{T_1}{T_3}=e^{2\mu\theta}=e^{2*0.35*230*3.14/180}=e^{2.80}=16.64$$

87. Given data,
Theoretical capacity of plate conveyor (T) = 400 te/hr
Speed of conveyor (V) = 3 m/s
Bulk density of coal (b) = 1 te/m^3
Dynamic angle of repose of material (θ) = 20°
Let the width of plate conveyor is B
Therefore, the height of coal pile is B/4



And, Width of coal pile is 0.8B
We have,
Capacity of conveyor (T) = a*b*V te/s
= a*b*V*3600 te/hr
Where, a = average cross-sectional area of the material
Therefore,

$$T=\left(\frac{1}{2}*0.8*B*\frac{B}{4}\right)*b*V*3600$$

$$\Rightarrow 400=\frac{1}{8}*0.8*B^2*1*3*3600$$

$$\Rightarrow B=\sqrt{\frac{400*8}{3*3600*0.8}}=\sqrt{0.37}=0.608m=608mm$$

95. Given data,
Diameter of impeller (D) = 250 mm = 0.25 m
Speed of impeller (N) = 1440 rev/min
Manometric efficiency (η) = 0.7 %
We have,

$$\text{Manometric efficiency}(\eta)=\frac{\text{Manometric head (H)}}{\text{Work head imparted by the rotor on the fluid or Euler Head}}$$

$$\Rightarrow \eta=\frac{H}{(VU/g)}$$

Where,
V = whirl velocity at outlet
U = tangential velocity of the impeller at outlet = $\pi DN/60$
Again,
$V = U - V_f \cot\beta$

For radial blade, [β is blade angle]
$\beta = 0$
And hence, $V = U - V_f*0 = U$
Therefore, we have,

$$\Rightarrow \eta=\frac{H}{(UU/g)}=\frac{g*H}{U^2}$$

$$\Rightarrow 0.7=\frac{9.81*H}{(\pi DN/60)^2}$$

$$\Rightarrow H=\frac{0.7*(3.14*0.25*1440/60)^2}{9.81}=\frac{0.7*354.9456}{9.81}=25.32m$$

96. Given data,
Pump head (H) = 10 bars = 10*105 Pa
Flow rate (Q) = 1500 l/min = (1500*10–3/60) m^3/s = $25*10^{-3}$ m^3/s
Efficiency of pump (η) = 90 % = 0.9

Let the power of pump motor is P
We have,

$$\text{Efficiency of Pump}(\eta)=\frac{\text{Actual Power}}{\text{Rated Power of Motor (P)}}$$

$$\Rightarrow 0.9=\frac{H*Q}{P}$$

$$\Rightarrow P=\frac{10*10^5*25*10^{-3}}{0.9}=27777.77\ W=27.78\ kW$$

97. Standard atmospheric pressure is equivalent to
$P = 1.013*10^5\ N/m^2$
Density of water (ρw) = 1000 kg/m^3
Let the water can be lifted to a maximum theoretical height of h m
Therefore, we have
$P = h*\ \rho w*g$
$\Rightarrow$ $1.013*10^5 = h*1000*9.81$
$\Rightarrow$ $h = 1.013*10^5/(1000*9.81) = 10.33$ m

99. Given data,
Pump efficiency (η) = 70% = 0.7
Discharge rate (Q) = 2100 L/min = $(2100*10^{-3}/60)\ m^3/s = 35*10^{-3}\ m^3/s$
Head (H) = 100 m
Pump motor efficiency (η_m) = 90% = 0.9
Let the input power required for the motor is P
We have,

$$\text{Pump efficiency}(\eta)*\text{Pump Motor efficiency }(\eta_m)=\frac{\text{Actual Power}}{\text{Input Power to Motor (P)}}$$

$$\Rightarrow \eta*\eta_m=\frac{H*\rho w*g*Q}{P}$$

$$\Rightarrow 0.7*0.9=\frac{100*1000*9.81*35*10^{-3}}{P}$$

$$\Rightarrow P=\frac{100*1000*9.81*35*10^{-3}}{0.63}=54500\ W=54.5\ kW$$

100. Given data,
Velocity of water in suction pipe = V_1 = 1.8 m/s
Velocity of water in suction pipe = V_2 = 2.5 m/s
Let the diameter of suction and delivery pipe are d_1 and d_2 respectively.
We have,
Rate of water discharge (Q) = $A_1V_1 = A_2V_2$
[A_1 and A_2 are the cross sectional area of suction and delivery pipe respectively]

$$\Rightarrow 0.035=3.14*\frac{d_1^2}{4}*1.8=3.14*\frac{d_2^2}{4}*2.5$$

$\Rightarrow d_1 = 0.1573m = 15.73cm$
And, $d_2 = 0.1335m = 13.35cm$

101. Given data,
Total head (H) = 200m
Flow rate (Q) = 2000 l/min = $(2000*10^{-3}/60)$ m^3/s = 1/30 m^3/s
Efficiency of Pump (η) = 75% = 0.75
Let the input power to the pump is P
We have,

$$\text{Efficiency of Pump }(\eta)=\frac{\text{Actual Power}}{\text{Rated Power of Motor (P)}}$$

$$\Rightarrow 0.75=\frac{H*\rho w*g*Q}{P}$$

$$\Rightarrow P=\frac{200*1000*9.81*1/30}{0.75}=87.2*10^3\,W=87.2kW$$

108. Given data,
Resistance to motion of a dumper = R = $(35 + 0.1\ V^2)$ N/te
Where, V is the velocity in m/s
Tractive effort (TE) = 325 N/te
Velocity of dumper (V) = 10 m/s
Let the acceleration of the dumper is a, and the mass is M kg
We have,
Draw bar Pull = Tractive effort (TE) – Resistance (R)
$\Rightarrow$ Ma = 325*M/1000 - $(35 + 0.1\ V^2)$*M/1000
$\Rightarrow$ 1000a = 325 – 35 – 0.1*102 = 325 – 45 = 280
$\Rightarrow$ a = 0.28 m/s^2

109. Give data,
Case I :
Production rate (Q_1) = 400 m^3/hr
Material digging resistance (R_1) = 25 N/mm
Case II :
Let the production rate is Q_2
Material digging resistance (R_2) = 45 N/mm
Again, it is given that

$$\text{Digging resistance }(R)\propto\frac{1}{\sqrt{\text{Production rate (Q)}}}$$

Therefore,

$$\frac{R_1}{R_2}=\sqrt{\frac{Q_2}{Q_1}}$$

$$\Rightarrow \frac{25}{45}=\sqrt{\frac{Q_2}{400}}$$

$$\Rightarrow Q_2=\left(\frac{25}{45}\right)^2*400=123.45\text{ te / hr}$$

111. It is given that torque is described by the relationship as
T = 1450 – 3.2ω
Where, ω is the angular speed of the motor in rad/s
Given that, motor speed (ω) = 1450 rpm = 1450*2π/60 rad/s = 151.76 rad/s
Let the power of the motor is P
Therefore, the torque can be determined as
T = 1450 – 3.2*151.76 = 964.35 N-m
Power of the motor can be determined as
Power = Torque (T) * Angular speed (ω)
= 964.35*151.76 =146349 W = 146.35 kW

112. Given data,
Coefficient of friction between each pulley and rope (μ) = 0.2
Weight of load (W) = 10 kN = 10000 N
Let the angle of wrap in bottom and top pulleys are θ_1 and θ_2 respectively.
Therefore, we have

$$\frac{T_1}{T_2}=e^{\mu\theta_1}\qquad\text{(i)}$$

$$\text{And }\frac{T_2}{T_3}=e^{\mu\theta_2}\qquad\text{(ii)}$$

θ_1 and θ_2 can be determined as

$$\theta_1=90^0-\text{tain}^{-1}\left(\frac{25}{16}\right)=90^0-57.38^0=32.62^0$$

$$\theta_2=180^0-\text{tain}^{-1}\left(\frac{16}{25}\right)=180^0-32.62^0=147.38^0$$

Putting the values of θ_1 and θ_2 in eqn. (i) and eqn. (ii), we get

$$\frac{T_1}{T_2}=e^{(0.2*32.62*3.14/180)}=e^{0.114}=1.12$$

$$\text{And }\frac{T_2}{T_3}=e\left(\frac{0.2\times147.38*3.14}{180}\right)=e^{0.514}=1.67$$

Again, T_3 = weight of load (W) = 10000N
Therefore, T_2 = 1.67*10000 = 16.7kN
T_1 = 1.12*16.7 = 18.73 kN

113. Given data,
Power of the dump truck (P) = 350 kW
Running speed (V) = 35 km/h = (35*1000/3600) m/s = 9.72 m/s
Transmission efficiency (η) = 85 % = 0.85
Let the rim pull of the truck is F

We have,

$$\text{Transmission efficiency}(\eta) = \frac{\text{Power available for rim pull}}{\text{Input Power}} = \frac{F*V}{P}$$

$$\Rightarrow 0.85 = \frac{F*9.72}{350000}$$

$$\Rightarrow F = 30606W = 30.6kW \approx 31\,kW$$

114. Given data,
Maximum speed of vehicle (V) = 60 km/h = (60*1000/3600) m/s =16.67 m/s
Centrifugal ratio (CR) = ¼
Let the maximum radius of curve is R
We have,

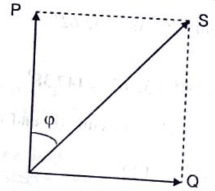
$$\text{Centrufugal Ratio (CR)} = \frac{V^2}{gR}$$

$$\Rightarrow \frac{1}{4} = \frac{(16.67)^2}{9.81*R}$$

$$\Rightarrow R = 113.26m$$

115. Given that,
Torque develop in the drive shaft (τ) = 500 N-m
Angular speed (ω) = 50 rpm = (50*2π/60) rad/s = 5.23 rad/s
We have,
Power = τ*ω = 500*5.23 = 2616.66 W ≈ 2.62 kW

116. It is common to define the Power Factor - PF - as the cosine of the phase angle between voltage and current - or the "cosϕ".



The power factor defined by IEEE and IEC is the ratio between the applied **active (true) power** - and the **apparent power**, and can in general be expressed as :

PF = P / S = P/ (VA)

Where,

PF = power factor
P = active (true or real) power (Watts)
S = apparent power (VA, volts amps)

For 3-Phase power supply

$$\text{Power factor} = P\left(\sqrt{3}\,V*I\right)$$

Thus, PF = 185000/ ($\sqrt{3}$*1100*115) = 0.84

---

117. Given data,
Line current (I) = 10 A
Power factor (PF) = 0.86
Voltage (V) = 400 V
Let input power to the rotor is P_i
Total Stator Losses (L_s) = $0.05P_i$
Rotor Copper Losses (L_c) = (1-0.05) P*0.04 = 0.95*0.04*P = 0.038 P
Mechanical Losses (L_m) = (1–0.05) P*0.03 = 0.95*0.03*P = 0.0285 P

The input power to the rotor for a 3-Phase power supply can be determined as

P = $\sqrt{3}$*Power factor (PF)*Apparent Power
= $\sqrt{3}$*PF*V*I
= 1.732*0.86*400*10 = 5958 W

118. The shaft output power is given by
$P_o = P_i - L_s - L_c - L_m$
$\Rightarrow P_o$ = 5958 (1 – 0.05 – 0.038 – 0.0285)
$\Rightarrow P_o$ = 5958 *0.8835 = 5263.89 ≈5264 W ■

7.0 Mine Systems Engineering

7.1 Linear Programming

Linear programming may be classified as the most popular optimization technique used ever in mine system engineering. Its popularity is partly because of the readily available software packages for problem solution apart from its potential for detailed modelling and analysis. There are various methods to solve the problem in linear programming such as graphical method and simplex method but the most commonly used algorithm is the simplex method.

The only significant disadvantage of LP is that, it can only be used for linear optimization models. A linar mathematical function f $(X_1, X_2 \ldots X_n) = a_1X_1 + a_2X_2 + a_3X_3 + \ldots\ldots a_nX_n$

Where, $X_1, X_2 \ldots X_n$, are variables, and $a_1, a_2 \ldots$an are constants.

Graphical Method : This is the method to optimize (maximum/ minimum) the solution graphically without any complex calculation.

Example :

Maximise $Z = 2x_1 + x_2$

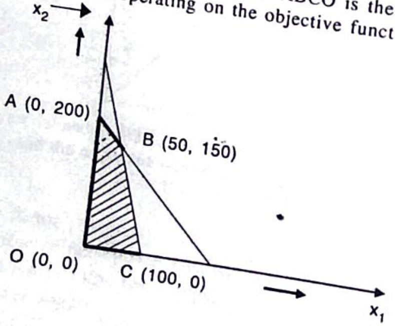
Subject to $3x_1 + x_2 = 300$

$x_1 + x_2 = 200$

$x_1, x_2 = 0$

Let the horizontal axis represents model x_1, and the vertical axis model x_2. Plot all the constraint lines and mark the common segments to determine the feasibility region as has been done in the graph shown.

Any point on the thick line or inside the shaded portion of Figure 7.1 will satisfy all the restrictions of the problem. Thus ABCO is the feasibility region carried out all the constraints operating on the objective function.



Point	Feasible solution or variables		Value of objective function z
	x_1	x_2	
A	0	200	200
B	50	150	250
C	100	0	200
O	0	0	0

From the above table, we can find that the value of the objective function z is maximum at B (50, 150), and this is equal to 250. Thus, this is the optimum solution out of the several feasible solutions.

Multiple Solutions : Multiple solutions are alternate solutions to a problem yielding the same optimal value of the objective function.

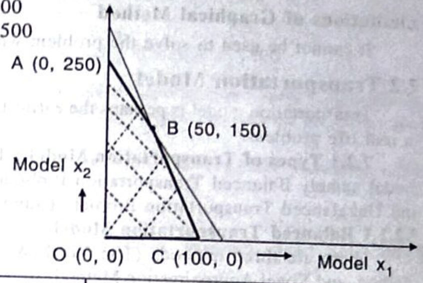
Example :

Maximise $Z = 2x_1 + x_2$

Subject to $3x_1 + x_2 = 300$

$4x_1 + 2x_2 = 500$

$x_1, x_2 = 0$



Point	Feasible solution or variables		Value of objective function z
	x_1	x_2	
A	0	250	250
B	50	150	250
C	100	0	200
O	0	0	0

From the above table, we find that the value of the objective function z is maximum at two points A (0, 250) & B (50, 150), and this is equal to 250. Thus, there are multiple solutions which give optimum value out of the several feasible solutions.

Unbounded Solution :

An unbounded solution is one in which the objective function value can be arbitrarily increased without violating the constraints.

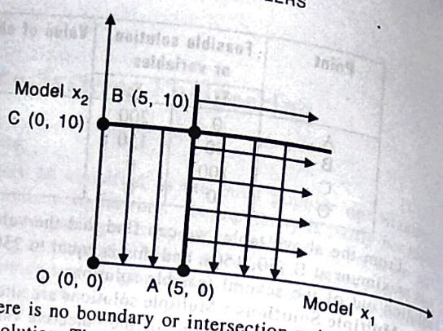
Example :

Maximise $z = x_1 + x_2$

Subject to $x_1 = 5$

$x_2 = 10$

$x_1, x_2 \geq 0$



In the above graph, there is no boundary or intersection points beyond the oint B to give the feasible solution. Thus, the above problem has the unbounded olution instead of optimal solution.

Limitations of Graphical Method

It cannot be used to solve the problem which has more than two variables.

7.2 Transportation Model

Transportation model is perhaps the earliest application of linear programming n real life problem.

7.2.1 Types of Transportation Model : There are two types transportation nodel namely Balanced Transportation Problem (Supply and Demand are equal) ınd Unbalanced Transportation Problem (Supply and Demand are unequal).

.2.1.1 Balanced Transportation Model

There are three methods (The North-West Corner Rule, Matrix Minimum Method, and Vogel Approximation Method) for finding initial basic feasible solution nd one optimum solution method (UV method) for finding optimal solution.

nitial Basic Feasible Solution of a Transportation Model

- **The North-West Corner Rule** : This rule is a simple method of obtaining a basic feasible solution of the transportation problem. In this method, the procedure for obtaining the solution is started from the north-west corner and approaches towards the south-east corner. The only variables required to obtain this type of solution are supply and demand and not the cost of transportation.
- **Matrix Minimum Method :** This is an efficient method for finding the initial basic feasible solution of a transportation problem. In this method, solution starts with, searching the cell of minimum cost and assign the units and so on. If there are multiple minimum, say $Cmn = Cpq$, then $C = Cmn$ = if $m < p$ or $m = p$, $n < q$
  $= Cpq$ = if $m > p$ or $m = p$, $n > q$

The above equation implies that if there are two or more minimum costs exists then we should have to select the cell corresponding to the lower numbered row. In the case, they appear in the same row, we should select the cell corresponding to the lower numbered column.

- **Vogel Approximation Method (VAM) :** This method requires a good deal of calculations for finding the initial basic feasible solution of the transportation problem. This method however, usually yields a starting solution which is very close, if not equal, to the optimum solution.

Example : The coal from three mines M1, M2 and M3 are to be transported to three power plants P1, P2 and P3. The cost of transportation of each tonne from mines to the plants along with normal capacities of mines and plants are indicated below :

Mines	Plants			
	P_1	P_2	P_3	a (capacity of mines)
M_1	25 (X_{11})	17 (X_{12})	25 (X_{11})	400 (a_1)
M_2	15 (x_{21})	10 (X_{22})	18 (X_{23})	300 (a_2)
M_3	16 (X_{31})	13 (X_{32})	16 (X_{33})	400 (a_3)
b (Capacity of plants	250 (b_1)	350 (b_2)	500 (b_3)	1100

[**Note :** The above problem is a balanced transportatin problems as the capacity of the supply by the mines is equal to the capacity of consumed by the plants.]

Thus, if the equation $\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j$ is holds, problem is a balanced transportation problem

And, if the equation $\sum_{i=1}^{n} a_i \neq \sum_{j=1}^{m} b_j$ is holds, problem is an unbalanced transportation problem

Solution :

➢North-West Corner Rule :

Step I : $X_{11} = \min (a_1, b_1) = b_1 = 250$; i.e. if $b_1 = a_1$; $X_{i1} = 0$ [for i >1]

Step II : $X_{12} = \min (a_1 - b_1, b_2) = (150, 350) = a_1 - b_1 = 150$; thus $X_{1j} = 0$ [for j >2]

Step III : $X_{22} = \min [b_2 - (a_1 - b_1), a_2)] = (200, 300) = 200$; $X_{i2} = 0$ [i > 2]

Step IV: $X_{23} = \min [a_2 - (b_2 - a_1 - b_1), b_3] = (100, 500) = 100$

Step V: $X_{33} = b_3 - a_2 - b_2 - a_1 - b_1 = 400$

Mines	Plants			
	P_1	P_2	P_3	a (capacity of mines)
M_1	25 (250)	17 (150)	25 (0)	400 (a)
M_2	15 (0)	10 (200)	18 (100)	300 (a_2)
M_3	16 (0)	13 (0)	16 (400)	400 (a_3)
b (Capacity of plants	250 (b_1)	350 (b_2)	500 (b_3)	1100

Therefore, the cost of transportation

= 25*250 + 17*150 + 10*200 + 18*100 + 16*400

= 19.000

➢ Matrix Minimum Method

Minimum cost cell = X_{22} (10) = min (a_2, b_2) = 300; X_{2j} = 0 [for j ≠ 2]

Next minimum cost cell = X_{32} (13) = min (b_2 − a_2, a_3) = 50; X_{i2} = 0 [for i ≠ 3]

The solution will proceed in the same way

Mines	Plants			
	P_1	P_2	P_3	a (capacity of mines)
M_1	25 (0)	17 (0)	25 (400)	400 (a_1)
M_2	15 (0)	10 (300)	18 (0)	300 (a_2)
M_3	16 (250)	13 (50)	16 (100)	400 (a_3)
b (Capacity of plants	250 (b_1)	350 (b_2)	500 (b_3)	1100

Therefore, the cost of transportation

= 16*250 + 10*300 +13*50 + 25*400 + 16*100

= 19250

➢ Vogel Approximation Method

Step I :

Mines	Plants				Penalty
	P_1	P_2	P_3	a (capacity of mines)	
M_1	25 ( )	17 (350)	25 ( )	400 (a_1)	8
M_2	15 ( )	10 (0)	18 (0)	300 (a_2)	5
M_3	16 ( )	13 (0)	16 ( )	400 (a_3)	3
b (Capacity of plants	250 (b_1)	350 (b_2)	500 (b_3)	1100	
Penalty	1	3	2		

Penalty = Minimum cost of any row – Next minimum cost of any row

= Min. cost of any column – Next minimum cost of any column

The maximum penalty found in row 1, i.e. C_{11} - C_{12}, and thus assigned the value to cell C_{12}. In the next step, column 2 has been eliminated.

Step II :

Mines	Plants			Penalty
	P_1	P_2	a (capacity of mines)	
M_1	25 (0)	25 ( )	50 (a_1)	0
M_2	15 (250)	18 ( )	300 (a_2)	3
M_3	16 (0)	16 ( )	400 (a_3)	0
b (Capacity of plants	250 (b_1)	500 (b_3)	1100	
Penalty	1	2		

Step III :

Mines	Plants		Penalty
	P_3	a (capacity of mines)	
M_1	25 (50)	50 (a_1)	
M_2	18 (50)	50 (a_2)	
M_3	16 (400)	400 (a_3)	
b (Capacity of plants)	500 (b_3)	1100	
Penalty	2		

Final Solution :

Mines	Plants			
	P_1	P_2	P_3	a (capacity of mines)
M_1	25 (0)	17 (350)	25 (50)	400 (a_1)
M_2	15 (250)	10 (0)	18 (50)	300 (a_2)
M_3	16 (0)	13 (0)	16 (400)	400 (a_3)
b (Capacity of plants	250 (b_1)	350 (b_2)	500 (b_3)	1100

Therefore, the cost of transportation

= 15*250 + 17*350 + 25*50 + 18*50 + 16*400

= 18250

➢ Optimal Solution of a Transportation Model

UV method :

The initial basic feasible solution has been derived from North-West corner method. The solution derived is as follows :

Mines	Plants			
	P_1	P_2	P_3	a (capacity of mines)
M_1	X_{11}= 250 (B.V.)	X_{12}= 150 (B.V.)	X_{13}= 0 (N.B.V.)	400 (a_1)
M_2	X_{21}= 0 (N.B.V.)	X_{22}= 200 (B.V.)	X_{23}= 100 (B.V.)	300 (a_2)
M_3	X_{31}= 0 (N.B.V.)	X_{32}= 0 (N.B.V.)	X_{33}= 400 (B.V.)	400 (a_3)
b (Capacity of plants	250 (b_1)	350 (b_2)	500 (b_3)	1100

[B.V. = Basic Variable; N.B.V. = Non-basic Variable]

Therefore, the cost of transportation = 25*250 + 17*150 + 10*200 + 18*100 + 16*400

= 19,000

Optimal Solution

Mines	Plants			
	P_1	P_2	P_3	
M_1	X_{11} = 250 (C_{11} = 25)	X_{12} = 150 (C_{12} = 17)	X_{13} = 0 (C_{13} = 15)	U_1
M_2	X_{21} = 250 (C_{21} = 15)	X_{22} = 200 (C_{12} = 17)	X_{23} = 100 (C_{23} = 18)	U_2
M_3	X_{31} = 250 (C_{11} = 16)	X_{32} = 0 (C_{32} = 13)	X_{33} = 400 (C_{33} = 15)	U_3
	V_1	V_2	V_3	

The determination of the entering variable from among the current non-basic variables (those that are not part of the starting basic solution, i.e., value is 0) is done by computing the non-basic coefficients in the objective function, using the method of multiplier.

In this method we associate the multipliers Ui and Vj with row i and column j of the transportation tableau. For each current basic variable xij, these multipliers satisfy the following equations:

$U_i + V_j = c_{ij}$ for each basic variable xij

Basic Variable	**(U, V) Equation**	**Solution**	
X_{11}	$U_1 + V_1 = 25$	Set $U_1 = 0$, $V_1 = 25$ [Arbitrarily,] zero value can be assigned to any one]	
X_{12}	$U_1 + V_2 = 17$	$U_2 = -7$,	$V_2 = 17$
X_{22}	$U_2 + V_2 = 10$	$U_3 = -9$,	$V_3 = 25$
X_{23}	$U_2 + V_3 = 18$		
X_{33}	$U_3 + V_3 = 16$		

Next we use U_i and V_j to evaluate the non-basic variables by computing $U_i + V_j - C_{ij}$ for each non-basic x_{ij}

Non-basic Variable	$\mathbf{U_i + V_j - C_{ij}}$
X_{13}	$U_1 + V_3 - C_{13} = 10$ [E.V. is Maximum Positive]
X_{21}	$U_2 + V_1 - C_{21} = 3$
X_{31}	$U_3 + V_1 - C_{31} = 0$
X_{32}	$U_3 + V_2 - C_{32} = -5$

X_{11} = 250	X_{12} 150 − θ	X_{13} = 0 + θ
X_{21} = 0	X_{22} 200 + θ	X23 = 100 − θ
X_{31} = 0	X_{32} = 0	X_{33} = 400

'θ' can be determined from the constrained derived from the above tableau

$$150 - \theta \geqslant 0$$
$$100 - \theta \geqslant 0$$

Thus, maximum value of θ is 100

The new solution after assigning value to entering cell and shifting the same corresponding to that value for balancing the demand and supply as follows :

Mines	Plants			
	P_1	P_2	P_3	
M_1	X_{11} = 250 (C_{11} = 25)	X_{12} = 50 (C_{12} = 17)	X_{13} = 0 (C_{13} = 15)	U_1
M_2	X_{21} = 0 (C_{21} = 15)	X_{22} = 300 (C_{12} = 10)	X_{23} = 100 (C_{23} = 18)	U_2
M_3	X_{31} = 0 (C_{11} = 16)	X_{32} = 0 (C_{32} = 13)	X_{33} = 400 (C_{33} = 16)	U_3
	V_1	V_2	V_3	

The leaving variable is x_{23} as the assigned value becomes zero in the new solution. Thus, x_{23} becomes non-basic variable and x_{13} is basic variable.

The new cost of transportation is

= 250*25 + 50*17 + 100*15 + 300*10 + 400*16 = 18000

Thus the solution improves from 1900 to 1800.

The above iteration continues till the value of all non-basic variables ($U_i + V_j - C_{ij}$) becomes negative.

7.2.1.2 Unbalanced Transportation Model : When the supply and demand are unequal, the model is called unbalanced transportation model. In this case, we need to add a dummy row (demand > supply) or dummy column (demand < supply) and assign zero cost to each cell. After this operation, same methods can be followed for solving the problems.

7.3 Assignment Problem : An assignment problem in the most general form may be stated as follows; given n individual and n jobs, we wish to assign each individual to exactly one job so that each job is covered by exactly one individual, in such a way that the total cost is minimized.

7.3.1 Solution Methods of Assignment Problem :

- Transportation method
- Simplex method
- Complete Enumeration method
- Hungarian method

7.3.2 The Hungarian Method

7.3.2.1 Balanced Model : When number of columns and number of rows are equal, the problem is balanced problem.

The assignment problem can be solved with this method as follows:

Step I : For the original cost matrix, identify each row's minimum, and subtract it from all the entries of the row.

Step II : For the matrix resulting from step I, identify each column's minimum, and subtract it from all the entries of the column.

Step III : Identify the optimal solution as the feasible assignment associated with the zero elements of the matrix obtained in step II.

Example : There are three jobs A, B, and C can be assigned to P, Q and R. A single person can finish a single job. The cost of completion of different people for different job is different and is given below.

Job ⇒ / ⇓ Person	A	B	C
P	15	10	9
Q	9	15	10
R	10	12	8

Determine the minimum cost to finish all the jobs.

Solution : Let p_i an q_j be the minimum costs associated with row i and column j as defined in step I and II respectively.

The row minimum for step I are computed from the original cost matrix as shown below :

Job ⇒ / ⇓ Person	A	B	C	Row Minimum
P	15	10	9	$P_1 = 9$
Q	9	15	10	$P_2 = 9$
R	10	12	8	$P_3 = 8$

Next, subtract the row minimum from each respective row to obtain the reduced matrix is given below :

Job ⇒ / ⇓ Person	A	B	C
P	6	1	0
Q	0	6	4
R	2	4	0
Column Minimum	$q_1 = 0$	$q_2 = 1$	$q_3 = 0$

The application of step II yields the column minimum. Subtracting these values from the respective columns, we get the reduced matrix as given below :

⇒ Job / ⇓ Person	A	B	C
P	6	0	0
Q	0	5	4
R	2	3	0

The cells with underscored zero entries provide the optimum solution. Only one cell should be chosedn in a particular row or colun.

Thus, the total cost of finish the jobs = 9 + 10 + 9 = 27

Example 2 :

Job ⇒ / ⇓ Person	A	B	C	D
P	1	4	6	3
Q	9	7	10	9
R	4	5	11	7
S	8	7	8	5

Step 1 :

Job ⇒ / ⇓ Person	A	B	C	D	Row Minimum
P	1	4	6	3	$P_1 = 1$
Q	9	7	10	9	$P_2 = 7$
R	4	5	11	7	$P_3 = 4$
S	8	7	8	5	$P_4 = 5$

Step II :

Job ⇒ / ⇩ Person	A	B	C	D
P	0	3	5	2
Q	2	0	3	2
R	0	1	7	3
S	3	2	3	0
Column Minimum	$q_1 = 0$	$q_2 = 0$	$q_3 = 3$	$q_4 = 0$

Step III :

Job ⇒ / ⇩ Person	A	B	C	D
P	0	3	2	2
Q	2	0	0	2
R	0	1		3
S	3	2	0	0

In the above tableau, the locations of the zero entries do not allow assigning unique chores to all the persons.

Step IV : Draw the minimum number of horizontal and vertical lines in the last reduced matrix that will cover all the zero entries.

Job ⇒ / ⇩ Person	A	B	C	D
P	0	3	2	2
Q	2	0	0	2
R	0	1	4	3
S	3	2	0	0

Step V : Select the smallest uncovered entry, subtract it from every uncovered entry, and then add it to every entry at the intersection of two lines.

Job ⇒ / ⇩ Person	A	B	C	D
P	$\underline{0}$	2	2	2
Q	3	0	$\underline{0}$	2
R	0	$\underline{0}$	4	3
S	4	2	0	$\underline{0}$

Thus the optimal cost of finishing the job = 1 + 5 + 10 + 5 = 21

Step VI : Still, if no feasible assignment can be found among the resulting zero entries, repeat step IV.

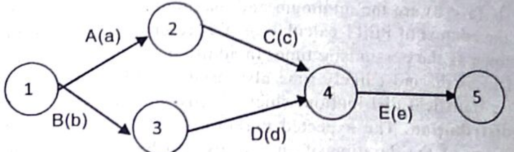
7.3.2.2 Unbalanced Model : When number of columns and number of rows are not equal, the problem is called an unbalanced problem. In such situations, dummy column/row, whichever is smaller in number, are inserted with zeros as the cost elements. After introducing the dummy row or column, the method is same as balanced model.

7.4 Network Analysis in Project Planning

7.4.1 Introduction : Network planning consists of arranging the precedence and sequence of project tasks appropriately to provide the road map of the project execution. It starts with the construction of a diagram, known as network diagram that reflects the interdependencies and time requirements of the individual tasks that constitute the project. The individual activities or tasks have the two fundamental properties:

- An event only occurs if and only if all the activities leading into it are complete
- No activity can start until its tail event has occurred

For example



In the above network,

1, 2, 3, 4 and 5 represents the nodes

A, B, C, D and E represents the 5 activities or tasks

a, b, c, d and e represents the time to complete the individuals tasks A, B, C, D and E respectively

Thus, the activity E cannot start until both the activities (C and D) leading into it complete.

The definitions of few related terms are given below :

- ➢ **Critical path :** The path that takes the most time to complete.
- ➢ **Critical task (critical activity) :** A task that resides on the critical path.
- ➢ **Non-critical path :** Any path that is not a critical path and thus takes less effort to complete than the critical path.
- ➢ **Non-critical task (non-critical activity) :** Any activity that resides on a non-critical path and not a critical path. These tasks may accept some delay in completion.

7.4.2 Methods of Determining the Duration of a Particular Project

7.4.2.1 Critical Path Method (CPM) or Critical Path Analysis (CPA) :

Since all the activities directed to the events must be completed before it occurs, the time at which an event occurs is the maximum duration of the paths directed to the event. Thus, in a complete network of the project, the path having the maximum of the elapsed times from the origin to the terminal node called the critical path and the activities on it are called critical activities. It is possible to have a multiple critical paths if there is more than one path with the longest duration.

CPM is based on the deterministic approach where the durations of the various activities are assumed to be known with certainty.

In the above figure, if a = 5 months; b = 6 months; c = 8 months; d = 4 months and e = 5 months then

Duration of the project along the path 1-2-4-5 = a + c + e = 18 months

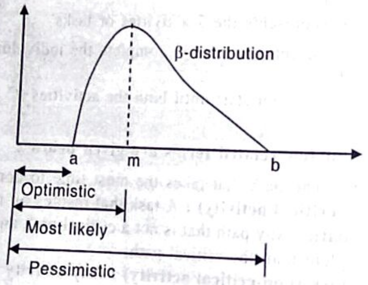
And, duration of the project along the path 1-3-4-5 = b + d + e = 15 months

Therefore, the longest duration path is 1-2-4-5 and this path is called the critical path.

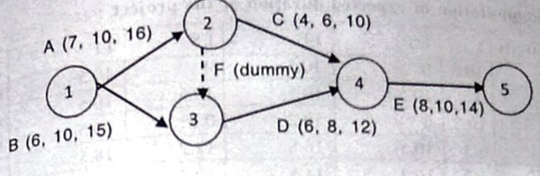
7.4.2.2 Programme Evaluation and Review Technique (PERT) : For a variety of reasons, durations of some of the activities are non-deterministic. It is thus necessary to use a probability distribution to represent the stochastic nature of the activity duration. It is assumed that this probability distribution is continuous, uni-modal with two non-negative abscissa intercepts. These intercepts denoted by 'a' and 'b' (a < b) are the minimum and maximum time necessary to perform the job. In the context of PERT calculation 'a' is referred to as the optimistic time and 'b' is known as the pessimistic time. In addition, the modal value m is commonly referred to as the most likely time also used in PERT calculation.

The statistical distribution which ideally suits these assumptions is known as a **β-distribution.** The expected duration (te) of an activity and the standard deviation (σ) of the duration of an activity can be approximated by

$$t_e = \frac{a+4m+b}{6}; \quad \sigma = \frac{b-a}{6}$$



Example : A mining project has 5 tasks (A, B, C, D, and E). The sequence and the time required to finish the individual taks are shown in the Fig. given below. Determine the expected time of completion of the project.



➢ **Computation of expected duration (t_e) and standard deviation (σ) of individual tasks :**

Activity	t_e = (a + 4m + b)/6	σ = (b − a)/6
A*	10.5	1.5
B	10.2	1.5
C	6.3	1.0
D*	8.3	1.0
E*	10.3	1.5

➢ **PERT Analysis Algorithm**

- **Step 1 :** Make a forward pass through the network as follows: For each of these activities, i, compute :

– Earliest Start (ES) Time = the maximum of all earliest finish times for all its immediate predecessors. (For starting node '1', this is 0.)

- ES_i = Maximum (EFj) for all immediate proceeding activities j.

– Earliest Finish (EF) Time = (Earliest Start Time) + (Time to complete activity i).

- EFi = $ES_i + t_i$

The project completion time is the Earliest Finish Times at the last node, '5'.

– This will also be used as Latest Finish Time at last node in the next step.

- **Step 2 :** Make a backwards pass through the network as follows: Move sequentially backwards from the last node, '5' to its immediate predecessors, etc. At a given node, j, consider all activities immediately following it and compute:

– Late Finish (LF) Time = the minimum of the latest start times for all activities that immediately follow j. (For node '5', this is the project completion time.)

– LF_j= Minimum (LSi) for all immediate following activities i.

– Late Start (LS) Time = (Latest Finish Time) - (Time to complete activity j).

– $LS_j = LF_j - t_j$

- **Step 3 :** Calculate the slack time for each activity by:

Slack = (Late Start) − (Earliest Start) or

= (Late Finish) − (Earliest Finish).

A critical path is a path of activities, from node starting node '1' to finish node '5', with 0 slack times.

➢ **Computation of expected duration of the project :**

Activity	t_e	ES	EF	LS	LF	Slack
A*	10.5	0	10.5	0	10.5	0
F*	0	10.5	10.5	10.5	10.5	0
B	10.2	0	10.2	0.3	10.5	0.3
C	6.3	10.5	16.8	12.2	18.5	1.7
D*	8.3	10.2	18.5	10.2	18.5	0
E*	10.3	16.8	27.1	18.5	28.8	0
Note : *Critical activity/task						

Thus, the expected duration of the project is 28.8 days

Variance of the completion time $(V_T) = \sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2 + \ldots\ldots\sigma_n{}^2$

$= 8.75\ (\text{days})^2$

Project time standard deviation $(\sigma) = \sqrt{V_T} = \sqrt{8.75} = 2.96\,(\text{days})$

➢ **Probability Analysis of the Project :** In the above example, what is the probability that the project will complete within 35 days?

Following things are determined for the above example:

Expectedor Mean duration of project completion (μ) = 28.8 days

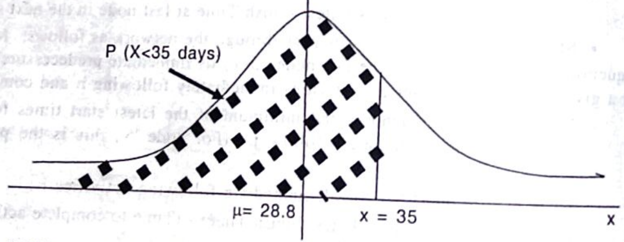
Project time standard deviation (σ) = 2.96 days

Probability that the project will finish within 35 days is given by

$$P(X \le 35) = P(X - \mu \le 35 - \mu) = P\left(\frac{X-\mu}{\sigma} \le \frac{35-\mu}{\sigma}\right) = P\left(Z \le \frac{35-28.8}{2.96}\right)$$

$$\Rightarrow P(Z \le 2.09) = 0.9817$$

[Refer the normal distribution table. The value is corresponding to 2.0 in column and 0.09 in row]



7.4.3 Network Crashing : It is sometimes desirable to accelerate the project progress. Thus, to reduce the duration of the project, duration of certain activities has to be crash or reduce. The network planning device can be used to identify the activities whose duration should be shortened so that the completion time of the project can be shortened in the most economic manner. The process is known as network crashing. The crashing of duration should be done only along the critical path. If there is more than one critical path, duration of the activities should be crashed equally in both the paths.

7.5 Reliability Analysis

7.5.1 Introduction : Reliability of equipments or a system is the probability that it will function properly without failure for a specified length of time under a given environmental condition. It is defined with respect to time horizon and combines the time dimension with the performance level of equipment. It is defined formally as the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered.

7.5.2 Statistical Concepts of Reliability : Let the life of a component be denoted by the random variable t with distribution function F(t) and density function f(t). In the context of reliability, F(t) and f(t) are also referred to as failure distribution.

The reliability R(t) is given by

$$R(t) = 1 - F(t)$$

Clearly, R(t) indicates the probability that the component will work at least up to time thereby adding the time dimension to the definition.

Probability that a component of age t will fail in the interval t and (t + dt)

$$= h(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1-F(t)}$$

If the failure distribution is exponential with parameter λ, then

$$h(t) = \frac{\lambda e^{-\mu}}{e^{-\mu}} = \lambda$$

7.5.3 General Reliability Function

The general equation of determining the reliability is given by

$$R(t) = e^{-\int_0^t \lambda(t)\,dt}$$

where,

R(t) is the reliability at time t,

λ(t) is the hazard rate or time-dependent failure rate.

It is used to obtain the reliability of an item whose times to failure are described by statistical distributions such as exponential, Rayleigh, Weibull, and normal, etc.

If the failure rate of the equipment is constant with time, the above equation can be written as

$$R(t) = e^{-\int_0^t \lambda\,dt} = e^{-\lambda t}$$

where λ is constant failure rate.

7.5.4 Mean Time Between Failure (MTBF)

Assuming, an equipment or a system is in operation. Let us assume, when the equipment fails, it is repaired and the repair time is negligible. Then the average failure time E(t) is identical with the average time between successive failures. This is referred to Mean Time Between Failure (MTBF).

$$\text{MTBF} = E(t) = \int_0^\infty 1 - F(t)dt = \int_0^\infty R(t)dt$$

Putting the value of R(t), for constant failure time and exponential distribution, we have

$$\text{MTBF} = \int_0^{\infty} e^{-\lambda t} dt = \left[\frac{e^{-\lambda t}}{-\lambda}\right]_0^{\infty} = \frac{1}{\lambda}$$

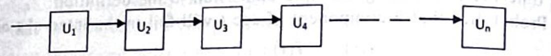
Thus, the reliability or the probability that the equipment run up to time t for exponential distribution failure rate $= e^{-\lambda t} = e^{-t/m}$

Where, $m = 1/\lambda$ = mean time between failure

7.5.5 Reliability of a System

A system can form various types of networks or configurations in performing reliability analysis. The reliability of the entire system is, obviously, a function of the reliability of its components.

7.5.5.1 Reliability of a Series Configuration/System : In series arrangement/ configuration, all the units must operate normally for the successful operation of the system. It is depicted in the figure given below :



Let X_j denote the event that the jth unit in above figure is successful, then the reliability of the series configuration/system is given by

$$R_s = P(X_1 X_2 \ldots\ldots X_n)$$

In case, the componets are independents to each other, we have

$$R_s = P(X_1)\, P(X_2)\, P(X_3)\, \ldots\ldots\, P(X_n)$$

Where,

$P(X_j)$ is the probability of occurrence of success event X_j, for $j = 1, 2, 3, \ldots, n$.

Let $R_j = P(X_j)$ for $j = 1, 2, 3 \ldots n$, the above equation becomes

$$R_S = R_1\, R_2\, R_3 \ldots\ldots\ldots R_n = \prod_{j=1}^{n} R_j$$

Where R_j is the reliability of jth unit

For exponential distribution and constant failure rate (λ_j) for jth unit, we have

$$R_j = e^{-\lambda_j t}$$

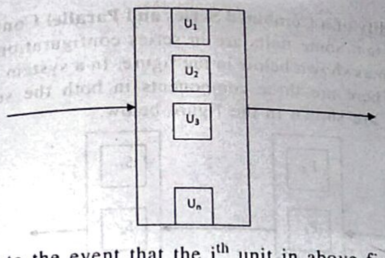
Putting the values of R_j, the reliability of the system becomes

$$R_S = e^{-\sum_{j=1}^{n} \lambda_j t} = e^{-(\lambda_1 + \lambda_2 \ldots\ldots\ldots \lambda_n)t}$$

The MTBF of the series systemcan be determined as

$$\text{MTBF}_s = \int_0^{\infty} e^{-(\lambda_1 + \lambda_2 \ldots\ldots \lambda_n)t} dt = \frac{1}{\lambda_1 + \lambda_2 \ldots\ldots \lambda_n}$$

7.5.5.2 Reliability of a Series Configuration/System : In this case, all n units are active and at least one of these units must operate normally for the successful operation of the system. The block diagram of an 'n' unit parallel configuration/system is shown in the Figure given below.



Let $\overline{X}_j$ denote the event that the jth unit in above figure is unseuccessful, then the probability of failure of the parallel configuration/system is given by

$$R_f = P\left(\overline{X}_1 \overline{X}_2 \ldots\ldots \overline{X}_n\right)$$

In case, the compenets are independents to each other, we have

$$R_f = P\left(\overline{X}_1\right) P\left(\overline{X}_2\right) \ldots\ldots P\left(\overline{X}_n\right) = [1 - P(X_1)][1 - P(X_2)] \ldots\ldots\ldots [1 - P(X_n)]$$

Let the reliability of jth unit, $R_j = P(X_j)$ for $j = 1, 2, 3, \ldots .n^{th}$ units, the above equation becomes

$$R_f = (1 - R_1)(1 - R_2)(1 - R_3) \ldots\ldots (1 - R_n) = \prod_{j=1}^{n} (1 - R_j)$$

Where R_f is the failure probability of the jth unit

Therefore, the reliability of the successful operation of the system is given by

$$R_s = 1 - R_f = 1 - \prod_{j=1}^{n} (1 - R_j)$$

For exponential distribution and constant failure rate (λ_j) for jth unit, we have

$$R_j = e^{-\lambda_j t}$$

Therefore, the reliability of the system is

$$R_s = 1 - \prod_{j=1}^{n} (1 - R_j) = 1 - \prod_{j=1}^{n} \left(1 - e^{-\lambda_j t}\right)$$

$$= 1 - \left(1 - e^{-\lambda_1 t}\right) * \left(1 - e^{-\lambda_2 t}\right) * \ldots\ldots\ldots \left(1 - e^{\lambda_n t}\right)$$

The MTBF of the parallel system can be determined as

$$\text{MTBF} = \int_0^{\infty} \left[1 - \left\{\left(1 - e^{-\lambda_1 t}\right) * \left(1 - e^{-\lambda_2 t}\right) * \ldots \left(1 - e^{-\lambda_n t}\right)\right\}\right] dt$$

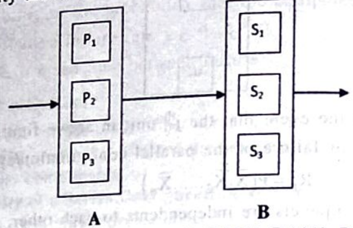
For identical units, $\lambda_1 = \lambda_2 = \lambda_3 = \ldots .\ \lambda_n = \lambda$

Therefore, the above equation reduces to

$$\text{MTBF}_P = \int_0^{\infty} \left[1 - \left\{\left(1 - e^{-\lambda t}\right) * \left(1 - e^{-\lambda t}\right) * \ldots \left(1 - e^{-\lambda t}\right)\right\}\right] dt$$

$$= \int_0^{\infty} \left[1 - \left(1 - e^{\lambda t}\right)^n\right] dt = \frac{1}{\lambda} \sum_{j=1}^{n} \frac{1}{j}$$

7.5.5.3 Reliability of a Combined Series and Parallel Configuration/System : In this type of system some units are in series configuration and some are in parallel configuration as shown below in the figure. In a system there are two sub-systems A and B. There are three components in both the sub-systems whose individual reliability are shown in the figure below.



Reliability of sub-system A = $1 - (1-P_1)*(1-P_2)*(1-P_3)$

Reliability of sub-system B = $1 - (1-S_1)*(1-S_2)*(1-S_3)$

Reliability of system = $[1 - (1-P_1)*(1-P_2)*(1-P_3)] * [1- (1-S_1)*(1-S_2)*(1-S_3)]$

7.5.5.4 Reliability of k-out-of-n Configuration : In this case, n numbers of units are active and at least k units must operate normally for the system success. The parallel and series configurations are special cases of this configuration for k=1 and k = n, respectively.

Using the binomial distribution, for independent and identical units, the k-out-of-n configuration reliability can be determined as :

$$R_{k/n} = \sum_{j=k}^{n} nC_j R^j (1-R)^{n-j}$$

Where, $R_{k/n}$ is the k-out-of-n configuration/system reliability,

and R is the reliability of each unit.

For a constant failure rates (λ) of units, the above equation can be reduces to

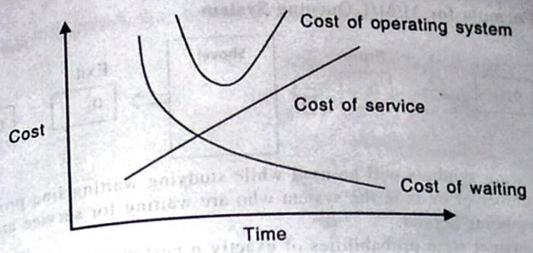
$$R_{k/n} = \sum_{j=k}^{n} = nC_j\, e^{-j\lambda t}\left(1-e^{\lambda t}\right)^{n-j}$$

The MTBF of the parallel system can be determined as

$$MTBF_{k/n} = \int_0^{\infty}\left[\sum_{j=k}^{n} nC_j\, e^{-jkt}\left(1-e^{-\lambda t}\right)^{n-j}\right] dt = \frac{1}{\lambda}\sum_{j=k}^{n}\frac{1}{j}$$

7.6 Waiting Line Problem

7.6.1 Introduction : This problem is applicable to optimize the combination of various machineries in a particular operation like number of dumper required for a single shovel so that the maximum utilization factor of the machineries can be achieved and the overall operating cost is reduced.



7.6.2 Characteristics of Queuing System : To characterize a queueing system we have to identify the probabilistic properties of the incoming flow of requests, service times, and service disciplines. The process is generally stochastic in nature. The components of in a queuing system are :

- **Arrival :** This denotes nature of arrival of the machineries. The arrival process can be characterized by the distribution of the interarrival times of the customers.
- **Service :** This indicates nature or capacity of service provided by the machine. The other random variable is the service time, sometimes it is called service request, work
- **Server :** This indicates nature of service facility. The structure of service and service discipline tell us the number of servers, the capacity of the system, that is, the maximum number of customers staying in the system including the ones being under service.

The most commonly used laws are

- ➢ FIFO - First In First Out: who comes earlier leaves earlier
- ➢ LIFO - Last Come First Out: who comes later leaves earlier
- ➢ RS - Random Service: the customer is selected randomly
- ➢ Priority : the customer is selected on priority basis

7.6.3 Kendall notation : Kendall originated a notation for queuing system as

A/ B / m / K / n / D

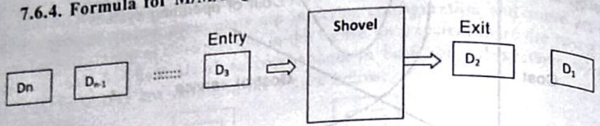
Where,

A : distribution function of the interarrival times,
B : distribution function of the service times,
m : number of servers,
K : capacity of the system, the maximum number of customers in the system including the one being serviced,
n : population size, number of sources of customers,
D : service discipline.

Note : Exponentially distributed random variables are notated by M, meaning Markovain. Furthermore, if the population size and the capacity are infinite, and the service discipline is FIFO, then they are omitted

Hence M/M/1 denotes a system with Poisson arrivals, exponentially distributed service times and a single server

7.6.4. Formula for M/M/1 Queuing System



The following symbols will be used while studying waiting line problem :

n = number of units in the system who are waiting for service and who are being served.

$P_n(t)$ = transient state probabilities of exactly n customers in the system at time t assuming that the system has started its operation at time zero.

P_n = steady state probability of having exactly n customers in the system.

λ = mean effective arrival rate (number of customers arriving per unit time)

μ = mean service rate per busy server (number of customers served per unit time)

c = number of parallel servers

W_s = expected waiting time per customer in the system

W_q = expected waiting time per customer in the queue

L_s = expected number of customers in the system

L_q = expected number of customers in the queue

ρ = proportion of time that the server is busy or traffic density

Under general conditions of arrival, departure and service discipline

$$\rho = \frac{\lambda}{\mu}$$

$$Ws = \frac{1}{\mu - \lambda}$$

$$Wq = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$Ls = \lambda * Ws = \frac{\lambda}{\mu - \lambda}$$

$$Lq = \lambda * Wq = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

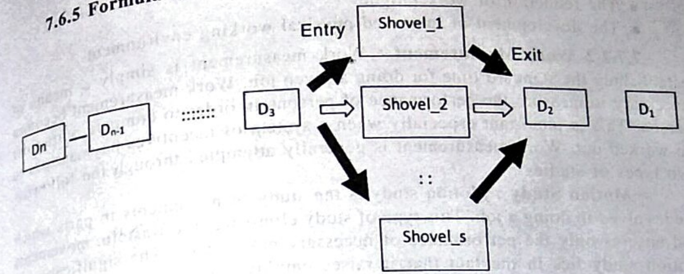
P_0 = Probability of 0 units in the system (that is, the service unit is idle)

$$= 1 - \frac{\lambda}{\mu}$$

$P_{n>k}$ = Probability of more than k units in the system (n = total number of units in the system)

$$= \left(\frac{\lambda}{\mu}\right)^k$$

7.6.5 Formula for M/M/S Queuing System



$$P_0 = 1 / \left\{ \sum_{n=0}^{S-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^S}{S!} \frac{1}{1-(\lambda/S\mu)} \right\}$$

$$L_q = P_0 \frac{(\lambda/\mu)^s}{S!} \frac{\rho}{(1-\rho)^2}$$

$$L_s = L_q + \frac{\lambda}{\mu} = P_0 \frac{(\lambda/\mu)}{S!} \frac{\rho}{(1-\rho)^2} + \frac{\lambda}{\mu}$$

$$W_q = \frac{L_q}{\lambda} = P_0 \frac{(\lambda/\mu)^s}{S!} \frac{\rho}{(1-\rho)^2} * \frac{1}{\lambda}$$

$$W_s = W_q + \frac{1}{\mu} = P_0 \frac{(\lambda/\mu)^S}{S!} \frac{\rho}{(1-\rho)^2} * \frac{1}{\lambda} + \frac{1}{\mu}$$

[s is the number of server]

7.7 Work Study

7.7.1 Introduction : Work study is a tool or technique of management involving analytical study of a job with a view to determine what exactly has to be done and the measurement of the work content of the job. Work study leads to the elimination of all wasteful, unnecessary and ineffective efforts; it establishes the best way of doing the given job by removing all unsatisfactory working conditions and standardizes the work method.

7.7.2 Basic Techniques of Work Study

7.7.2.1 Method Study : Method study is concerned with the way in which the work is performed. It may be defined as the systematic recording, analysis and critical examination of existing and proposed ways of doing work and is generally concerned with the development of easier and more effective methods.

Objectives of Method Study

- The improvement of procedures so that the time involved in doing a work may be reduced

- The improvement of work place layout
- The reduction of work fatigue
- The development of improved physical working environment

7.7.2.2 Work Measurement : Work measurement is simply a means of establishing the standard time for doing a given job. Work measurement becomes necessary to measure the performance of persons in order to compare output per person. This is important especially when a system of incentives payment has to be worked out. Work measurement is generally attempted through the following two types of studies :

➢ **Motion Study :** Motion study is the study of movements in parts which are involved in doing a job. This type of study eliminates the wasteful movements and ensures only the performance of necessary movements. The significance of motion study lies in the fact that it raises employee productivity and reduces fatigue.

➢ **Time Study :** Time study refers to the analysis and determination of the time necessary to perform a given task. Time study is a scientific and reliable method and provides a sound basis for introducing incentive scheme in mining industry.

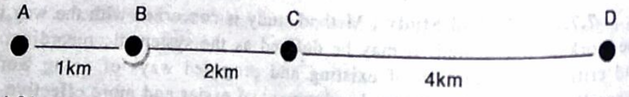
There are three methods of time study as follows :

- **Job time study :** this study is applied to set an allowed time for each piece of work turned out at a given standardized operation wherein the parts made are identical.
- **Operation time study :** This is one that applied to establish allowed times for an operation wherein the parts made vary in size and composition.
- **Production time study :** It is an all day study of a given task for the purpose of acquiring detailed job information.

EXERCISE - 7

1. For an LP problem, identify the incorrect statement
   a. Optimal point lies in one of the corner points
   b. Objective function is linear
   c. All the constraints are linear
   d. Optimal point lies in any one of the interior points of the feasible region.
   [GATE 2012]

2. Four mines A, B, C, and D are located along a road as shown with production in Mt per year 1, 2, 1 and 3 respectively. In order to handle total coal produced, the ideal distance of a coal washery along the road from the mine A in Km is



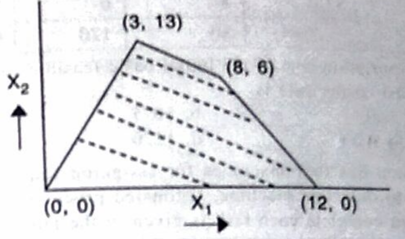
a. 4.01 b. 3.91
c. 3.81 d. 3.71 [GATE 2008]

3. The solution of the following system of linear equations is

$$x + 4y + 3z = 0$$
$$3x + 5y + 2z = 0$$
$$8x + 10y + 12z = 0$$

a. (0, 0, 0) b. (1, −1, 1)
c. (2, −1, −2) d. (−3, 0, 1) [GATE 2008]

4. The feasible region of an LP problem is shown as given below. The maximum value of the objective function $Z = 1600X_1 + 1200X_2$ is



a. 20400 b. 20000
c. 19200 d. 16800 [GATE 2008]

5. The feasibility region of an LP problem in variable x and y is given by the following constraints, in addition to the non- negativity constraints.

$y \geq 60;\ x \leq 90;\ x + y \leq 70$

The numbers of corner point feasible solutions for this problem are

a. 3 b. 4
c. 5 d. 6 [GATE 2010]

*6. There are 50 lemon trees in a reclaimed mine area. Each tree produces 800 lemons per year. For each additional tree planted in this area, considering all trees, the output number of fruits per tree drops by 10 lemons in a year. The number of trees that to be added to the existing reclaimed area in order to maximise the total number of lemons in the year is

a. 10 b. 15
c. 16 d. 26 [GATE 2009]

7. A balanced transportation problem is characterised by
   a. total supply exceeds total demand
   b. total demand exceeds total supply
   c. total demand is equal to total supply
   d. total supply is either equal to or more than total demand [GATE 2009]

*8. Consider the following linear programming problem :

$$\text{Maximize } z = 3x + 2y$$
$$\text{s.t. } 3x + 2y \geq 15$$
$$2x + 3y \leq 6$$
$$x \geq 0,\ y \geq 0$$

The above linear programming problem has

a. Unique optimal solution　b. Multiple optimal solution
c. Unbounded solution　d. Infeasible solution
[GATE 2009]

9. The unit cost of transportation of a balanced transportation problem is shown below :

Destination →		D1	D2	D3	Supply
Source s	S1	7	3	6	60
	S2	5	4	9	~~60~~ [illegible]
	S3	8	6	7	80
Demand		50	120	~~30~~ [illegible]	

The transportation cost of the initial basic feasible solution obtained by the North-West corner rule is
a. 1025　b. 1075
c. 1310 → 1130　d. 1226
[GATE 2010]

10. A workshop has four machines for assigning four jobs. Each job can be assigned to only one machine. Estimated processing cost (in Rs.) at each machine to complete each task is given in the table below

		Machine			
		M1	M2	M3	M4
Tasks	T1	61	92	52	72
	T2	42	49	69	85
	T3	47	59	80	71
	T4	65	70	68	72

The minimum value of total processing cost based on an optimal assignment schedules is
a. 214　b. 220
c. 215　d. 262

11. A company has three mines at locations A, B and C, which supply coal to power plants at D, E, F and G. Mines A, B, and C produce 1800 t/day, 1700 t/day, and 2900 t/day respectively. Daily raw coal requirements at the power plants D, E, F, and G from these mines are 2050 t, 1600 t, 1300 t, and 1450 t respectively. Unit transportation costs are as follows :

Mines ↓	Power Plants			
	D	E	F	G
A	3.5	4.0	2.5	5.5
B	4.5	3.7	6.0	4.7
C	4.2	3.8	5.2	2.9

The company has to minimize the total cost of transportation to the power plants. The problem is
a. an unbalanced transportation problem

b. a balanced transportation problem
c. an assignment problem
d. none of the above
[GATE 2002]

12. Unit cost of matrix of a transportation problem is given below in certain monetary units.

Destination / Source	1	2	3	
1	2	3 → 2	8	15
2	1	5	7	40
3	6	4	3	20
	10	25	40	Supply / Demand

The total cost of transportation based on the initial basic feasible solution obtained by the North-West corner rule is
a. 250　b. 290
c. 330　d. 360
[GATE 2008]

13. The optimal solution for the transportation problem has allocation as shown below : → above question

Destination / Source	1	2	3	
1		15		15
2	10	10	20	40
3			20	20
	10	25	40	Supply / Demand

When compared to initial basic feasible solution from the above, the optimal allocation results in savings of
a. 10　b. 20
c. 30　d. 40
[GATE 2008]

14. Which one of the following operation research problems is solved by the Branch and Bound algorithm?
a. transportation problem　b. assignment problem
c. integer programming problem　d. queuing problem

15. A mining machinery company has four plants at locations A, B, C, and D and three warehouses at destinations X, Y and Z. The number of machine units available at the plants A, B, C and D are 5, 8, 7 and 14 respectively. The demands at X, Y and Z are 7, 9 and 18 respectively. The unit costs of transportation in rupees as follows :

		Destinations		
		X	Y	Z
Locations	A	20	70	40
	B	30	30	10
	C	50	40	70
	D	10	60	20

The total cost of transportation in rupees based on the initial basic feasible solution obtained by the north-west corner rule is

a. 760 b. 800
c. 1020 d. 1110

16. In the above question, the total number of basic variables obtained based on the initial basic feasible solution is

a. 3 b. 4
c. 6 d. 7 [GATE 2006]

17. A mine workshop has 4 lathe machines and 4 tasks for completion. Each of the machines can perform each of the 4 tasks. Each task can be assigned to one and only one machine. Estimated cost in rupees to complete each task is given in the matrix below.

Machine / Tasks	M1	M2	M3	M4
T1	61	92	52	72
T2	42	49	69	85
T3	47	59	80	71
T4	65	70	68	72

The total optimum cost in Rupees for assigning the tasks to the machine is

a. 210 b. 215
c. 220 d. 286 [GATE 2009]

18. A mining company has three mines (M1, M2 and M3) that supply coal to three power plants (P1, P2 and P3). The three mines produce 900, 1000 and 1200 te of coal per day respectively. The power plant requirements from these three mines are 1200, 1000 and 900 te per day respectively. The unit cost of transporting coal from the three mines to the three power plants in Rs. is given below

		Power Plants		
		P1	P2	P3
Mines	M1	8	10	12
	M2	12	13	12
	M3	14	10	11

Based on the initial basic feasible solution, using Vogel's approximation method, the total transportation cost in Rs. is

a. 31200 b. 31400
c. 32800 d. 40000 [GATE 2012]

19. Three jobs A, B, and C are to be assigned to three machines X, Y and Z. the processing costs are given below :

		Machines		
		X	Y	Z
Job	A	19	28	31
	B	11	17	31
	C	12	15	13

The minimum total cost of assigning the jobs to the machines is

a. 60 b. 54
c. 51 d. 49 [GATE 2007]

20. In PERT analysis, the expected time to complete an activity is computed from the three time estimates : a most likely time (m), an optimistic time (a), and pessimistic time (b) using the expression

a. (a + 4m + b)/6 b. (4a + m + b)/6
c. (a + m + 4b)/6 d. (a + m + b)/3

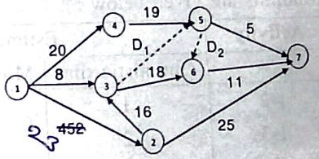
21. The probabilistic nature of activity times in PERT network is described by which one of the following distributions?

a. normal b. exponential
c. beta d. uniform

22. In PERT network, the activity duration is assumed to follow

a. Beta distribution b. Normal distribution
c. Binomial distribution d. Weibull distribution [GATE 2008]

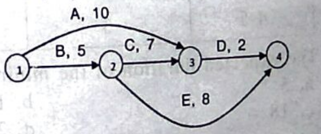
23. The following project network is given with arrows as activities and the numbers by each arrow representing job durations in days. The network has two dummy activities D_1 and D_2. The minimum completion time of the project in days is



23 ~~452~~

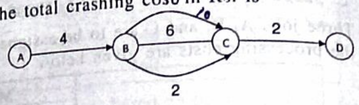
a. 13
b. 19
c. 75
d. 68

24. In the project network shown activity durations are given in months. If the activity can be crashed to expedite the project, which one would you crash and by how many months?



a. C, 1
b. C, 2
c. D, 1
d. E, 1

25. A project network comprises five activities as shown below. the activity durations, in days, are as indicated. Crashing of any activity costs Rs. 1000 per day. If the project is crashed to the shortest possible duration without changing the critical path the total crashing cost in Rs. is

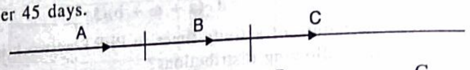


a. 5000
b. 4000
c. 3000
d. 2000

26. The three time estimates of PERT network are assumed to follow
a. exponential
b. normal
c. beta
d. gamma

27. Activity durations in PERT and CPM are decided on the following consideration
a. PERT – stochastic CPM - deterministic
b. PERT – stochastic CPM – stochastic
c. PERT – deterministic CPM – deterministic
d. PERT – deterministic CPM – stochastic

28. A mining project comprising of A, B, and C activities is scheduled for 90 days at a cost of Rs. 1200 million. The manager of the project decides to reduce the time for completion of the project to 85 days. The decision was taken after 45 days.



Activity	A	B	C
Duration (days)	40	15	35
Crashing cost/day (million rupees)	15	25	20

The minimum cost of the project in million rupees after crashing 5 days is
a. 100
b. 1300
c. 1475
d. 1825
[GATE 2008]

29. A mining project is composed of five activities whose three time estimates in months are given below :

Activity	Estimated duration (months)		
	Optimistic time	Most likely time	Permissible time
1-2	1	1	7
1-3	2	5	8
2-4	1	1	7
3-4	2	5	14
4-5	3	6	15

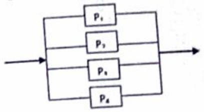
The expected duration of the mining project in months is
a. 5
b. 16
c. 18
d. 29
[GATE 2012]

30. In the above question, the standard deviation of the project length in months is
a. 2
b. 3
c. 6
d. 9
[GATE 2012]

31. In a PERT network, the activities on the critical path are a, b and c. The standard deviations of the durations of these activities are 2, 2 and 1 respectively. The variance of the project duration is
a. 3
b. 5
c. 9
d. 12
[GATE 2014]

32. In an opencast mine with shovel-dumper combination have 1 shovel and 6 dumpers. The reliability of the shovel is 0.9 and all the dumpers have equal reliability of 0.8. What is the system reliability?
a. 0.899
b. 0.576
c. 0.720
d. 0.160

33. A pumping station in a mine has 4 pumps, P_1, P_2, P_3 and P_4 which are connected functionally in parallel configuration as shown in the figure. The individual reliability of the pumps P_1, P_2, P_3 and P_4 are 0.80, 0.90, 0.85 and 0.95 respectively. For an effective dewatering operation in the mine denoting system success, at least three pumps should be in operation. The reliability of the system is



a. 0.19
b. 0.93
c. 0.99
d. 0.58
[GATE 2004]

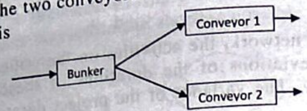
34. A system consists of four elements A, B, C and D which are connected functionally in a parallel configuration. The individual reliability of the elements is 0.80, 0.82, 0.85 and 0.90 respectively. The reliability of the system is
a. 0.498
b. 0.602
c. 0.750
d. 0.999
[GATE 2012]

35. The lifetime of a shovel is exponentially distributed with a mean life of 10 years. If the shovel has worked for 10 years in an opencast mine, the probability that it will be working for an additional 10 years is
a. 0.003
b. 0.135
c. 0.367
d. 0.500
[GATE 2006]

36. The failure data analysis of a longwall mining system consisting of three major subsystems revealed that the failure rate of shearer, face conveyor and stage loader are 0.15/ hour, 0.2/ hour and 0.1/ hour respectively. If all the subsystems must be running for a satisfactory system operation, the reliability of the longwall system for a period of 4 hours is
a. 0.165
b. 0.259
c. 0.405
d. 0.630
[GATE 2006]

37. A bunker-conveyor system as shown in the figure works under the following scheme :
i. Bunker and two conveyors running

ii. Bunker and either of the two conveyors running
If the bunker and the two conveyors have identical reliability of 0.9, the system reliability is



a. 0.729
b. 0.810
c. 0.999
d. 0.891

[GATE 2005]

38. A system of two identical mine pumps were to connected in series has reliability 0.49. If the pumps were to be connected in parallel, the system reliability would be

a. 0.21
b. 0.60
c. 0.91
d. 0.95

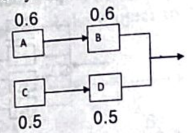
[GATE 2008]

39. The failure and the repair rates of a shovel are 0.06 hr-1 and 0.04 hr-1 respectively. The availability of the shovel in percentage is

a. 40 %
b. 60 %
c. 10 %
d. 80 %

[GATE 2014]

40. The individual reliability values of four sub-systems are given in the figure below. The reliability of the system is



a. 0.42
b. 0.22
c. 0.61
d. 0.52

[GATE 2014]

41. The variance of failure time (time to failure) of an electric motor in shovel is 1600 hr^2. If the failure time follows an exponential distribution, the expected mean time between failure in hr is

a. 40
b. 80
c. 800
d. 1600

42. The failure data of equipment follows an exponential distribution. If the mean time between failure is 3000 hours, the reliability of the equipment for 750 hours is

a. 0.24
b. 0.36
c. 0.40
d. 0.78

[GATE 2015]

43. In an opencast mine loading and transporting has been carried out by shovel dumper combination. There is only one shovel is in operation. Dumpers arrive at the rate of 10 dumpers per hour and it takes shovel on an average 5 minutes to load a dumper. Assume arrival is Poisson and service time is exponentially distributed. Find the proportion of time the shovel is busy.

a. 5/6
b. 7/8
c. 3/4
d. 1/2

44. In the above question, find the probability that the shovel is sitting idle.

a. 1/6
b. 1/8
c. 1/4
d. 1/2

---

45. In the above question, find the expected number of dumpers in the system.

a. 2
b. 3
c. 4
d. 5

46. In the above question, find the expected number of dumpers waiting to be load in the system.

a. 100/24
b. 100/12
c. 2
d. 4

47. In the above question what is expected waiting time of the dumper in the queue?

a. 5/7 hour
b. 5/ 12 hour
c. 12/ 7 hour
d. 1/2 hour

48. In the above question what is expected idle time of a dumper in the system?

a. 30 min
b. 60 min
c. 45 min
d. 15 min

49. In an opencast mine, dumpers arrive at a shovel location with an average arrival rate of 12 per/ hour. The inter-arrival times of the dumpers follow an exponential distribution. The service times of the shovel also follow an exponential distribution with an average of 3 minute. The average waiting time of a dumper in the queue in minutes is

a. 2.70
b. 6.00
c. 4.50
d. 7.50

[GATE 2005]

50. A queue of trucks at a crusher plant hopper is known to be M/M/1 queue. The probability that there is no truck to unload is 0.3. Due to rains the mean service time at the hopper is increased by 30%. As a consequence, the expected number of trucks in the queuing system (including the one possibly unloading) becomes

a. 10
b. 12
c. 14
d. 16

[GATE 2010]

51. In industrial work study, work sampling is a technique of

a. method study
b. work measurement
c. time study
d. motion study

[GATE 2005]

52. The time study data of an equipment deployed in a mine during a calendar month is given below.

Total working hours = 400
Total maintenance hours = 100
Total hours of actual work = 240
The percentage of utilization of the equipment is

a. 85
b. 80
c. 65
d. 60

[GATE 2011]

53. Workmen arrive at a mine workshop to receive tools for maintenance. The inter-arrival time of workmen at the service counter is exponentially distributed with an average time of 10 min. The service time at the counter is also distributed exponentially with a mean time of 6 min. The probability that there is a queue (more than one workman) at the service counter is

a. 0.24 b. 0.36

c. 0.40 d. 0.60

54. In the above problem, the average time spent by a workman waiting for his turn to be served in min is [GATE 2009]

a. 9 b. 12

c. 15 d. 18

55. ISO 9000 Quality Systems DO NOT contain

a. legal provisions b. measurement

c. document control d. standardization

[GATE 2011]

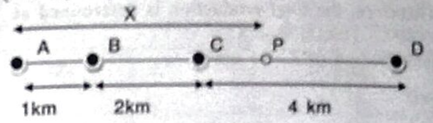
Answer

1. d	2. d	3. a	4. a	5. b	6. b	7. c
8. d	9. c	10. b	11. b	12. c	13. d	14. c
15. c	16. c	17. c	18. b	19. d	20. a	21. c
22. a	23. d	24. a	25. d	26. c	27. a	28. b
29. c	30. b	31. c	32. a	33. b	34. d	35. c
36. a	37. d	38. c	39. a	40. d	41. a	42. d
43. a	44. a	45. d	46. a	47. b	48. a	49. c
50. a	51. b	52. a	53. b	54. a	55. a	

Solutions of Exercise 7

2. Given data,

Coal produced in mines A, B, C, and D are M_a (= 1 te), M_b (= 2 te), M_c (= 1 te), and M_d (= 3 te) respectively.



Let the ideal distance of washery (P) from mine A is X to minimise the transportation cost.

Therefore, $M_a*d_a + M_b*d_b + M_c*d_c = M_d*d_d$

Where, d_a, d_b, d_c, and d_d are the distances of mines A, B, C, and D respectively from coal washery P.

$\Rightarrow$ $1*X + 2*(X-1) + 1*(X-3) = 3*(7-X)$

$\Rightarrow$ $7X = 26$

$\Rightarrow$ $X = 26/7 = 3.71$ km

4. The given objective function is

Max Z = 1600 X1 + 1200 X2

The value of objective function at four corner points

Z (0, 0) = 1600*0 + 1200*0 = 0

Z (12, 0) = 1600*12 + 1200*0 = 19200

Z (8, 6) = 1600*8 + 1200*6 = 20000

Z (3, 13) = 1600*3 + 1200*13 = 20400

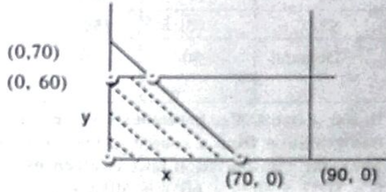
Therefore, Maximum value of Z is 20400

5. Given constraints :

$y \leq 60$

$x \geq 90$

$x + y \leq 70$



The feasible region of the LP problem is shown by dotted lines. The number of corner point feasible solutions for the given problem is 4.

6. Given data,

Existing number of trees = N = 50

Rate of lemon production = Q = 800 lemon per tree per year

Let the number of trees added to the existing number is X for maximize the total number of lemon production.

Give that change in lemon production with plantation of new trees is 10 lemon per tree per year

Therefore, the number of trees after plantation of new trees = $N' = (50 + X)$

And, rate of lemon production after plantation of new trees = $Q' = 800 - 10X$

Therefore, the total production is determined as

$Z = (800 - 10X)*(50 + X)$

For maximization of Z, we have

$$\frac{dz}{dx} = 0$$

$\Rightarrow (800 - 10X) * 1 + (50 + X) * (-10) = 0$

$\Rightarrow 800 - 10X - 500 - 10X = 0$

$\Rightarrow 20X = 300$

$\Rightarrow X = 15$

8. Given linear programming problem

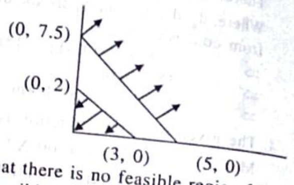
Maximize $Z = 3x + 2y$

s. t. $3x + 2y \geqslant 15$

$2x + 3y < 6$

$x \geq 0$

$y \geq 0$



The above diagram clearly indicates that there is no feasible region for the problem and thus the problem has infeasible solution.

9.

Destination / Source	D1	D2	D3	Supply
S1	(50) 7	(10) 3	(0) 6	60
S2	(0) 5	(60) 4	× (20) 9	60
S3	(0) 8	(50) 6	(30) 7	80
Demand	50	120	30	Supply / Demand

∴ In the above table, parenthesis value represents the material assigned for transportation from a source to the corresponding destination.

Thus, the transportation cost is given by

$C = 50*7 + 10*3 + 60*4 + 50*6 + \underset{\times}{\underline{20*9}} + 30*7$

$= 1310 \rightarrow 1130$

10. Given table :

		Machines			
		M1	M2	M3	M4
Tasks	T1	61	92	52	72
	T2	42	49	69	85
	T3	47	59	80	71
	T4	65	70	68	72

Let pi and qi be the minimum costs associated with row i and column j as defined in step I and II respectively in page 325 and 326.

The row minimums for step I are computed from the original cost matrix as shown below :

		Machines				Row
		M1	M2	M3	M4	Minimum
Task	T1	61	92	52	72	$P_1 = 52$
	T2	42	49	69	85	$P_2 = 42$
	T3	47	59	80	71	$P_3 = 47$
	T4	65	70	68	72	$P_4 = 65$

Next, subtract the row minimum from each respective row to obtain the reduced matrix is given below :

		Machines			
		M1	M2	M3	M4
Task	T1	9	40	0	20
	T2	0	7	27	43
	T3	0	12	33	24
	T4	0	5	3	7
Column Minimum		$q_1 = 0$	$q_2 = 5$	$q_3 = 0$	$q_4 = 7$

The application of step II yields the column minimum. Subtracting these values from the respective columns, we get the reduced matrix as given below :

		Machines			
		M1	M2	M3	M4
Task	T1	9	35	0	13
	T2	0	2	27	36
	T3	0	7	33	17
	T4	0	0	3	0

Step IV : Draw the minimum number of horizontal and vertical lines in the last reduced matrix that will cover all the zero entries.

Step V: If the number of lines is equal to the number of rows/column then optimal point reached. Since, the number of lines is less than the number of rows/column, select the smallest uncovered entry, subtract it from every uncovered entry, and then add it to every entry at the intersection of two lines.

		Machines			
		M1	M2	M3	M4
Task	T1	9	33	0	11
	T2	0	0	27	34
	T3	0	5	33	15
	T4	2	0	5	0

Now, the number of lines is less than the number of rows/column, thus optimality reached.

Thus, the assignment of tasks is as follows :

T1 → M3, T2 → M2, T3 → M1, T4 → M4

Cost of processing = 47 + 49 + 52 + 72 = 220

11.Given data :

Mines **Source**	**D**	**E**	**F**	**G**	**Supply (t/d)**
A	3.5	4.0	2.5	5.5	1800
B	4.5	3.7	6.0	4.7	1700
C	4.2	3.8	5.2	2.9	2900
Demand (t/d)	2050	1600	1300	1450	Supply Demand

We have, total supply from three mines

= 1800 + 1700 + 2900 = 6400 t/d

Total demand = 2050 + 1600 + 1300 + 1450 = 6400 t/day

Since, total supply = total demand, the problem is a balanced transportation problem

2. Given data :

Destination **Source**	**1**	**2**	**3**	**Supply**
1	**$(X_{11} = 10)$** $\boxed{2}$	**$(X_{12} = 5)$** $\boxed{2}$	$(X_{13} = 10)$ $\boxed{8}$	15
2	$(X_{21} = 0)$ $\boxed{1}$	**$(X_{22} = 20)$** $\boxed{5}$	**$(X_{23} = 20)$** $\boxed{7}$	40
3	$(X_{31} = 0)$ $\boxed{6}$	$(X_{32} = 0)$ $\boxed{4}$	**$(X_{33} = 20)$** $\boxed{3}$	20
Demand	10	25	40	Supply Demand

The initial basic feasible solution using North-West corner method is represented in above table.

In the above table, Xij represents the material transported from source i to destination j.

Let C_{ij} represents the cost of transportation of per unit material from source i to source j.

Thus, the solution is

$X_{11} = 10,\quad X_{12} = 5,\quad X_{13} = 0$

$X_{21} = 0,\quad X_{22} = 20,\quad X_{23} = 10$

$X_{31} = 0,\quad X_{32} = 0,\quad X_{33} = 10$

Therefore, total cost of transportation is given by

$$TC_I = \sum_{i,j=1\text{ to }3} X_{ij} * C_{ij}$$

$$= X_{11} * C_{11} + X_{12} * C_{12} + X_{13} * C_{13} + X_{21} * C_{21} + X_{22} * C_{22} + X_{23} * C_{23} + X_{31} * C_{31} + X_{32} * C_{32} + X_{33} * C_{33}$$

$$= 10*2 + 5*2 + 0*8 + 0*1 + 20*5 + 20*7 + 0*6 + 0*4 + 20*3$$

$$= 330$$

13. The Optimal solution of the above transportation problem is :

Destination **Source**	**1**	**2**	**3**	**Supply**
1	$(X_{11} = 10)$ $\boxed{2}$ 2	**$(X_{12} = 15)$** $\boxed{2}$	$(X_{13} = 0)$ $\boxed{8}$	15
2	**$(X_{21} = 10)$** $\boxed{1}$	**$(X_{22} = 10)$** $\boxed{5}$	**$(X_{23} = 20)$** $\boxed{7}$	40
3	$(X_{31} = 0)$ $\boxed{6}$	$(X_{32} = 0)$ $\boxed{4}$	**$(X_{33} = 20)$** $\boxed{3}$	20
Demand	10	25	40	Supply Demand

Total cost of transportation for optimal allocation is

$$TC_O = X_{11} * C_{11} + X_{12} * C_{12} + X_{13} * C_{13} + X_{21} * C_{21} + X_{22} * C_{22} + X_{23} * C_{23} + X_{31} * C_{31} + X_{32} * C_{32} + X_{33} * C_{33}$$

$$= 0*2 + 15*2 + 0*8 + 10*1 + 10*5 + 20*7 + 0*6 + 0*4 + 20*3$$

$$= 290$$

Therefore, the saving is

$$= TC_I - TC_O$$

$$= 330 - 290 = 40$$

15. The unit cost of transportation of machineries from a particular location to destination is given as :

		Destination		
		X	Y	Z
Location	A	20	70	40
	B	30	30	10
	C	50	40	70
	D	10	60	20

The availability of machines in various locations and their demand at various destinations are given as :

		Destination			Availability
		X	Y	Z	
Location	A	20	70	40	5
	B	30	30	10	8
	C	50	40	70	7
	D	10	60	20	14
	Demand	7	9	18	

In the above table, it is clear that sum of machine's availability is equal to sum of machine's demand. Therefore, the problem is a balanced transportation problem.

Let X_{ij} is the number of machine units transported from location i to destination j.

And, C_{ij} is the unit cost of transportation of machineries from a location i to destination j.

The initial basic feasible solution using north-west corner method can be determined as :

		Destination			Availability
		X	Y	Z	
Location	A	$X_{11} = 5$ $\boxed{20}$	$X_{12} = 0$ $\boxed{70}$	$X_{13} = 0$ $\boxed{40}$	5
	B	$X_{21} = 2$ $\boxed{30}$	$X_{22} = 3$ $\boxed{40}$	$X_{33} = 4$ $\boxed{70}$	7
	C	$X_{31} = 0$ $\boxed{50}$	$X_{32} = 3$ $\boxed{40}$	$X_{33} = 4$ $\boxed{70}$	7
	A	$X_{41} = 0$ $\boxed{10}$	$X_{42} = 0$ $\boxed{60}$	$X_{43} = 0$ $\boxed{20}$	14
Demaind		7	9	18	

Total cost of transportation (TC)

$= X_{11} * C_{11} + X_{21} * C_{21} + X_{22} * C_{22} + X_{32} * C_{32} + X_{3}3 * C_{33} + X_{43} * C_{43}$

$= 5*20 + 2*30 + 6*30 + 3*40 + 4*70 + 14*20$

$= 100 + 60 + 180 + 120 + 280 + 280 = 1020$

16. Total number of basic variables is equal to the total number of allocated cells. In the above table, number of allocated cell is 6 and hence number of basic variable is 6.

---

17. Given data :

Machines / **Task**	M1	M2	M3	M4
T1	61	92	52	72
T2	42	49	69	85
T3	47	59	80	71
T4	65	70	68	72

Let p_i and q_i be the minimum costs associated with row i and column j. The row minimums are computed from the original cost matrix as shown below :

M/c / **Task**	M1	M2	M3	M4	**Row Minimum**
T1	61	92	$\underline{52}$	72	$P_1 = 52$
T2	$\underline{42}$	49	69	85	$P_2 = 42$
T3	$\underline{47}$	59	80	71	$P_3 = 47$
T4	$\underline{65}$	70	68	72	$P_4 = 65$

Next, subtract the row minimum from each respective row to obtain the reduced matrix is given below :

Machines / **Task**	M1	M2	M3	M4
T1	9	40	$\underline{0}$	20
T2	0	$\underline{7}$	27	43
T3	0	12	33	24
T4	$\underline{0}$	$\underline{5}$	3	$\underline{7}$
Column minimum	$q_1 = 0$	$q_2 = 5$	$q_3 = 0$	$q_4 = 7$

The application of step II yields the column minimum as identified in above table. Subtracting these values from the respective columns, we get the reduced matrix as given below :

Machines / **Task**	M1	M2	M3	M4
T1	9	40	$\underline{0}$	20
T2	$\underline{0}$	2	27	36
T3	$\underline{0}$	7	33	17
T4	0	0	3	0
Column minimum	$q_1 = 0$	$q_2 = 5$	$q_3 = 0$	$q_4 = 7$

In the above tableau, the locations of the zero entries do not allow assigning unique chores to all the tasks.

Thus, draw the minimum number of horizontal and vertical lines in the last reduced matrix that will cover all the zero entries.

Machines **Task**	M1	M2	M3	M4
T1	9	35	0	13
T2	0	2	27	36
T3	0	7	33	17
T4	0	0	3	0

In the next step, selects the smallest uncovered entry, subtract it from every uncovered entry, and then add it to every entry at the intersection of two lines.

Machines **Task**	M1	M2	M3	M4
T1	9 +2 = 11	35	0	13
T2	0	2 −2 = 0	2 7−2=25	36−2=34
T3	0	7 − 2 = 5	3 3−2 = 31	17−2 = 15
T4	0 + 2 = 2	0	3	0

The resultant table is as :

M/c **Task**	M1	M2	M3	M4
T1	11	35	$\underline{0}$	13
T2	0	$\underline{0}$	25	34
T3	$\underline{0}$	5	31	15
T4	2	0	3	$\underline{0}$

The optimal allocation is T1 -> M3, T2 -> M2, T3 -> M1, T4 -> M4

Thus, the optimal cost = 52 + 49 + 47 + 72 = Rs. 220

18. The unit cost of transporting coal from three mines to the three power plants, the production in mines, and demand of power plants are given as :

		Power Plants			**Supply**
		P1	P2	P3	
Mines	M1	X_{11} $\boxed{8}$	X_{12} $\boxed{10}$	X_{13} $\boxed{12}$	900
	M2	X_{21} $\boxed{12}$	X_{22} $\boxed{13}$	X_{23} $\boxed{12}$	1000
	M3	X_{31} $\boxed{14}$	X_{32} $\boxed{10}$	X_{33} $\boxed{11}$	1200
Demand		1200	1000	900	

Applying Vogel approximation method for initial basic feasible solution, the penalty for each row and column can be determined as :

1st Iteration :

		Power plants			**Supply**
		P1	P2	P3	
Mines	M1	X_{11}=900 $\boxed{8}$	X_{12}=0 $\boxed{10}$	X_{13}=0 $\boxed{12}$	2
	M2	X_{21} $\boxed{12}$	X_{22} $\boxed{13}$	X_{33} $\boxed{12}$	0
	M3	X_{31} $\boxed{14}$	X_{32} $\boxed{10}$	X_{33} $\boxed{11}$	1
Demaind		1200	1000	900	
Penalty		4	0	1	

Penalty = Minimum cost of each row/column − Next minimum cost of corresponding row/column

The maximum penalty was found to be 4 for 1st column. Therefore, first allocation should be assigned to cell X_{11}

2nd Iteration

		Power plants			**Supply**	**Penalty**
		P1	P2	P3		
Mines	M2	X_{21} $\boxed{12}$	X_{22} =0 $\boxed{13}$	X_{23} $\boxed{12}$	1000	0
	M3	X_{31} $\boxed{14}$	X_{32} =1000 $\boxed{10}$	X_{33} $\boxed{11}$	1200	1
Demaind		1200	1000	900		
Penalty		2	3	1		

The maximum penalty was found to be 3 for 1st column. Therefore, next allocation should be assigned to cell X_{32}

3^{rd} Iteration

		Power plants		**Supply**	**Penalty**
		P1	P3		
Mines	M2	X_{21} $\boxed{12}$	$X_{23}=0$ $\boxed{12}$	1000	0
	M3	X_{31} $\boxed{14}$	X_{33} =900 $\boxed{11}$	1200	3
Demaind		1200	900		
Penalty		**2**	**1**		

The maximum penalty was found to be 3 for 2^{nd} row. Therefore, next allocation should be assigned to cell X_{33}. The remaining cells should be allocated for balancing the supply and demand.

Thus, the final allocation table is :

		Power Plants			**Supply**
		P1	P2	P3	
Mines	M1	$X_{11} = 900$ $\boxed{8}$	$X_{12} = 0$ $\boxed{10}$	$X_{13} = 0$ $\boxed{12}$	900
	M2	$X_{21} = 300$ $\boxed{12}$	$X_{22} = 0$ $\boxed{13}$	$X_{23}=700$ $\boxed{12}$	1000
	M3	$X_{31} = 0$ $\boxed{14}$	$X_{32} = 1000$ $\boxed{40}$	$X_{33}=200$ $\boxed{11}$	1200
Demaind		**1200**	**1000**	**900**	

Total transportation cost (TC)

$= X_{11} * C_{11} + X_{21} * C_{21} + X_{32} * C_{32} + X_{23} * C_{23} + X_{33} * C_{33}$

$= 900*8 + 300*12 + 1000*10 + 700*12 + 200*11$

$= 7200 + 3600 + 10000 + 8400 + 2200$

$= 31400$

19. Given table :

		Machines		
		X	Y	Z
Tasks	A	19	28	31
	B	11	17	31
	C	12	15	13

Let p_i and q_i be the minimum costs associated with row i and column j as defined in step I and II respectively

The row minimums for step I are computed from the original cost matrix as shown below :

		Machines			**Row Minimum**
		X	Y	Z	
Tasks	A	19	28	31	$\mathbf{P_1 = 19}$
	B	11	17	31	$\mathbf{P_2 = 11}$
	C	12	15	13	$\mathbf{P_3 = 12}$

Next, subtract the row minimum from each respective row to obtain the reduced matrix is given below :

		Machines		
		X	Y	Z
Job	A	0	9	12
	B	0	6	20
	C	0	3	1
Column Minimum		$q_1 = 0$	$q_1 = 0$	$q_1 = 0$

The application of step II yields the column minimum. Subtracting these values from the respective columns, we get the reduced matrix as given below :

		Machines		
		X	Y	Z
Job	A	0	6	11
	B	0	3	19
	C	0	0	0

Step IV : Draw the minimum number of horizontal and vertical lines in the last reduced matrix that will cover all the zero entries.

Step V : If the number of lines is equal to the number of rows/column then optimal point reached. Since, the number of lines is less than the number of rows/column, select the smallest uncovered entry, subtract it from every uncovered entry, and then add it to every entry at the intersection of two lines.

Step IV : Draw the minimum number of horizontal and vertical lines in the last reduced matrix that will cover all the zero entries.

Step V : If the number of lines is equal to the number of rows/column then optimal point reached. Since, the number of lines is less than the number of rows/column, select the smallest uncovered entry, subtract it from every uncovered entry, and then add it to every entry at the intersection of two lines.

	Machines			
		X	Y	Z
Job	A	0	3	8
	B	0	0	16
	C	3	0	0

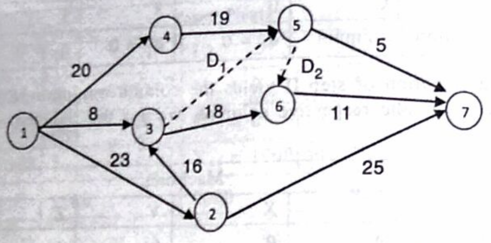
Now, the number of lines is less than the number of rows/column, thus optimality reached.

Thus, the assignment of tasks is as follows :

A → X, B → Y, C → Z

Minimum total cost of assigning the jobs = 19 + 17 + 13 = 49

23. Given network :

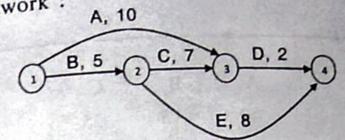


Minimum completion time of the project is the longest path of the network. The following paths can be identified from given network. The time required to complete the dummy paths is zero.

Sl. No.	Path	Time
1.	1-2-7	23 + 25 = 48
2.	1-2-3-6-7	23 + 16 + 18 + 11 = 68
3.	1-2-3-5-6-7	23 + 16 + 0 + 0 + 11 = 50
4.	1-2-3-5-7	23 + 16 + 0 + 5 = 44
5.	1-3-6-7	8 + 18 + 11 = 37
6.	1-3-5-6-7	8 + 0 + 0 + 11 = 19
7.	1-4-5-7	20 + 19 + 5 = 44
8.	1-4-6-7	20 + 19 + 0 + 5 = 44

Longest duration path is 1-2-3-6-7. Thus, minimum completion time of the project is 68 days.

24. Given projet network :



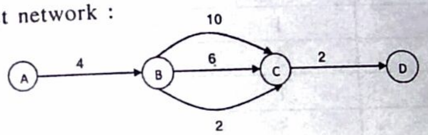
The project network has three paths. These paths and the corresponding time are as follows :

Sl. No.	Path	Time
1.	1-2-3-4	5 + 7 + 2 = 14
2.	1-3-4	10 + 2 = 12
3.	1-2-4	5 + 8 = 13

Therefore, the critical path is 1-2-3-4 and thus normal time required to finish the project is 14 months.

To expedite the project, the activity along the critical path should be crashed. The preferable crashing is 1 month for activity C. The time required to finish the project after crashing should not be less than any other non-critical paths.

25. Given project network :



The project network has three paths. These paths and the corresponding time are as follows :

Paths	Time
Path 1	4 + 10 + 2 = 16
Path 2	4 + 6 + 2 = 12
Path 3	4 + 2 + 2 = 8

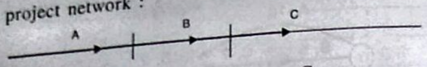
The normal time required to finish the project is the maximum duration paths. Therefore, normal time require finishing the project is 16 days.

To expedite the project, the activity along the critical path should be crashed first. The crashing possible in Path 1 is 4 days without changing the critical path.

Therefore, crashing cost is

= 1000 × 4 = 4000

28. Given project network :



Acivity	A	B	C
Duration (days)	40	15	35
Crashing cost/day	15	25	20

The duration of the project is crashed to 85 days from 90 days. The decision was taken after 45 days of working. Therefore, there is no scope of crashing the activity A. Again, the cost of crashing the activity C is less than activity B. Thus, to minimize the crashing cost of the project, activity C should be crashed by 5 days.

Cost of completion the project after crashing

= Normal cost of project completion + Crashed cost

= 1200 + 5*20 = 1300

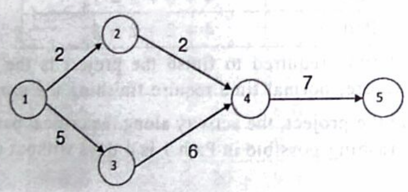
29. Three time estimates of five activities of the mining project is given as :

Activiy	Estimated duration (months)			
	Optimistic time (a)	Most likely time (m)	permissible time (b)	Expected time (t_e)
1-2	1	1	7	2
1-3	2	5	8	5
2-4	1	1	7	2
3-4	2	5	14	6
4-5	3	6	15	7

The expected duration for each activity can be determined as

= (a + b + 4m)/6

The project network can be drawn as :



Durations along various paths :

1-2-4-5 : 11 months

1-3-4-5: 18 months

Therefore, critical path is 1-3-4-5, and the expected duration of completion of the project is 18 months.

30. Standard deviation (s) of each path can be determined as = (b − a)/6

Activity	Estimated duration (months)			Expected time (t_e)	Standard deviation (σ^2)
	Optimistic time (a)	Most likely time (m)	Permissible time (b)		
1-2	1	1	7	2	1
1-3*	2	5	8	5	1
2-4	1	1	7	2	1
3-4*	2	5	14	6	2
4-5*	3	6	15	7	2
* indicate critical activities along critical path					

Variance of the project or critical path can be determine as

$$\text{Variance of the project} = \sum_i \text{Variance of critical activites} = \sum_i \sigma_i^2$$

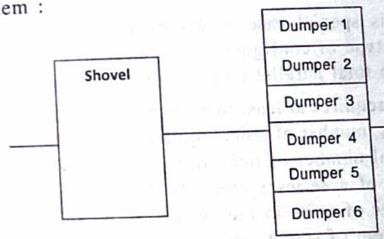
$$= 1 + 4 + 4 = 9$$

$$\text{Standard deviation } (\sigma) \text{ of the project} = \sqrt{\text{Variance of the project}} = \sqrt{9} = 3$$

31. Variance of the project $= \sum_i \text{Variance of critical activities} = \sum_i \sigma_i^2$

$= 4 + 4 + 1 = 9$

32. Given System :



Reliability of Shovel (R_s) = 0.9

Reliability of each dumper (R_d) = 0.8

In the given system, six dumpers are in parallel condition. It is assumed that the system will work in the following condition:

1: 1 Shovel works + 1 Dumper works

2: 1 Shovel works + 2 Dumpers work

3: 1 Shovel works + 3 Dumpers work

4: 1 Shovel works + 4 Dumpers work

5: 1 Shovel works + 5 Dumpers work

6: 1 Shovel works + 6 Dumpers work

Therefore, the reliability of the system can be determined as

$Rs = 0.9*6C_1*0.8^1*(1-0.8)^5 + 0.9*6C_2*0.8^2*(1-0.8)^4 + 0.9*6C_3*0.8^3*(1-0.8)^3 + 0.9*6C_4*0.8^4*(1-0.8)^2 + 0.9*6C_5*0.85*(1-0.8)^1 + 0.9*6C_6*0.86$

$= 0.0013824 + 0.013824 + 0.073728 + 0.221184 + 0.3538944 + 0.2359296$

$= 0.8999$

Or

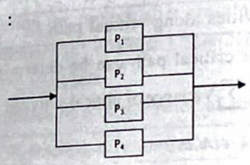
The equivalent reliability of dumpers can be determined as

$R_{de} = 1 - (1-R_d)^6 = 1 - (1-0.8)^6 = 1 - 0.000064 = 0.999936$

This equivalent systme works in series arrangement, and thus the equivalent reliability of the system is

$R = R_s * R_{de} = 0.9 * 0.999936 = 0.8999424$

33. Given System :



The reliabilities of four pumps are given as :

P1: $R_1 = 0.80$
P2: $R_2 = 0.90$
P3: $R_3 = 0.85$
P4: $R_4 = 0.95$

The above case is special case of k-out-of- n configuration of parallel redundancy. This type of configuration requires that at least k components succeed out of the total parallel components for the system to succeed.

The above system requires at least three pumps should work for functioning of the system. Thus, number of units required to keep the system functioning approaches the total number of units in the system, the system's behaviour tends towards that of a series system. If the number of units required is equal to the number of units in the system, it is a series system. In other words, a series system of statistically independent components is an n-out-of-n system and a parallel system of statistically independent components is a 1-out-of-n system.

The following operational combinations are possible for system success :

1. All 4 pumps operate.
2. Pump 1 fails while Pump 2, 3 and 4 continue to operate.
3. Pump 2 fails while Pump 1, 3 and 4 continue to operate.
4. Pump 3 fails while Pump 1, 2 and 4 continue to operate.
5. Pump 4 fails while Pump 1, 2 and 3 continue to operate.

The probability of success for the system (reliability) can now be expressed as :

$R_s = R_1*R_2*R_3*R_4 + (1-R_1)*R_2*R_3*R_4 + R_1*(1-R_2)*R_3*R_4 + R_1*R_2*(1-R_3)*R_4 + R_1*R_2*R_3*(1-R_4)$

This equation for the reliability of the system can be reduced to :

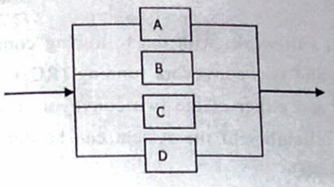
$R_s = R_2*R_3*R_4 + R_1*R_3*R_4 + R_1*R_2*R_4 + R_1*R_2*R_3 - 3*R_1*R_2*R_3*R_4$

$\Rightarrow R_s = 0.9*0.85*0.95 + 0.8*0.85*0.95 + 0.8*0.9*0.95 + 0.8*0.9*0.85 - 3*0.8*0.9*0.85*0.95$

$\Rightarrow Rs = 0.72675 + 0.646 + 0.684 + 0.612 - 1.7442 = 0.92455$

34. Reliabilities of four components are given as :

A : $R_1 = 0.80$
B : $R_2 = 0.82$
C : $R_3 = 0.85$
D : $R_4 = 0.90$



Since the arrangement is parallel. The system reliability can be determined as :

$Rs = 1- \{(1-R_1)*(1-R_2)*(1-R_3)* (1-R_4)\}$

$= 1 - \{(1-0.80)* (1-0.82)* (1- 0.85)* (1- 0.90)\}$

$= 1 - 0.2*0.18*0.15*0.10$

$= 1- 0.00054 = 0.99946$

35. Given data,

Mean life of the shovel (m) = 10 years

It is given that the lifetime of shovel is exponentially distributed. If the shovel is worked for 10 years, the probability that the shovel will work for additional t (= 10) years is given by

$P (t > 20/t > 10) = P (t > 10) = e^{-t/m}$

$\Rightarrow P = 2.718^{(-10/10)} = 0.367$

36. Given data,

Failure rate of shearer (λ_1) = 0.15/hr.

Failure rate of face conveyor (λ_2) = 0.20/hr.

Failure rate of stage loader (λ_3) = 0.10/hr.

It is suggested that all the subsystem must be running for a satisfactory system operation. Therefore, these three subsystems are in series configuration.

Reliability of the system for a period of t (= 4) hour satisfactory operation is given by

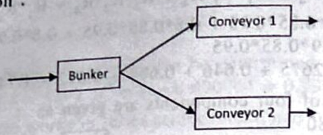
$Rs = e^{-(\lambda 1 + \lambda 2 + \lambda 3)t}$

$Rs = e^{-(0.15+0.20+0.10)*4} = e^{-1.8} = 0.165$

37. Given data :

Reliability of bunker and both the conveyor belts are equal. Let the reliability of bunker, and two conveyor belts are R1, R2, and R3 respectively.

Given configuration :



The system will works with the following condition:

⇒ Bunker and two conveyors running (RC_1)

⇒ Bunker and either of the two conveyors running (RC_2)

Therefore, reliability of the system can be determined as

$R_s = RC_1 + RC_2$

Where, R_I is the reliability for bunker and two conveyors running

R_{CI} is the reliability for bunker and either of the two conveyors running

⇒ $Rs = R_1*R_2*R_3 + \{R_1*(1-R_2)*R_3 + R_1*R_2*(1-R_3)\}$

⇒ $Rs = 0.9*0.9*0.9 + 0.9*(1-0.9)*0.9 + 0.9*0.9*(1-0.9)$

[Reliability of each component is equal and this is equal to 0.9]

⇒ $Rs = 0.729 + 0.081 + 0.081 = 0.891$

38. Let the reliability of each pump is R

For series configuration, the system reliability can be determined as

$Rs = R*R$

⇒ $0.49 = R^2$

⇒ $R = 0.7$

When the pumps are connected in parallel, the system reliability can be determined as

$R_{sI} = 1- \{(1-R)*(1-R)\} = 1 - \{(1-0.7)*(1-0.7)\} = 1- 0.09 = 0.91$

39. Given data,

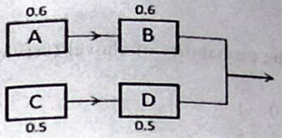
Failure rate of shovel (λ) = 0.06/hr.

Repair rate of shovel (μ) = 0.04/hr.

Availability of the shovel can be determined as

$$\text{Availability} = \frac{\text{Repair Rate}}{\text{Failure Rate} + \text{Repair Rate}} = \frac{\mu}{\lambda+\mu} = \frac{0.04}{0.06+0.04} = 0.4 = 40\%$$

40. Given network configuration :



Reliabilities of various subsystems are given as :

$R_a = 0.6,\ R_b = 0.6,\ R_c = 0.5,\ R_d = 0.5$

In the above network, subsystem A and B are in series. Equivalent reliability of A and B is

$R_{ab} = R_a*R_b = 0.6*0.6 = 0.36$

Similarly, subsystem C and D are in series. Equivalent reliability of C and D is

$R_{cd} = R_c*R_d = 0.5*0.5 = 0.25$

Further, the equivalent subsystems are in parallel arrangement. Therefore, the equivalent reliability of the system is

$Rs = 1- \{(1-R_{ab})*(1-R_{cd})\} = 1- \{(1-0.36)*(1-0.25)\} = 1- 0.64*0.75 = 1- 0.48 = 0.52$

41. Given data,

Variance of failure time of an electric motor (V) = 1600 hr^2

It is given that failure time follows exponential distribution.

For exponential distribution,

Variance $(V) = E(t^2) - \{E(t)\}^2 = 1/\lambda^2 = 1600$

⇒ $1/\lambda = 40$ hr.

Expected Mean time between failure is given by

$E(t) = 1/\lambda = 40$ hr.

42. Given data :

Mean time between failure (m) = 3000 hrs.

Failure data follows exponential distribution.

The reliability of the equipment for running t (=750 hrs.) hrs. is given by

$R = e^{-\lambda t} = e^{-t/m} = e^{-750/3000} = e^{-0.25} = 0.78$

43. Given data :

Arrival rate (λ) = 10/hr.

Service rate (μ) = 60/5 = 12/hr.

Given that the arrival pattern follows Poisson distribution and service time follows Exponential distribution.

Utilisation of shovel can be determined as

$$\rho = \frac{\lambda}{\mu} = \frac{10}{12} = \frac{5}{6}$$

44. In the above problem, the probability of shovel (service unit) is sitting idle can be determined as

$$\rho' = 1 - \rho = 1 = \frac{\lambda}{\mu} = 1 - \frac{10}{12} = \frac{1}{6}$$

45. Expected number of dumpers in system is given by

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{10}{12-10} = 5$$

46. Expected number of dumpers waiting to be load in the queue is given by

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{10-10}{12(12-10)} = \frac{100}{24}$$

47. Expected waiting time of the dumper in the queue is given by

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{10}{12(12-10)} = \frac{5}{12}\ \text{hr.}$$

48. Expected waiting/idle time of the dumper in the system is given by :

$$W_q = \frac{1}{(\mu - \lambda)} = \frac{10}{(12-10)} = \frac{1}{2}\ \text{hr} = 30\ \text{min}$$

49. Given data :

Arrival rate (λ) = 12/hr.

Average service time (t) = 3 min

Service rate (μ) = 60/3 = 20 /hr.

Expected waiting time of the dumper in the queue is given by

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{12}{20(20-12)} = \frac{12}{160}\ \text{hr.} = 12 * \frac{60}{160}\ \text{min} = 4.5\ \text{min utes}$$

50. Given data :

Probability that there is no truck to unload (that is, server is sitting idle) is 0.3

Let the service rate is μ, and the arrival rate is λ

Therefore, in normal condition

The probability of service unit is sitting idle is given by

$$\rho' = 1 - \frac{\lambda}{\mu} = 0.3$$

$$\Rightarrow \frac{\lambda}{\mu} = 0.7$$

It is given that the service time is increased by 30% due to rains. Therefore, the new service rate is μ' ($=\mu/1.3$).

The arrival rate remains constant in new condition.

The expected number of trucks in the queuing system in new condition is given by

$$L_s = \frac{\lambda}{\mu' - \lambda} = \frac{\lambda}{\mu/1.3 - \lambda} = \frac{1.3(\lambda/\mu)}{1 - 1.3(\lambda/\mu)} = \frac{1.3*0.7}{1 - 1.3*0.7}$$

$$= 10.1 \approx 10$$

52. Given data :

Total working hours (t_w) = 400

Total maintenance hours (t_m) = 100

Total hours of actual work (t_a) = 240

Percentage of utilization of the equipment is given by

$$\eta = \frac{t_a + t_m}{t_w} * 100\% = \frac{240+100}{400} * 100\% = 85\%$$

53. Given data :

Average arrival time of workmen (t_a) = 10 min

Average service time (t_s) = 6 min

Therefore, Arrival rate (λ) = 60/10 /hr. = 6 /hr.

Service rate (μ) = 60/6 /hr. = 10 /hr.

The probability of more than K workman at the service counter is given by

$$P_{n>k} = \left(\frac{\lambda}{\mu}\right)^{k+1}$$

For K = 1,

$$P_{n>1} = \left(\frac{\lambda}{\mu}\right)^{1+1} = \left(\frac{6}{10}\right)^2 = 0.36$$

54. In the above problem, the average time spent by a workman for his turn to be served is equal to the average time spent in the queue.

Average time spent by the workman in the queue is given by

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{6}{10(10-6)} = \frac{6}{40}\ \text{hr.} = 6 * \frac{60}{40}\ \text{min utes} = 9\ \text{min utes}$$

■

8.0 Mine Economics

8.1 Definiiton

Economics is the social science that analyzes the production, distribution, and consumption of goods and services as needed for mankind/ the efficient allocation of scarce resources.

Mineral Economics is the study of the business and economic aspects of natural resource extraction and use.

Micro Economics – The study of the economic behavior of individual units of an economy (such as a person, household, firm, or industry) and not of the aggregate economy.

Macro Economics – The study of the behavior of the whole (aggregate) economies or economic systems instead of the behavior of individuals, individual firms, or markets.

Natural Resource Economics (including Mineral Economics) – The study of economics as it applies to the natural resource industry and environment, including Micro and Macro Economics.

8.2 The Essence of Economics : The essence of economics is Supply, Demand, Price, and Price equilibrium.

Supply - A fundamental economic concept that describes the total amount of a specific good or service that is available to consumers. Aggregate supply is the supply from all suppliers. The law of supply states that the higher the price of the good, the higher the quantity supplied to the market.

Demand - An economic principle that describes a consumer's desire and willingness to pay a price for a specific good or service. Aggregate demand is the demand from all consumers. The law of demand states that, if all other factors remain equal, the higher the price of a good, the less people will demand that good.

Price and Price Equilibrium : A value that will purchase a finite quantity, weight, or other measure of a good or service. The equilibrium price is the price where the goods and services supplied by the producer equal the goods and services demanded by the customers. When supply & demand are equal, the economy is said to be at equilibrium. At this point, the allocation of goods is optimum because the amount of goods being supplied is exactly the amount of goods being demanded.

8.3 Appraisal Techniques : There are several appraisal techniques for judging the profitability of new investments in assets. They may be classified as Non-discounting and Discounting techniques. These are as follows:

8.3.1 Non-discounting Techniques : The non-dicounting techniques do not consider the rate of interest or money devaluation with time. These are of various types as :

- **Payback period (PBP) :** The payback period is the number of years required to recover the money invested in a project. It can be calculated

by dividing the Intial investment to average annual cash flow in the project.

$$\text{Payback period (PBP)} = \frac{\text{Initial Investment}}{\text{Constan t annualcash flow in a project}}$$

For example, a company invests Rs. 400,000 in more efficient equipment. The cash savings from the new equipment is expected to be Rs. 100,000 per year for 10 years. **The payback period is 4 years** (Rs. 400,000 divided by Rs.100,000 per year).

- Average rate of return (ARR) : The **ARR** calculation may have divided a division's operating income by the average amount of operating assets being utilized by the division.

A drawback of ARR is that the accounting amounts ignore the time value of money.

8.3.2 **Discounting techniques :** The dicounting techniques consider the rate of interest or money devaluation with time. These are of various types as :

- **Net Present Value (NPV) :** This is the present value of each of the individual cash flows. The net present value assuming a minimum acceptable discount rate of i% is

$$NPV = \frac{X_0}{(1+i)^0} + \frac{X_1}{(1+i)^1} + \frac{X_2}{(1+i)^2} + \frac{X_3}{(1+i)^3} + \text{..........} \frac{X_n}{(1+i)^n}$$

Where, X_0, X_1, X_2, $X_3 \ldots X_n$ are the cash flows in the year of 0, 1, 2, 3.....n respectively.

- **Profitability Index (P.I) :** The profitability index (also known as benefit/cost ratio) is the ratio of the present value of the future net cash flows to the initial outlay of the project. This index provides a relative measure for judging desirability and evaluating the worth of an investment proposal. It can be written as

$$PI = \frac{\sum_{r=1}^{n} \frac{CF_i}{(1+i)^r}}{I_0}$$

Where, CF_i = net cash flow in time period i
i = rate of discount
n = the project's expected life
I_0 = initial cash outlay

The accept-reject criterion in case of P.I. is as under:
Accept the project if it's P.I. > 1
Reject the project I its P.I < 1

- **Rate of Return on an Investment :** The interest rate 'i' which makes the future worth equal [Case 1:Rs. X is invested for n number of years; Case 2: Annual profit after tax of Rs. Y will be generated through initial investment of Rs. X] is called the rate of return (ROR) on an investment.

Where, i is rate of return on an investment

For case 1 : $FW = X(1+i)^n$

For case 2 : $FW = Y\left[\frac{(1+i)^n - 1}{i}\right]$

In this case, $X(1+i)^n = Y\left[\frac{(1+i)^n - 1}{i}\right]$

Where, i is rate of return on an investment.

- **Internal Rate of Return (IRR) :** Discounted cash flow rate of return (DCFROR) or internal rate of return (IRR) may be defined as the discount rate that equates present value of the project's future net cash flows with the projects initial cash outlay. In other words, we can say that the net present value at the end of the project will be equal to zero.
Thus in the above NPV equation, if i is the DCFROR or IRR then NPV = 0.
- **Payback Period :** Time required to repay the loan with equal yearly payments (annuity) called the payback period.
The loan has been repaid when the net present value (NPV) is equal to zero.

$$NPV = -X + A * \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 0$$

Where, i = interest rate compounded annually
n = payback period
X = loan amount
A = annual payment

8.4 Determination of Future Worth : If the future worth is determined on the basis of non-discounting factors then, it will be simply the aritmatic sum of all the cash flow. On the other hand, if it is determined on the basis of discouting factors, this can be determined as:

- **For Annual Compound Interest Rate 'i' :**
Future worth = FW = PV$(1 + i)^n$
Where, FW = Future worth after nth year
i = interest rate compounded annually
n = number of years
- **For Half Yearly Compound Interest Rate 'i' :**
Future Worth = FW = PV $(1+ i/2)^{2n}$
Where, FW = Future worth after nth year
i = interest rate compounded half yearly
n = number of years
- **For Equal Yearly Payments (annuity) :**

$$FW = A * \left[\frac{(1+i)^n - 1}{i}\right]$$

Where, FW = future worth after n^{th} year
i = interest rate compounded annually
n = number of years
A = yearly payments

8.5 Depreciation : The amount of wear and tear (of a property etc.) for which a reduction may be made in a valuation. Standard straight line depreciation yields a yearly depreciation value as

Yearly Depreciation Value = $\frac{\text{Investement} - \text{salvage value after completion of the project}}{\text{Life of the project}}$

8.6 Redemption : Recover the capital by expenditure of effort or by a stipulated payment.

8.7 Depletion : When natural resources are being consumed in producing products or services, the term depletion is used to indicate the decrease in value of the resource base that has occurred. The term is commonly used in connection with non-renewable resources like mineral, oil etc. The basic difference between the depreciation and depletion is that in the case of depreciation the property involved usually may be replaced with similar property when it has become fully depreciated whereas in the case of depletion of mineral resources such replacement usually is not possible.

There are two ways to compute depletion allowances :

- The cost method
- The percentage method

EXERCISE - 8

1. Payback period is the time required
   a. For the cash income from a project to get back the initial cash investment
   b. From the start of the project to the time to recover the total initial investment
   c. From the start of the project to the start of the production
   d. To the period during which internal rate of return is generated
[GATE 2010]

2. What will be the effective rate of interest for nominal rate of interest 10 %, when money is compounded quarterly?
   a. 10.381% b. 09.381%
   c. 11.381% d. 12.381%

3. The cash flow of a project is given as follows :

Year	0	1	2	3	4
Cash flow (Rs.)	–5000	15000	15000	20000	22500

The internal rate of return (IRR) in percent lies between
   a. 15 and 16 b. 14 and 15
   c. 16 and 17 d. 13 and 14
[GATE 2002]

4. A mining company is considering the following project :
Cash Flows (Rs) for five years

Year	0	1	2	3	4	5
Cash flow (Rs.)	–50000	+11300	+12769	14428	+16305	+18421

What will be the NPV for the project if the cost of capital is 10%.

a. Rs. 4240.12
b. Rs. 4554.90
c. Rs. 4207.12
d. Rs. 4449.90

5. A firm, having a cost of capital of 12 %, evaluates a small mining project of initial investment of Rs. 100 lakhs. For the benefits shown in the table below, the benefit-cost ratio of the project is

Year	1	2	3	4
Benefits (Rs. in lakhs)	25	40	40	50

a. 0.985
b. 1.145
c. 1.384
d. 1.550
[GATE 2005]

6. At a discount rate of 12 %, uniform annual savings producing Rs. 6 lakhs at the end of 10 years is
a. Rs. 105740
b. Rs. 52850
c. Rs. 19320
d. Rs. 34190
[GATE 2001]

7. Four alternative investment opportunities are available in a mining project each offering different revenues for the next two years as shown in the table. If the interest rate is 10 % annually, which alternative provides the best investment opportunity based on the net present value?

	Case I	Case II	Case III	Case IV
Initial Investment (Rs.)	40,000	1,00,000	80,000	1,20,000
Return in Year 2	30,000	75,000	65,000	80,000
Return in Year 3	30,000	75,000	65,000	80,000

a. I
b. II
c. III
d. IV
[GATE 2004]

8. The probability distribution of the outcome of an investment is

Outcome (Rs.)	1000	700	300
Probability	0.2	0.5	0.3

The measure of risk of the investment based on standard deviation (σ) is given as
a. 183.20
b. 204.00
c. 249.80
d. 260.82
e. 498.80
[GATE 2004]

9. Hoskold's formula is used for
a. Salting
b. Speculative rate of return
c. Depreciation
d. Ore reserve calculation

10. Sum of the years digit method is used for
a. Salting
b. Speculative rate of return
c. Depreciation
d. Ore reserve calculation

11. Using Hoskold's formula, the present worth, in rupees of annuity of Rs. 1 per year for the 10 years after the deferment period of 5 years and allowing the purchaser 10 % interest is

a. 3.81
b. 5.22
c. 5.57
d. 8.41

12. A company producing bearing plates for rock bolts has the selling price Rs. 20/unit and the variable cost has Rs. 12/unit. If the total fixed cost is Rs. 5,60,000, then the break even output units of bearing plates is
a. 17,500
b. 28,000
c. 45,500
d. 70,000

13. A company producing bearing plates for rock bolts has the selling price Rs. 20/unit and the variable cost has Rs. 12/unit. If the total fixed cost is Rs. 5,60,000, then the profit earned for an output of 1,00,000 units of bearing plates is
a. Rs. 2,40,000
b. Rs. 6,40,000
c. Rs. 8,00,000
d. Rs. 14,40,000

14. A company producing bearing plates for rock bolts has the selling price Rs. 20/unit and the variable cost has Rs. 12/unit. If the total fixed cost is Rs. 5,60,000, then for a target profit of Rs. 4,00,000, the total output units of bearing plates is
a. 30,000
b. 50,000
c. 1,20,000
d. 1,60,000

15. A bank lends a mine owner Rs. 1, 00,000 today to purchase hand-held drills. If a note of agreement is signed to pay the bank Rs. 1,76,234 at the end of 5 years, the rate of interest in percent compounded yearly on the loan becomes
a. 6
b. 8
c. 10
d. 12

16. The net present value of Rs. 400 at the end of 5 years at 10 % discount rate is
a. Rs. 273
b. Rs. 226
c. Rs. 248
d. Rs. 465
[GATE 2000]

17. The uniform end of year payment, R, which can be realized for n years from a single present investment, P, at i rate of interest is

(a) $R = P(1+i)^n\left[\dfrac{i}{(1+i)^n - 1}\right]$

(b) $R = P(1+i)^n\left[\dfrac{(1+i)^n - 1}{i}\right]$

(c) $R = P\left[\dfrac{(1+i)^n - 1}{i}\right]$

(d) $R = \dfrac{P}{(1+i)^n}\left[\dfrac{i}{(1+i)^n - 1}\right]$

[GATE 1999]

18. A company takes a loan of Rs. 25,000 at 8 % interest for the extension of a mine. To repay the loan, an additional saving of Rs. 2500 per annum is necessary. The period taken for repayment is
    a. 25 years　　b. 28 years
    c. 21 years　　d. 30 years　　[GATE 1999]

19. As per DCF analysis a project report is acceptable if
    a. Discounting rate >IRR　　b. Discounting rate < IRR
    c. Discounting rate = IRR
    d. There exists no relation between the two　　[GATE 1998]

20. The present value of a 4 year annuity of Rs. 10,000 discounted at 10 % is
    a. 35,600　　b. 33,500
    c. 32,400　　d. 31,700　　[GATE 1998]

21. For an open pit operation the value of metal is Rs. 210/kg, and recoverable grade is 1.2 %. Production cost per tonne of ore inclusive of mining and processing but excluding stripping is Rs. 2000. If the break even stripping ratio is 3.49 m^3/te, then the stripping cost is (1te = 1000 kg)
    a. Rs. 149 /m^3　　b. Rs. 140 /m^3
    c. Rs. 72 /m^3　　d. Rs. 60 /m^3　　[GATE 1997]

22. For a mine production t per year, the total cost of production is given by $(at^2 + b)$. The revenue from sale is given by ct. If a, b, and c, are constants, the breakeven value of t is
    a. $\left[c \pm \sqrt{(c^2 - 4ab)}\right]/(2a)$　　b. $\left[\sqrt{(c^2 - 4ab)}\right]/(2a)$
    c. $\left[-c \pm \sqrt{c^2 - 4ab}\right]/(2a)$　　d. $\left[c \pm \sqrt{c^2 - 4ab}\right]/(2a)$　　[GATE 2010]

23. The net present value in Rs. of a 3 year annuity of Rs. 10,000 discounted at 10 % is
    a. 9,091　　b. 17,355
    c. 24,869　　d. 26,446

24. A company invested Rs. 4 lakhs in a machine. The net income expected from the operation of the machine is Rs. 80,000 per annum. The payback period for the machine in years is
    a. 5　　b. 6
    c. 7　　d. 8

25. An O.C. mine requires 16 blocks of overburden of 3000 te each to mine 4 blocks of coal of 2000 te each. The mining cost of both overburden and coal is Rs. 50 per tonne. The cost of transportation of overburden to dump areas is Rs. 50 per tonne. Coal transportation cost to the selling point is Rs. 75 per tonne. The price of coal is Rs. 800 per tonne. If the royalty and tax to the state is Rs. 50 per tonne, the nominal profit in rupees is
    a. 2,00,000　　b. 20,00,000
    c. 30,00,000　　d. 50,00,000　　[GATE 2005]

26. A person borrows a sum of Rs 50,000 @ 6 % interest rate for 10 years. Determine the equal annual sum to be paid at the end of each year to repay the loan.
    a. 6795　　b. 5672
    c. 6749　　d. 6592

27. An equipment cost Rs. 8,00,000 today and has a service life of 15 years. The salvage value at the end of 15 years is Rs. 2,00,000. What will be the value at the end of 10 years?
    a. 4,00,000　　b. 6,60,000
    c. 4,50,000　　d. 5,00,000

28. An investment of Rs. 10,000, compounded annually, is estimated to return Rs. 20,000 after 6 years from the date of investment. The expected rate of return on this investment in percentage is
    a. 8.75　　b. 10.50
    c. 12.25　　d. 16.6　　[GATE 2012]

29. What will be the profitability index with discount rate of 12 % for the cash flows given below?

Year	0	1	2	3	4	5
Net Cash Flow in (Rs.)	–40000	15000	14000	13000	12000	11000

    a. 1.192　　b. 2.186
    c. 2.193　　d. 0.292

30. The ABC copper company recently bought an ore-bearing parcel of land for Rs. 2,000,000. The recoverable reserves in the mine were estimated to be 5,00,000 tons. If 75,000 tons of ore were mined during the first year and 50,000 tons sold, what was the depletion allowance for one year?
    a. Rs. 2,00,000　　b. Rs. 2,50,000
    c. Rs. 3,00,000　　d. Rs. 4,00,000

31. The probability distribution of the outcome of an investment is

Outcome (Rs.)	1000	700	300
Probability	0.2	0.5	0.3

    The expected rate of return is
    a. 540　　b. 640
    c. 740　　d. 840

32. A mining industry has the following total cost and total revenue functions:
    Total cost = TC = $50 + 100q - 6q^2 + 1/3\ q^3$
    Total revenue = TR = $64\ q$
    Where q is the output of mine in tonne/day
    The profit will be maximized for what level of production?
    a. 4　　b. 6
    c. 8　　d. 10

33. An investment at 10% yearly interest rate, compounded quarterly, accumulates to a sum of Rs. 120,000 in 5 years. The present value of the sum in rupees is

a. 73,233
b. 74,511
c. 88,232
d. 106,063
[GATE 2012]

34. The total cost C (lakh rupees) of a longwall face of length Lin m is given by the equation C= 0.1L+ (1562.5/L) + 300. Length of the face in m for the minimum total cost is

a. 40
b. 125
c. 156
d. 300
[GATE 2011]

35. A coal mine receives two bids for purchase of a new dragline. The first bid quotes Rs. 150 crore as a price to be paid in full on delivery. The second bid quotes Rs. 180 crore as a price payable at the end of the third year after delivery. If the discount rate is 12%, the difference in NPV between the first and second bids in crore of rupees is

a. 19.68
b. 21.88
c. 31.88
d. 20.68
[GATE 2014]

36. A mining equipment has a life of 5 years with no salvage value. Assuming that the depreciation of the equipment is calculated by the straight line method, the average annual value of the equipment percentage of its original value is

a. 20
b. 40
c. 50
d. 60
[GATE 2009]

37. The cash flow table of a manganese mine for a particular year is shown below :

Item Amount	(Rs. in lakhs)
Revenue	900
Cost (other than depreciation)	300
Depreciation	100
Profit before tax	500

If the corporate tax is 50 % of the profit before tax, operating cash inflow in lakhs of rupees is

a. 400
b. 350
c. 250
d. 200
[GATE 2009]

38. In a mine site, the cost of shaft sinking in lakhs of rupees is given as 2.64D + 34.8, where D is the shaft depth in m. In the same site, the corresponding cost of driving an incline is 0.96 L, where L is the length of the incline in m. Assuming L by D ratio is 3.0, the depth in m beyond which the shaft sinking becomes more economical is

a. 43
b. 48
c. 145
d. 155
[GATE 2009]

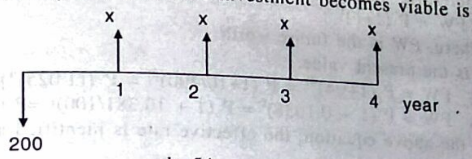
39. The following information is provided for an ore deposit :

Number of waste blocks = 10
Number of ore blocks = 5
Volume of each waste block, m^3 = 600
Total cost of waste handling per m^3 = Rs. 100
Tonnage of each ore block = 400
Total cost of ore handling per ton = Rs. 150
Sale price of ore per ton = Rs. 500

The net cash flow of mining the deposit in lakhs of rupees, is

a. 3.4
b. 2.5
c. 1.0
d. 0.8
[GATE 2009]

40. A cash flow diagram is shown below. based on NPV, at 10 % rate of interest, the minimum annuity x at which the investment becomes viable is



a. 63
b. 54
c. 42
d. 35
[GATE 2008]

41. A mining company is considering the following project :

Cash Flows (Rs) for five years

Year	0	1	2	3	4	5
Net Cash Flow in (Rs.)	–50000	15000	15000	15000	15000	15000

What is the project's IRR?

a. 10 %
b. 11 %
c. 12 %
d. 13 %

Answer

1. b 2. a 3. a 4. a 5. b 6. d 7. c
8. e 9. b 10. c 11. a 12. d 13. a 14. c
15. d 16. c 17. a 18. c 19. b 20. d 21. a
22. a 23. c 24. a 25. a 26. a 27. a 28. c
29. a 30. c 31. b 32. b 33. a 34. b 35. b
36. a 37. b 38. c 39. c 40. a 41. d

Solution of Exercise 8

2. Given that,

Nominal rate of interest (i) = 10 %

Let the effective rate of interest is r, when money is compounded quarterly.

When compounded quarterly,

Resultant Interest rate (i') = i/4

Resultant time (t') = 4t

Therefore,

$$FW = P\,(1+i')^{t'}$$

Where, FW is the future worth.

P is the present value.

$\Rightarrow$ $FW = P\,(1+i/4)^{4t} = P\,(1+10/400)^{4t} = P\,\{(1.025)^4\}^t = P\,(1.10381)^t$

$\Rightarrow$ $FW = P\,(1 + 0.1038)^t = P\,(1 + 10.381/100)t = P\,(1 + r/100)^t$

In the above equation, the effective rate is identified as 10.381%

3. The cash flow of a project is given as :

Year	0	1	2	3	4
Cash Flow in (Rs.)	–50000	15000	15000	20000	22500

Let the internal rate of return is i%. At internal rate of return, the net cash flow will be equal to zero.

It is easier to do the back calculation rather than solving the fourth degree equation.

$$\text{Net Cash Flow} = -50000+\frac{15000}{(1+i)}+\frac{15000}{(1+i)^2}+\frac{15000}{(1+i)^3}+\frac{15000}{(1+i)^4}$$

$$\Rightarrow\ 0 = -50000+\frac{15000}{(1+i)}+\frac{15000}{(1+i)^2}+\frac{15000}{(1+i)^3}+\frac{15000}{(1+i)^4}$$

$$\Rightarrow\ \frac{15000}{(1+i)}+\frac{15000}{(1+i)^2}+\frac{15000}{(1+i)^3}+\frac{15000}{(1+i)^4}=50000$$

$$\Rightarrow\ \frac{15}{(1+i)}+\frac{15}{(1+i)^2}+\frac{15}{(1+i)^3}+\frac{22.5}{(1+i)^4}=50$$

It is easier to do the back calculation rather than solving the fourth degree equation.

For i = 13 %, Left hand side (LHS) = 13.27 + 11.75 + 13.86 + 13.80 = 52.67

For i = 14 %, LHS = 13.16 + 11.54 + 13.50 + 13.32 = 51.52

For i = 15 %, LHS = 13.04 + 11.34 + 13.15 + 12.86 = 50.39

For i = 16 %, LHS = 12.93 + 11.14 + 12.81 + 12.42 = 49.30

Therefore, IRR lies between 15 and 16 percent.

4. The cashflow data are :

Year	0	1	2	3	4	5
Cash Flow in (Rs.)	–50000	+11300	+12769	+14428	+16305	+ 118421

Intereset rate (i) = 10 %

The NPV for the project can be determined as

$$NPV = -50000+\frac{11300}{(1+i)}+\frac{12769}{(1+i)^2}+\frac{14428}{(1+i)^3}+\frac{16305}{(1+i)^4}+\frac{18421}{(1+i)^5}$$

$$= -50000+\frac{11300}{(1+0.1)}+\frac{12769}{(1+0.1)^2}+\frac{14428}{(1+0.1)^3}+\frac{16305}{(1+0.1)^4}+\frac{18421}{(1+0.1)^5}$$

$$= 50000 + 10272.73 + 10552.89 + 10839.97 + 11136.53 + 11437.99 = 4240.12$$

5. Given data :

Discounting factor (r) = 12 %

The benefits achieved ¢in each year :

Year	1	2	3	4
Benefits (Rs. in lakhs)	25	40	40	50

Initial Investment (P) = 100 Lakhs

Net present value of aggregated benefits is given by

$$\text{Benifits} = \frac{25}{(1+r)}+\frac{40}{(1+r)^2}+\frac{40}{(1+r)^3}+\frac{50}{(1+r)^2}$$

$$= \frac{25}{(1+0.12)}+\frac{40}{(1+0.12)^2}+\frac{40}{(1+0.12)^3}+\frac{50}{(1+0.12)^4}$$

= 22.32 + 31.82 + 28.47 + 31.77 = 114.45

Therefore, Benefit-cost ratio = 114.45/100 = 1.1445

6. Given data :

Discount rate (r) = 12 %

Number of years (n) = 10

Total saving at the end of 10 years (FW) = 6 lakhs = Rs. 6,00,000

Let the annual saving is A

Therefore, future worth for annuity, A, and discounting rate, r, is given by

$$FW = A * \left[\frac{(1+r)^n - 1}{i}\right]$$

$$\Rightarrow\ 600000 = A * \left[\frac{(1+0.12)^{10} - 1}{0.12}\right]$$

$$\Rightarrow\ A = \frac{600000*0.12}{3.10-1} = \frac{72000}{2.106} = \text{Rs.}\,34190$$

7. Four investment opportunities are given as :

	Case I	Case II	Case III	Case IV
Initial Investment (Rs.)	40,000	1,00,000	80,000	1,20,000
Return in Year 2	30,000	75,000,	65,000	80,000
Return in Year 3	30,000	75,000	65,000	80,000

Interest rate (i) = 10 %

Net present value in each case can be determined as:

Case I :

$$NPV1 = -40000 + \frac{30000}{(1+i)} + \frac{30000}{(1+i)^2} = -40000 + \frac{30000}{(1.1)} + \frac{30000}{(1.1)^2} = 12066$$

Case II :

$$NPV2 = -100000 + \frac{75000}{(1+i)} + \frac{75000}{(1+i)^2} = -100000 + \frac{75000}{(1.1)} + \frac{75000}{(1.1)^2} = 30165$$

Case III :

$$NPV3 = -80000 + \frac{65000}{(1+i)} + \frac{65000}{(1+i)^2} = -80000 + \frac{65000}{(1.1)} + \frac{65000}{(1.1)^2} = 32809$$

Case IV :

$$NPV4 = -120000 + \frac{80000}{(1+i)} + \frac{80000}{(1+i)^2} = -120000 + \frac{80000}{(1.1)} + \frac{80000}{(1.1)^2} = 18842$$

The highest NPV value is observed for Case III and this is equal to 32809. Therefore, alternative III is most profitable.

8. The probability distribution of the outcome of an investment is given as

Outcome (Rs.)	1000	700	300
Probability	0.2	0.5	0.3

The expected return can be determined as :

E(r) = 1000*0.2 + 700*0.5 + 300*0.3 = 640

Variance can be determined as

$$Var = \{1000 - E(r)\}^2 + \{700 - E(r)\}^2 + \{300 - E(r)\}^2$$

$$\Rightarrow \quad Var = \{360\}^2 + \{60\}^2 + \{340\}^2 = 248800$$

Standard deviation of the risk is given by

$$\sigma = \sqrt{Var.} = \sqrt{248800} = 498.8$$

11. Given data :

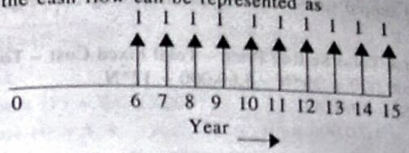
Annuity (A) = Rs 1

Period (n) = 10 years

Interest rate (i) = 10%

Deferment period (g) = 5 years [A time during which a borrower does not have to pay interest or repay the principal on a loan]

Therefore, the cash flow can be represented as



The present worth (NPV) can be determine as

$$NPV = \frac{A}{(1+i)^6} + \frac{A}{(1+i)^7} + \frac{A}{(1+i)^8} \cdots\cdots \frac{A}{(1+i)^{15}}$$

$$\Rightarrow \quad NPV = \frac{1}{(1.1)^6} + \frac{1}{(1.1)^7} + \frac{1}{(1.1)^8} \cdots\cdots \frac{1}{(1.1)^{15}}$$

$$\Rightarrow \quad NPV = \frac{1}{(1.1)^6}\left(1 + 1.1 + \ldots\ldots \quad 1.1^9\right)$$

$$\Rightarrow \quad NPV = \frac{1}{(1.1)^6}\left[1\frac{(1.1^{10} - 1)}{(1.1 - 1)}\right] = 3.81$$

12. Given data :

Selling Price (P) = Rs. 20/unit

Variable Cost (C) = Rs. 12/unit

Total Fixed Cost (FC) = Rs. 5,60,000

Let the break-even output units of bearing plates is N

At break-even point,

Total Selling Price = Total Fixed Cost + Total Variable Cost

⇒ 20*N = 560000 + 12*N

⇒ 8*N = 560000

⇒ N = 70000

13. Given data :

Selling Price (P) = Rs. 20/unit

Variable Cost (C) = Rs. 12/unit

Total Fixed Cost (FC) = Rs. 5,60,000

Output units of bearing plates (N) = 1,00,000

Therefore, profit from selling all the units can be determined as

Profit = Total Selling Price – Total Fixed Cost – Total Variable Cost

⇒ Profit = 20*1,00,000 – 5,60,000 – 12*1,00,000 = 2,40,0000

14. Given data :

Selling Price (P) = Rs. 20/unit

Variable Cost (C) = Rs. 12/unit

Total Fixed Cost (FC) = Rs. 5,60,000

Target Profit (TP) = 4,00,000

Let the total output units required for achieving the target profit is N

We have,

Target Profit = Total Selling Price – Total Fixed Cost – Total Variable Cost

$\Rightarrow$ 4,00,000 = 20*N - 5,60,000 – 12*N

$\Rightarrow$ 8N = 9,60,000

$\Rightarrow$ N = 1,20,000 units

15. Given data :

Present Value (PV) = 1,00,000

Future Worth (FW) = 1,76,234

Number of years (n) = 5

Let the rate of interest is i

We have,

$$FW = PV\,(1 + i)^n$$

$$\Rightarrow 176234 = 100000\,(1 + i)^5$$

$$\Rightarrow (1 + i)^5 = 1.76234$$

$$\Rightarrow 1 + i = 1.76234^{1/5} = 1.12$$

$$\Rightarrow i = 0.12 = 12\%$$

16. Given data,

Future Worth (FW) = 400

Number of years (n) = 5

Discount Rate (r) = 10%

Let the net present value of the money is NPV

We have,

$$NPV = \frac{FW}{(1+i)^n}$$

$$\Rightarrow NPV = \frac{400}{(1+10/100)^5} = \frac{400}{1.1^5} = 248.37$$

17. Given data :

Present, Value of the investment = P

Yearly payment or annuity = R

Number of years payment realised = n

Rate of interest = i

Therefore, we have

$$P = \frac{R}{(1+i)} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} \ldots\ldots \frac{R}{(1+i)^n}$$

$$\Rightarrow P = \frac{R}{(1+i)}\left[\frac{\{1/(1+i)\}^n - 1}{\{1/(1+i)\} - 1}\right] = \frac{R}{(1+i)} * \left\{\frac{(1+i)^n - 1}{i*(1+i)^n}\right\} * (1+i) = R * \left\{\frac{(1+i)^n - 1}{i*(1+i)^n}\right\}$$

$$\Rightarrow R = P\frac{i*(1+i)^n}{(1+i)^n - 1}$$

18. Given data :

Loan amount (P) = Rs. 25,000

Interest rate (i) = 8 %

Annual payment (A) = Rs. 2500

Let the period taken for repayment the loan is n years

We have,

$$P = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \ldots\ldots \frac{A}{(1+i)^n} = A * \left\{\frac{(1+i)^n - 1}{i*(1+i)^n}\right\}$$

$$\Rightarrow 25000 = 2500 * \left\{\frac{(1+0.08)^n - 1}{0.08*(1+0.08)^n}\right\}$$

$$\Rightarrow 10 = \frac{(1.08)^n - 1}{0.08*(1.08)^n}$$

$$\Rightarrow 0.8*(1.08)^n = (1.08)^n - 1$$

$$\Rightarrow 0.2*(1.08)^n = 1$$

$$\Rightarrow (1.08)^n = 5$$

Taking logarithm on both sides, we have

$$n \log 1.08 = \log 5$$

$$\Rightarrow n = 20.91$$

Therefore, number of years taken for repaying the loans is 21.

20. Given data :

Annuity (A) = Rs. 10,000

Discounting rate (i) = 10 %

Number of years (n) = 4

Let the present value of 4-year annuity is P

We have,

$$P = A * \left\{\frac{(1+i)^n - 1}{i*(1+i)^n}\right\}.$$

$$\Rightarrow P = 10000 * \left\{\frac{(1+0.1)^4 - 1}{0.1*(1+0.1)^4}\right\} = 10000 * \left\{\frac{(1.1)^4 - 1}{0.1*(1.1)^4}\right\} = 31698.65 \approx 31700$$

21. Given data :

Selling price of metal (S) = Rs. 210/kg

Recoverable grade (g) = 1.2%

Production cost of ore (C) = Rs. 2000/te

Break even stripping ratio (BESR) = 3.49 m^3/te

Let the stripping cost is SC.

We have,

Break – even Stripping Ratio

$$= \frac{(\text{Selling price per te of ore}) - (\text{Production cost per te ore})}{\text{Stripping Cost per cubic meter or overbrden material}}$$

$$\Rightarrow 3.49 = \frac{210*1000*\frac{1.2}{100} - 2000}{SC}$$

$$\Rightarrow SC = \frac{2520-2000}{3.49} = \frac{520}{3.49} = \text{Rs.}149/\text{m}^3$$

22. Given data :

Total cost of production (C) = $at^2 + b$

Revenue from sale (S) = ct

Total production per year = t

At breakeven point,

Total cost of production (C) = Revenue from sale (S)

$$\Rightarrow at^2 + b = ct$$

$$\Rightarrow at^2 - ct + b = 0$$

$$\Rightarrow t = \frac{c \pm \sqrt{c^2 - 4ab}}{2a}$$

23. Given data :

Annuity (A) = Rs. 10,000

Discounting rate (i) = 10 %

Number of year (n) = 3

Let the net present value is P

We have,

$$P = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} = A*\left\{\frac{(1+i)^3 - 1}{i*(1+i)^3}\right\} = 10000*\left\{\frac{(1.1)^3 - 1}{0.1*(1.1)^3}\right\} = 24{,}869$$

24. Given data :

Initial Investment (P) = 4 lakhs = Rs. 4,00,000

Net expected income (A) = Rs. 80,000 per annum

Let the payback period is n years

Assuming a zero discounting rate, the payback period can be determined as

Initial Investment = Total Income during payback period

$\Rightarrow$ 4,00,000 = 80,000*n

$\Rightarrow$ n = 5

25. Given data :

Weight of overburden material extraction (Wo) = 16*3000 te = 48000 te

Weight of coal produced (Wc) = 4*2000 te = 8000 te

Mining cost of both overburden and coal (C) = Rs. 50/te

Coal transportation cost to selling point (C_t) = Rs. 75/te

Overburden material transportation cost from mine to dump (O_t) = Rs. 75/te

Selling Price of coal (S_c) = Rs. 800/te

Royalty and tax to the state (C_r) = Rs. 50/te

Nominal Profit = Selling Price of Coal – Cost of coal produced – Cost of overburden removal – transportation cost of coal –transportation cost of overburden – Royalty and tax

$\Rightarrow$ Nominal profit = 800*8000 – 50*8000 – 50*48000 – 75*8000 – 50*48000 – 50*8000

$\Rightarrow$ Nominal profit = 64,00,000 – 4,00,000 – 24,00,000 – 6,00,000 – 24,00,000 – 4,00,000

$\Rightarrow$ Nominal profit = 2,00,000

26. Given data :

Loan amount (P) = Rs. 50,000

Interest rate (i) = 6 %

Repayment duration (n) = 10 years

Let the annual sum to be paid is A

We have,

$$P = A*\left\{\frac{(1+i)^n - 1}{i*(1+i)^n}\right\}$$

$$\Rightarrow 50000 = A*\left\{\frac{(1.06)^{10} - 1}{0.06*(1.06)^{10}}\right\}$$

$$\Rightarrow A = \frac{50{,}000*0.06*(1.06)^{10}}{(1.06)^{10} - 1} = \frac{5372.54}{0.79} = 6793.4$$

27. Given data :

Initial cost of the equipment (Ci) = Rs. 8, 00,000

Salvage value at the end of 15 years (cf) = Rs. 2,00,000

Yearly Depreciation rate = (C_i – c_f)/Number of years

= (8,00,000 – 2,00,000)/15 = 40,000

Therefore, value of the equipment after 10 years

$= 8,00,000 - 40,000*10 = 8,00,000 - 4,00,000 = 4,00,000$

28. Initial Investment (P) = Rs. 10,000

Estimated return after 6 years (FW) = Rs. 20,000

Let the expected rate of return is i

We have,

$FW = P(1 + i)^n$

$\Rightarrow 20000 = 10000 (1 + i)^6$

$\Rightarrow (1 + i)^6 = 2$

$\Rightarrow 1 + i = 2^{1/6} = 1.122$

$\Rightarrow i = 0.122 = 12.2\%$

29. Given cash flow data :

Year	0	1	2	3	4	5
Cash Flow in (Rs.)	-40000	15000	14000	13000	12000	11000

Discount rate (i) = 12%

The profitability index can be determined as

$$PI = \frac{\text{Cash Flow}}{\text{Initial Cash outlay}} = \frac{\sum_{r=0}^{n}\left\{CF_i/(1+i)^r\right\}}{I_0}$$

$$= \frac{-40,000 + \frac{15000}{(1.12)} + \frac{14000}{(1.12)^2} + \frac{13000}{(1.12)^3} + \frac{12000}{(1.12)^4} + \frac{11000}{(1.12)^5}}{40000}$$

$$= \frac{-40000 + 13392.86 + 11160.7 + 9253.1 + 7626.2 + 6241.7}{40000}$$

$$= \frac{47674.55}{40000} = 1.192$$

30. Given data :

Initial Cost of ore bearing parcel of land (P) = Rs. 20,00,000

Estimated recoverable reserve (Q) = 5,00,000 te

Extraction in 1st year (q) = 75,000 te

Weight of ore sold (qs) = 50,000 te

Depletion allowance for one year = (q/Q)*P

$= (75,000/5,00,000)*20,00,000 = \text{Rs. } 3,00,000$

31. The probability distribution of the outcome of an investment is given as

Outcome (Rs.)	1000	700	300
Probability	0.2	0.5	0.3

The expected rate of return is given by

$E(r) = 1000*0.2 + 700*0.5 + 300*0.3 = 640$

32. Total cost and total revenue functions :

$TC = 50 + 100q - 6q^2 + 1/3\ q^3$

$TR = 64\ q$

Where q is the output of mine in te/day

Profit can be determined as

Profit = P = TR − TC

$\Rightarrow P = 64q - 50 - 100q + 6q^2 - 1/3q^3$

For maximization, we have

$$\frac{dP}{dq} = 0$$

$$\Rightarrow \frac{d}{dq}\left(64q - 50 - 100q + 6q^2 - \frac{1}{3}q^3\right) = 0$$

$$\Rightarrow 64 - 0 - 100 + 12q - \frac{1}{3}*3q^2 = 0$$

$\Rightarrow -q^2 + 12q - 36 = 0$

$\Rightarrow q^2 - 12q + 36 = 0$

$\Rightarrow q = 6$ te/day

33. Given data :

Future worth of the investment (FW) = Rs. 1,20,000

Interest rate (i) = 10%

Number of years invested (n) = 5 years

Let the present value of the sum is P

It is given that the invested amount compounded quarterly. Therefore, the effective rate of interest and duration are given as :

$i' = i/4$

And, $n' = 4n$

We have,

$FW = p(1 + i')n'$

$$\Rightarrow 120000 = P\left(1 + \frac{10}{400}\right)^{4*5}$$

$$\Rightarrow P = \frac{120000}{(1.025)^{20}} = \frac{120000}{1.64} = \text{Rs. } 73233$$

34. The cost function of longwall face is given as

$C = 0.1\ L + 1562.2/L + 300$

Where, L is the length of the face in m

For cost minimization,

$$\frac{dC}{dL} = 0$$

$$\Rightarrow \frac{d}{dq}\left(0.1L + \frac{1562.2}{L} + 300\right) = 0$$

$$\Rightarrow 0.1 + \frac{1562.2}{L^2}(-1) + 0 = 0$$

$$\Rightarrow L = \sqrt{\frac{1562.2}{0.1}} = \sqrt{15622} = 125$$

35. Given data :

Bid 1 : Quotes Rs. 150 crores as a price to be paid in full on delivery

Bid 2 : Quotes Rs. 180 crores as a price to be paid at the end of third year after delivery

Discount rate (r) = 12 %

The first bid to be paid in full on delivery. Therefore, net present value will be same as the bidding price.

NPV1 = 150 crore

The second bid to be paid in full after three years of delivery. Therefore, the net present value can be determined as :

$$NPV2 = \frac{180}{(1+r)^3} = \frac{180}{(1+0.12)^3} = \frac{180}{(1.12)^3} = 128.12 \text{ crore}$$

Thus, difference in NPV between the two bids is

Difference = NPV1 – NPV2 = 150 – 128.12 = 21.88 crore

36. Average life of the equipment is 5 years. The depreciation money of the equipment follows straight line method. So, each year the depreciation value will be same.

Annual depreciation = (1/5)*100 = 20%

37. The cash flow of manganese mine is given as :

Item Amount	(Rs. in lakhs)
Revenue	900
Cost (other than depreciation)	300
Depreciation	100
Profit before tax	500

Corporate Tax (50% profit before tax) = 500*50/100 = 250 lakhs

Operating cash inflow = Revenue – Cost – Corporate Tax

= 900 – 300 – 250 = Rs. 350 lakhs

[Operating cash flow does not consider depreciation cost]

38. The cost of shaft sinking in lakhs of rupees is given as

$$C_s = 2.64D + 34.8$$

Where, D is the shaft in m

The cost of driving an incline is given by

$$C_i = 0.96\ L$$

Where, L is the length of incline

For shaft excavation become more economical, cost of shaft sinking should be less than cost of driving incline. That is,

$C_s < C_i$

$\Rightarrow 2.64D + 34.8 < 0.96L$

$\Rightarrow 2.64D + 34.8 < 0.96*3D$ [for L/D = 3]

$\Rightarrow 0.24D < 34.8$

$\Rightarrow D > 145$

39. Given data :

Number of waste blocks (Nw) = 10

Number of ore blocks (No) = 5

Volume of each waste block (Vw) = 600 m^3

Total cost of waste handling (Cw) = Rs. 100/m^3

Tonnage of each ore block (Q) = 400

Total cost of ore handling (Co) = Rs. 150/te

Sale price of ore per ton (S) = Rs. 500

The net cash flow in mining the deposit is given by

Net cash flow

= Selling price of ore – Cost of waste handling – Cost of ore handling

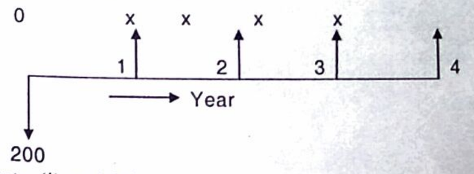
⇒ Net cash flow = No*Q*S – Nw*Vw*Cw – No*Qo*Co

⇒ Net cash flow = 5*400*500 – 10*600*100 – 5*400*150

⇒ Net cash flow = 10,00,000 – 6,00,000 – 3,00,000

⇒ Net cash flow = Rs. 1,00,000 = 1 lakh

40. The cash flow diagram is given as :



Interest rate (i) = 10%

For the investment becomes viable,

$$NPV \geqslant 0$$

$$\Rightarrow -200 + \frac{x}{(1+i)} + \frac{x}{(1+i)^2} + \frac{x}{(1+i)^3} + \frac{x}{(1+i)} \geq 0$$

$$\Rightarrow x*\left\{\frac{(1+i)^4-1}{i*(1+i)^4}\right\} \geq 200$$

$$\Rightarrow x*\left\{\frac{(1+i)^4-1}{0.1*(1.1)^4}\right\} \geq 200$$

$$\Rightarrow x*3.17 \geq 200$$

$$\Rightarrow x \geq 63$$

41. The cash flows (Rs) of the project is given as :

Year	0	1	2	3	4	5
Cash Flow in (Rs.)	–50000	+15000	+15000	+15000	+15000	+ 15000

Let the internal rate of return (IRR) is i

At internal rate of return, the NPV is equal to zero.

That is, NPV = 0

$$\Rightarrow -5000+\frac{x}{(1+i)}+\frac{x}{(1+i)^2}+\frac{x}{(1+i)^3}+\frac{x}{(1+i)^4}+\frac{x}{(1+i)^5}=0$$

$$\Rightarrow \frac{15000}{(1+i)}\left\{\frac{(1+1)^5-1}{i}\right\}=50000$$

$$\Rightarrow i = 13\%$$

■

9.0 Mineral Dressing

9.1 Classification

Classification is a method of separating mixtures of minerals into two or more products on the basis of the velocity with which the grains fall through a fluid medium.

9.2 Crushers

9.2.1 Introduction

Crushing is the first mechanical stage in the process of comminution in which the main objective is the liberation of the valuable minerals from the gangue. Crushing may be in open or closed circuit depending on product size. In open circuit crushing, undersize material from the screen is combined with the crusher product and is then routed to the next operation.

In closed circuit crushing operation, undersize from the screen is the finished product. The crusher product is returned to the screen so that any over over-size material will be recirculated. The open circuit and closed circuit crushing systems are shown in Figure 9.1 and Figure 9.2 respectively.

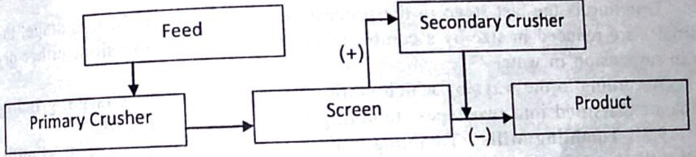


Fig. 9.1 Open-circuit crushing

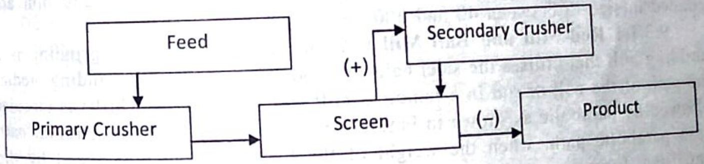


Fig. 9.2 Closed-circuit crushing

9.2.2 Types of Crushers

9.2.2.1 **Primary Crushers :** Primary crushers are heavy-duty machines, used to reduce the run-of-mine ore as large as 1.5 m across down to a suitable size in the order of 10 -20 cm for transport and for feeding the secondary crushers. They are always operated in open circuit.

There are mainly two types of primary crushers :

➢ **Jaw Crushers :** Jaw crushers are further classified by the method of pivoting the swing jaw.

- **Blake Crusher :** In the Blake crusher the jaw is pivoted at the top and thus has a fixed receiving area and a variable discharge opening.
- **Dodge Crusher :** In the Dodge crusher the jaw is pivoted at the bottom, giving it a variable feed area but fixed delivery area. It is never used for heavy duty crushing as it chokes very easily.
- **Universal Crusher :** The Universal crusher is pivoted in an intermediate position, and thus has a variable delivery and receiving area.

➢ **Gyratory Crushers :** Gyratory crusher consists essentially of a long spindle, carrying a hard steel conical grinding element, the head, seated in an eccentric sleeve. Gyratory crushers are principally used in surface- crushing plants, although a few currently operate underground. It has higher capacity in comparison to Jaw crushers.

9.2.2.2 **Secondary Crushers :** Secondary crushers further used to reduce the product ore from primary crusher to a size of 0.5 – 2 cm in diameter.

➢ Cone Crusher
➢ Gyradisc Crusher
➢ Roll Crushers

9.2.2.3 **Tertiary Crushers or Impact Crushers :** In this class of crusher, comminution is by impact rather than compression, by sharp blows applied at high speed to free falling rock. One of the tertiary crusher types is Hammer mill.

9.3 Grinding Mills

Grinding is the last stage in the process of comminution. In this stage, the particles are reduced in size by a combination of impact and abrasion, either dry or in suspension in water.

According to the ways by which motion is imparted to the charge, grinding mills are classified into two types: tumbling mills and stirred mills.

9.3.1 **Tumbling Mills :** Tumbling mills are of three basic types: Rod mills, Ball mills, Autogeneous mills. Tumbling mills are employed in the mineral industry for coarse grinding processes, in which particles between 5 and 250 mm are reduced in size to between 40 and 300 μm.

9.3.1.1 **Rod Mill and Ball Mill :** A rod mill and ball mill grinding is a tumbling mill that utilises the steel balls and rods respectively as grinding media. The path of the ball or rod in a tumbling mill (Ball mill or Rod Mill) is circular followed by parabolic as shown in Figure 9.3. The rod abandons its circular path for a parabolic path, when the weight of the ball/rod is just balanced by the centrifugal force.

i.e., $m\omega^2 R = mg \cos\alpha$

where, m = mass of the ball or rod
ω = angular velocity
g = acceleration due to gravity
R = radius of the mill inside liners = (D - d)/2
D = diameter of the mill
d = diameter of the ball or rod

$$\omega = \sqrt{\frac{g\cos\alpha}{R}}$$

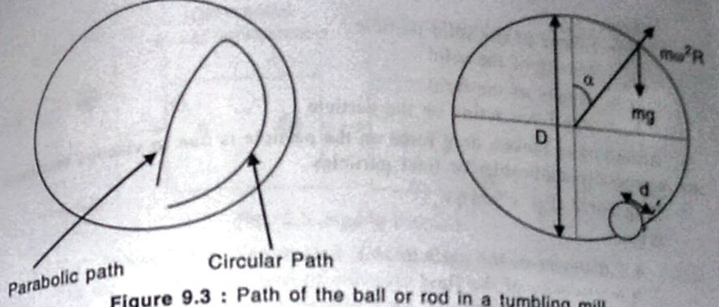


Figure 9.3 : Path of the ball or rod in a tumbling mill

The critical speed of the mill occurs when α = 0, i.e., the medium abandons its circular path at the highest vertical point. At this point, cos α = 1.

$$\text{Critical speed of the mill} = N_c = \frac{\sqrt{g}}{r}\ \text{m/s}$$

$$\Rightarrow N_c = \frac{\sqrt{2g}}{(D-d)} * \frac{60}{2\pi}\ \text{rev/min}$$

9.3.1.2 **Autogenous Mills :** An autogenous grinding is a tumbling mill that utilises the ore itself as grinding media. The semi autogenous mill is an autogenous mill that utilises steel balls in addition to the natural grinding media.

9.3.2 **Stirred Mills :** These employ stirrers comprising a shaft with pins or disks. Stirred mills find application in fine (15–40 μm) and ultra-fine (<15 μm) grinding. Finer grinding is normally achieved with stirred mills.

9.4 Principles of Classification in Liquid Medium

Classifiers consist essentially of a sorting column in which a fluid is rising at a uniform rate. Particles introduced into the sorting column either sinks or rise according to whether their terminal velocities are greater or lesser than the upward velocity of the fluid.

9.4.1 **Free Settling :** Free settling refers to the sinking of particles in a volume of fluid which is large with respect to the total volume of particles, hence particles crowding is negligible. The force diagram and the movement of particles is shown in Figure 9.4.

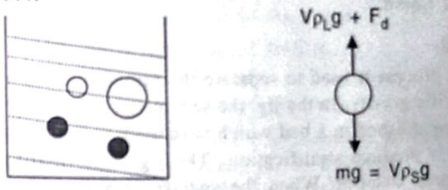


Figure 9.4 : Free settling of Particles in Liquid

The equation of the motion of the particle for zero acceleration is
$V_{\rho s} g - V\rho_L g - F_d = \text{mass*acceleration} = 0$

Where,

V = volume of the solid particle
ρ_S = density of the solid
ρ_L = density of the fluid
F_d = drag force acting on the particle

According to Stokes, drag force on the particle is due to viscous resistance and is generally applicable for finer particles.

Drag Force $(F_d) = 3\pi d \eta v$

Where,

d = diameter of the particle
η = viscosity of the fluid
v = terminal velocity

$$\text{Therefore, } v = \frac{gd^2(\rho_s - \rho_L)}{18\eta}$$

According to Newton, drag force acting on the particle is due to turbulent resistance and is generally applicable to coarser particles.

$$\text{Drag force} = F_d = 0.055\pi d^2 v^2 \rho_L = V\rho_s g - V\rho_L g$$

$$\text{Therefore, } v = \left[\frac{3gd(\rho_s - \rho_L)}{\rho_L}\right]^{1/2}$$

9.4.2 **Hindered Settling :** As the proportion of solids in the pulp increases, the effect of particle crowding becomes more apparent and the falling rate of the particles begins to decrease. Hindered settling condition begins when the proportion of solids in the pulp increases and density of the fluid is that of the pulp rather than that of the carrier liquid.

Therefore for hindered settling the settling velocity is given by

$$v = \frac{gd^2(\rho_s - \rho_P)}{18\eta} \qquad \text{[Stoke's Law]}$$

$$v = \left[\frac{3gd(\rho_s - \rho_P)}{\rho_L}\right]^{1/2} \qquad \text{[Newton's Law]}$$

The above expressions depends on the density of the pulp rather than density of the carrier liquid.

9.5 Jigging

Jigging process is used to separate the minerals from gangue materials with different specific gravity. In the jig, the separation of minerals of different specific gravity is accomplished in a bed which is rendered fluid by a pulsating current of water so as to produce stratification. The jig is normally used to concentrate relatively coarse material. When the specific gravity difference is large, good concentration is possible with a wider size range. An Idealised jigging process can be described as in Figure 9.5.

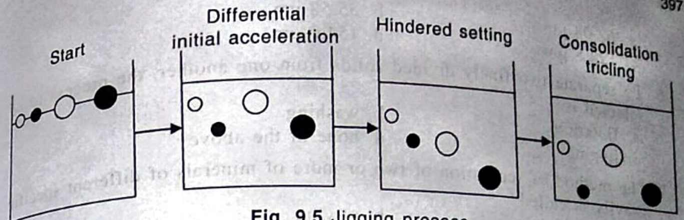


Fig. 9.5 Jigging process

9.6 S.G. Dense Media Separation (DMS)/ Heavy Media Separation : DMS is applied to the pre-concentration of minerals, i.e. the rejection of gangue prior to grinding for final liberation. It is also used in coal preparation to produce a commercially graded end-product, clean coal being separated from the heavier shale or high-ash coal. In this process, heavy liquids of suitable density are used, so that those minerals lighter than the liquid float while those denser than it sinks.

$$\begin{tikzcd} & \text{Feed} \arrow[d] & \\ \text{Minerals (S.G.} < x) & \text{Fluid Medium (S.G.} = x) \arrow[l] \arrow[r] & \text{Minerals (S.G.} > x) \\ \textbf{Float} & & \textbf{Sink} \end{tikzcd}$$

9.7 Froth Floatation : Flotation is a physico-chemical separation process that utilises the difference in surface properties of the valuable minerals and the unwanted gangue minerals.

9.8 Washing : Washing of run-of-mine ore can be carried out to facilitate sorting by removing obscuring dirt from the surfaces of the ore particles. Washing is normally performed after primary crushing as the ore is then of a suitable size to be passed over washing screens.

EXERCISE - 9

1. In-situ ore of 1.5 % Cu got diluted with waste rock of value 0.2 % Cu. If the resultant extracted ore has 1.2 % Cu, what is the percentage of dilution?
   a. 30 b. 25 c. 23 d. 20
2. For heavy duty crushing on hard material, the type of jaw crusher normally selected is
   a. blake type double toggle b. dodge type
   c. single toggle type d. none of the above
3. The mill which accepts the lowest size of feed is
   a. rod mill b. ball mill
   c. autogeneous mill d. none of the above
4. The diameter of a ball mill is 3 m and the diameter of the balls in the ball mill is 50 cm. What is the critical speed of the ball mill?
   a. 26.76 rev/min b. 53.52 rev/min
   c. 25.81 rev/min d. 30.45 rev/min
5. What will be the value of d50 for the specified overflow of 60% passing 74 m when the value of the multiplier is 2.08?

a. 62.08 μm b. 35.52 μm
c. 124.8 μm d. 154.0 μm

6. To separate two finely divided solids from one another, the process to be selected is
a. flotation b. washing
c. jigging d. none of the above

7. The method of separation of two or more of minerals of different specific gravity is called
a. flotation b. sieving
c. washing d. HMS

8. As the ball load increases, the energy consumed by ball mill
a. decreases b. increases
c. first increases and then decreases
d. first decreases and then increases

9. The path of a typical ball mill in a ball mill is
a. circular b. elliptical
c. parabolic d. partly circular and partly parabolic

10. When the particles of same or different specific gravity come to rest at different instants, the phenomenon is termed as
a. stratification b. hindered settling
c. consolidation trickling d. differential acceleration

11. Froth flotation is used in ore treatment plant to separate
a. two minerals in one b. mineral and gangue material
c. none of the above d. both (a) and (b)

12. Hydrocyclones are used in ore treatment plant to separate
a. coarse and fine particle b. mineral and gangue
c. concrete and tailings d. none of the above

13. Magnetic separation is
a. a dry method b. an wet method
c. combination of two d. none of the above

14. A concentrator pilot plant is fed with 1 tonne of copper ore at ROM grade of 1.5 % Cu. Metal recovery in the concentrator pilot plant is 90% and the grade of copper in concentrate is 20%. The amount of copper in concentrate in kg is
a. 13.5 b. 14.0
c. 14.5 d. 15.0 [GATE 2011]

15. In the above question, the amount of concentrate produced from 1 tonne of ore in kg is
a. 75.0 b. 72.0
c. 70.0 d. 67.5 [GATE 2011]

Answer

1. a 2. a 3. d 4. a 5. d 6. a 7. d
8. d 9. d 10. d 11. a 12. a 13. c 14. a
15. d

Solutions of Exercise 9

1. Given data :
Grade value of in-situ ore (gi) = 1.5 %
Grade value of waste rock (gw) = 0.2 %
Grade value of resultant ore (gr) = 1.2 %
Let X te of waste rock diluted with Y te of in-situ ore.
The grade of the diluted ore can be determined as

$$g_r = \frac{g_w * X + g_i * Y}{X+Y}$$

$$\Rightarrow 1.2 = \frac{0.2 * x + 1.5 * Y}{X+Y}$$

$$\Rightarrow 1.2X + 1.2Y = 0.2X + 1.5Y$$

$$\Rightarrow X = 0.3Y$$

The percentage of dilution is

$$= \frac{X}{X+Y} * 100 = \frac{30Y}{1.3Y} = 23\%$$

4. Given data :
Diameter of the ball mill = D = 3 m
Diameter of the balls in mill = d = 50 cm = 0.5 m
Let the critical speed of the mill is Nc
We have,

$$N_c = \sqrt{\frac{2g}{(D-d)}} * \frac{60}{2\pi} \text{ rev/min}$$

$$\Rightarrow N_c = \sqrt{\frac{2*9.81}{(3-0.5)}} * \frac{60}{2\pi} = 26.76 \text{ rev/min}$$

5. Given data :
Specified overflow (Q) is 60 % passing at d (= 74 μm) size.
Multiplier factor (f) = 2.08
The value of d50 size for the specified overflow at 74 μm size is given by
= d*f = 74*2.08 = 153.98 microns or μm

14. Given data :
Feeded quantity = (Q) = 1 te = 1000 kg
ROM grade (g) = 1.5 %
Metal recovery in the concentrator pilot plant (η) = 90 %
Grade of copper in concentrate (G) = 20 %

Amount of copper in the concentrate $= Q * \frac{g}{100} * \eta$

$$= 1000 * \frac{1.5}{100} * 0.9 = 13.5 \text{ kg}$$

15. Let the amount of concentrate produced from 1 te ore is X kg
Therefore, we have
X*0.20 = 13.5
=> X = 67.5 kg

■

10. Geostatistics

10.1 Arithmetic Mean (AM)

If the variable x takes the n values $x_1, x_2 \ldots x_n$, the arithmetic mean is given by

$$\overline{X} = \frac{x_1 + x_2 + \ldots\ldots x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

If the the variable $x_1, x_2, \ldots\ldots\ldots x_n$ occur with respective frequencies $f_1, f_2, \ldots\ldots\ldots\ldots f_n$, the AM is given by

$$\overline{X} = \frac{f_1x_1 + f_2x_2 + \ldots\ldots f_nx_n}{f_1 + f_2 + \ldots\ldots f_n} = \sum_{i=1}^{n} f_ix_i / \sum_{i}^{n} f_i$$

The above formula gives weighted arithmetic mean.

10.2 Median

Median is the value of the middle variable when variables are ascending in increasing or decreasing order. If the number of the variables is odd, say 2n+1, the value of the (n+1)th variables gives the median value, while if they are even, say 2n, then the mean of the nth and (n+1)th values gives the median. In the case of grouped frequency distribution, the median by simple interpolation is given by

$$M_d = L + \frac{\frac{N}{2} - F}{f} * i$$

Where, M_d = median

L = lower limit of the median class

f = frequency of the median class

F = total of all the frequencies before the median class

i = width of class interval of the median class

10.3 Mode

Mode is the value of the variable which has the maximum frequency. It is the maximum ordinate of the ideal curve which gives the closest possible fit to the actual distribution. For a grouped distribution, the mode is given by the formula

$$\text{Mode} = L + \frac{f - f_{-1}}{2f - f_1 - f_{-1}} * i$$

Where, L = lower limit of the modal class.

i = width of the modal class

f_1 and f_{-1} = frequencies of the class following and preceding the modal class respectively.

f = frequency of the modal class

Empirical relationship among mean, median and mode

For moderately symmetrical distribution, the following relationship approximately holds.

Mode = Mean − 3 (Mean − Median)

For a symmetrical distribution, the mean, median and mode coincide.

10.4 Standard Deviation : If we take our measure of dispersion as the sum of the deviations of the values of variate from the mean, the sum will be zero even though the actual dispersion from the central value may be quite large. In order to remove this discrepancy, either we take the mean of the absolute deviations from the average value or square the deviations to do away the negative sign of the deviations. Standard Deviation (σ) is the positive square root of the mean of the squares of the deviations from the mean. It is given by

$$\text{i.e., } \sigma = \left\{\frac{1}{N}\sum_{i}(x_i - \overline{x})^2\right\}$$

Where, N is the total frequency.

In case of frequency distribution, the standard deviation (σ) is given by

$$\sigma = \left\{\frac{1}{N}\sum_{i} f_i(x_i - \overline{x})^2\right\}$$

10.5 Mean Deviation or the Average Deviation : Another measure of dispersion in which the absolute values of the deviations are considered is used and is known as mean deviation. The mean deviation about the mean

$$\text{Mean Deviation} = \frac{1}{N}\sum f_i / |x_i - \overline{x}|$$

10.6 Coefficient of Variation (cv) : The coefficient of variation is the ratio of the standard deviation to the mean.

Coefficient of Variation = $\sigma/M*100$

10.7 Probability Distribution Function : For a discrete random variable, there are spikes of probability associated with the values that the random variable assumes. In case of discrete random variables, the probability distribution is called a probability mass function and in case of continuous random variable it is called a probability density function. The probability density function curve is shown in Figure 10.1.

The cumulative distribution function, F(x), represents the probability that X is less than or equal to x_k, for a discrete random variable is given by

$$F(X_k) = P(X \le x_k) = \sum_{k=1}^{n} P(x_k)$$

The cumulative probability density function curve is shown in Figure 10.2.

Probability distributions of continuous random variables are smooth curves. The probability density function (pdf) of a continuous random variable is denoted by f(x). The cumulative distribution function (cdf) of a continuous random variable is denoted by F(x). It is a non-decreasing function with a maximum value of 1. The cdf represents the probability that X is less than or equal to x,

$$\text{i.e., } F(X) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$$

Where f(x) is a continuous function of x called the probability density function and is measured by the ordinate of the probability curve.

The probability that a value of 'x' lies within the interval a, b is given by

$$P(a \le x \le b) = \int_{a}^{b} f(x)\,dx$$

Which is represented by the area under the probability curve y = f(x) between the values x = a and x = b under the following conditions

$f(x) \ge 0$ for all values of x

$\int_{\alpha}^{\beta} f(x)\,dx = 1$, if (α, β) is in the range of x

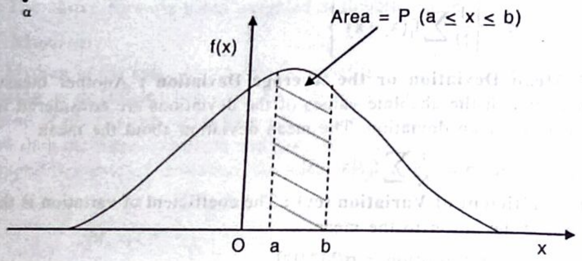


Fig. 10.1 Probability Density Function

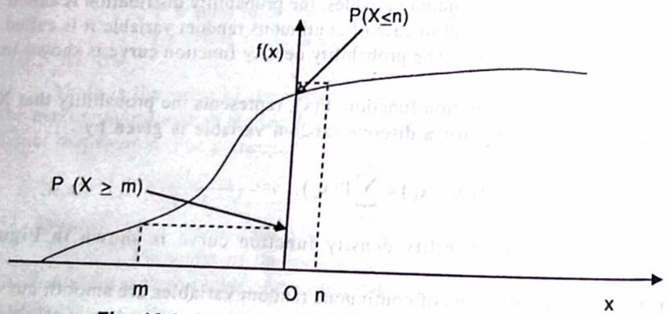


Fig. 10.2 Cumulative Probability Distribution Function

Probability of the random variable taking a value exactly equal to a given value is zero, because

$$P(x = m) = P(m \le x \le m) = \int_{m}^{m} f(x)\,dx = 0$$

The probability density is given by $f(x) = \frac{d}{dx}F(x)$

10.8 Normal Distribution : Amongst all the probability distribution, the normal probability distribution is by far the most important and frequently used continuous probability distribution. The distribution is shown in Figure 10.3.

The probability function of this distribution is generally written as under

$$f(X_i = x) = \left(\frac{1}{\sqrt{2\pi\sigma}}\right) e^{-\left(\frac{1}{2}\right)(x-\mu)^2/\sigma^2}$$

Where,

μ = mean of the distribution

σ^2 = variance of the distribution

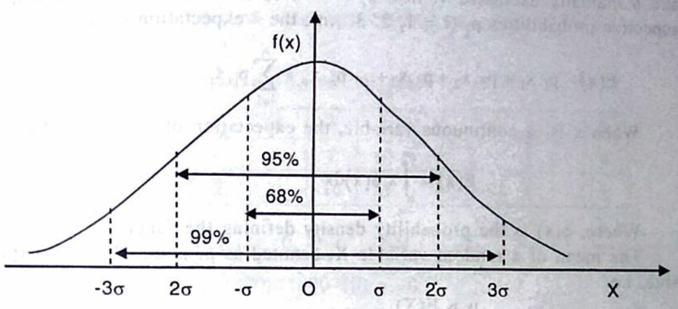


Fig. 10.3 Probability Density Function

If the mean is μ and the standard deviation is σ, then about 68 % of the values lies within $\mu \pm \sigma$ and 95 % values lies within $\mu \pm 2\sigma$ in a normal distribution.

10.9 **Poisson distribution :** The random variable of interest in Poisson distribution is the number of occurrences of a given event during a given interval.

The probability function of this distribution is generally written as under

$$f(X_i = x) = \frac{\lambda^x e^{-\lambda}}{!x}$$

Where,

λ = average number of occurrences per specified interval or the mean of the distribution = n*p.

x = number of occurrences of a given event. (= 0, 1, 2.......)

n = number of trials.

p = probability of success in a single trial.

10.10 Exponential distribution : Exponential distribution is the probability distribution of time, (say t), between events and such it is a continuous probability distribution concerned with the continuous random variable that takes on any value between zero and positive infinity.

The probability function of the exponential distribution is given by

$$f(x) = \mu e^{-\mu x}$$

Where, $x \geq 0$

μ = the average length of the interval between two occurrences

The expected value or mean of the exponential distribution is $1/\mu = \lambda$, and its variance is $1/\mu^2$

The cumulative distribution of the exponential distribution is

$$F(x) = P\ (X \leq x) = 1-e^{-\mu x};\ x \geq 0$$

$= 0$; elsewhere

10.11 Expectation of a Variate : When x is a discrete variable which may take n mutually exclusive values, $x_i = (i = 1, 2, 3.....n)$ and no others, with respective probabilities p_i $(i = 1, 2, 3....n)$, the expectation of x is

$$E(x) = p_1.x_1 + p_2.x_2 + p_3.x_3 + ...p_n.x_n = \sum_{i=1}^{n} p_i.x_i$$

When x is a continuous variable, the expectation of x is given by

$$E(x) = \int_{-\infty}^{\infty} x.\phi(x)dx$$

Where, $\phi(x)$ is the probability density defining the function.

The mean of a random variable X, denoted as μ, is equal to the expected value, i.e.

$$\mu = E(X)$$

The variance, $\sigma^2 = E(X-\mu)^2 = \int_{-\infty}^{\infty}(x-\mu)^2.f(x)dx$

[For continuous random variable]

The variance of a constant time random variable is the constant squared times the variance of random variable and is generally denoted simply as

$$\text{Var}\ (cX) = c^2\ \text{Var}\ (X)$$

The mathematical expectation of a sum of a number of random variables is equal to the sum of their expectations.

$$E(x_1 + x_2 + ...x_n) = E(x_1) + E(x_2) + ...\ E(x_n)$$

The mathematical expectation of the product of a number of independent random variables is equal to the product of their expectations.

$$E(x_1 * x_2 * .....x_n) = E(x_1) * E(x_2) * ....\ E(x_n)$$

10.12 Skewness : Skewness is the lack of symmetry and is termed as positive if the longer tail of the frequency curve is towards the higher value of the variate or in other words the mean is greater than the mode or median and negative in the reverse case. **For a symmetrical distribution, the mean, mode and median coincide.**

$$\text{Coefficient of Skewness} = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$$

10.13 Correlation Coefficient : The relationship between two variables such that a change in one variable result in a positive or negative change in the other and also greater change in one variable results in a corresponding greater change in the other is known as correlation.

10.14 Rank Correlation : It can be defined as the degree of agreement between two judges who have graded or ranked the same individuals regarding the same characteristics, for which there is no known objective standard of measurement.

$$\text{Rank correlation (R)} = 1 - \frac{6\sum d_i^2}{n^3 - n}$$

Where, n = number of character

d_i = differences in agreement of the two judges.

For example,

Rank correlation between the two judges :

x	1	6	5	10	3	2	4	9	7	8
y	3	5	8	4	7	10	2	1	6	9
d^2	4	1	9	36	16	25	4	64	1	1

In the above example, $n = 10$, $\sum d^2 = 200$

Therefore, $R = 1 - (6*200)/(1000-10) = -0.2$

10.15 **Probability : If there are S exhaustive, mutually exclusive and equally likely outcomes of a random experiment and 'r' of them are favourable to the happenings of an event A, the probability of the happening of A is**

$$P(A) = \frac{r}{S}$$

It is sometimes expressed as odds in favour of A are r to S-r or the odds against A are S-r to r.

10.15.1 Additional Theory of Probability

If vents A and B are mutually exclusive (i.e. both the events are not happened simultaneously), the probabilities of A and B or both happening is given by

$$P\ (AUB) = P(A) + P\ (B)$$

In general,

$$P\ (A_1 + A_2 + A_3....+A_n) = P\ (A_1) + P\ (A_2) + ...\ P\ (A_n)$$

If the events are not mutually exclusive, then

$$P\ (AUB) = P\ (A) + P\ (B) - P\ (AB)$$

In case of three events :

P (A U B U C) = P (A) + P (B) + P (C) – P (AB) – P (BC) – P (AC) + P (ABC)

Where, P (A) = chances of occurring the event A
P (B) = chances of occurring the event B
P (C) = chances of occurring the event C
P (AB) = chances of occurring both the events A and B simultaneously
P (BC) = chances of occurring both the events C and B simultaneously
P (AC) = chances of occurring both the events A and C simultaneously
P (ABC) = chances of occurring all the three events A, B and C simultaneously

10.15.2 Theorem of Total Probability for Compound Events :

If A is any event in a sample space S, then

$$P(A) = \sum_{i-1}^{n} P(A_i)\,P(A/A_i) = \sum_{i=1}^{n} P\ (A \cap A_i)$$

Provided P $(A_i) \neq 0$, i = 1, 2, 3...... n

10.16 Bayes' Theorem

If the events A_1, A_2...An are pair-wise independent and $A_1 U A_2$..........UA_n = S, then for any event A

$$P(A_k/A) = \frac{P(A_k)P(A/A_k)}{P(A)} = \frac{P(A_k)P(A/A_k)}{\sum_{i=1}^{n} P(A_i)P(A/A_i)}$$

Bayes' theorem serves to connect the conditional probability of A_k, given P(Ak), with the conditional probability of A, given Ai and the probability of A_i themselves. If the evens A_i are called causes then the above relation can be regarded as a formula for the probability that the event A, which has occurred, is the result of the cause A_k.

The probabilities P (A_i), i = 1, 2, 3.........n are called 'a priori probabilities' because they exist before we get any information from the experiments itself. The probabilities P (A/A_i), i = 1, 2 ...n are called 'posteriori' probabilities, because they are determined after the results of the experiment are known.

10.17 Determination of Average Grade of Ore in a Vertical Section : The average grade of the core of vertical section is shown in Figure 10.4.

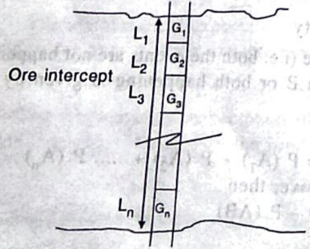


Figure 10.4 : Length and Grade of Section

Length	Grade	Length*Grade
L_1	G_1	L_1*G_1
L_2	G_2	L_2*G_2
L_3	G_3	L_3*G_3
....	....	....
....	....	....
L_n	G_n	L_n*G_n
$\sum_n L_i$	$\sum_n G_i$	$\sum_n L_i*G_i$

The average grade is given by

$$\bar{g} = \frac{\sum_n L_i G_i}{\sum_n L_i}$$

10.18 **Tonnage Factor :** In mining, payment is normally made on the basis of the weight of the valuable material contained. The conversion from volume 'V' to weight 'W' and vice versa is done with the help of a tonnage factor (TF).

TF = Volume (V)/ Weight (W)

If the specific gravity of the mined material is 'S', the density of the mined material is given by

$$\rho_m = S*\rho_w$$

Where, ρ_w = density of water
ρ_m = density of mined out material

$$\text{Average grade of the ore actually mined} = \frac{X_1G_1 + X_2G_2 - X_3G_3}{X_1 + X_2 + X_3}$$

Where,
X_1 = Tonnage of ore in place
X_2 = tonnage of diluted ore
X_3 = tonnage of ore loss
G_1 = grade of ore in place
G_2 = grade of diluted ore
G_3 = grade of loss ore

10.19 Determination of Average Grade of Ore in a Horizontal Section :

10.19.1 **Triangular Metod :** The average grade of horizontal section shown in Figure 10.5 can be determined using triangular method.

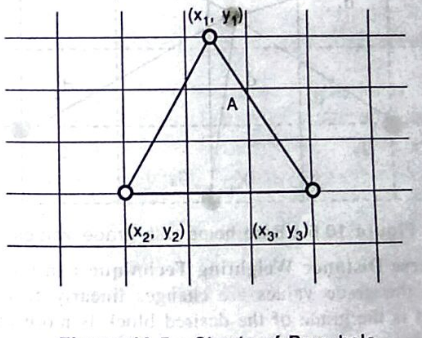


Figure 10.5 : Simple of Bore hole

The average grade of the triangular is given by

$$\bar{g} = \frac{g_1 + g_2 + g_3}{3} \quad \text{[For the same borehole thickness]}$$

If the thickness of the borehole is different

$$\bar{g} = \frac{g_1 t_1 + g_2 t_2 + g_3 t_3}{t_1 + t_2 + t_3}$$

Average thickness is given by

$$\bar{t} = \frac{t_1 + t_2 + t_3}{3}$$

10.19.2 **Polygons :** In the polygon method, each drill hole lies within the centre of a polygon. The polygon is constructed such that its boundary is always equidistant from the nearest neighbouring hole. Within the polygon, the grade is assumed constant and equal to that of the hole it includes.

10.19.3 **Constant Distance Weighting Technique :** The grade of any point can be calculated on the basis the grade of the neighbourhood holes (Shown in Figure 10.6) and the distance between the desired point and the neighbourhood holes. It is given by

$$\text{Average grade of the desired block (P)} = \bar{g} = \frac{\sum_{i=1}^{n} \frac{g_i}{d_i}}{\sum_{i=1}^{n} \frac{1}{d_i}}$$

Where,

g_i is the given grade at a distance d_i away from the desired point, P

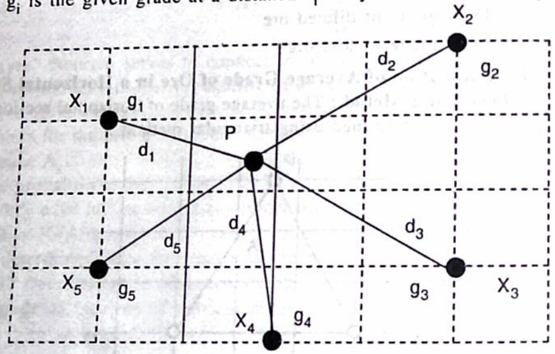


Figure 10.6 : Bore holes with grade values

10.19.4 **Inverse Distance Weighting Technique :** In the above formula, it was assumed that the grade values are changes linearly from the hole to the desired point. That is the grade of the desired block is more close to the nearer points than those far away. To emphasize this dependence, the weighting with distance can be increased. This is done by changing the power of di. If the dependence varies inversely with the square of the distance rather than linearly, the method is called inverse distance weighting method. Thus the grade of the desired block can be calculated by

$$\text{Average grade of the desired block (P)} = \bar{g} = \frac{\sum_{i=1}^{n} \frac{g_i}{d_i^2}}{\sum_{i=1}^{n} \frac{1}{d_i^2}}$$

10.20 Applications of Statistics on the Orebody

Average grade is given by $\bar{g} = \frac{1}{n}\sum_{i=1}^{n} g_i$

Variance in the grade values $= \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(g_i - \bar{g}_i)^2$

Therefore, Standard Deviation $= \sqrt{\text{Variation}} = \sigma$

Determination of range of influence, lag distance and variogram: The boreholes with corresponding grades are shown in Figure 10.7.

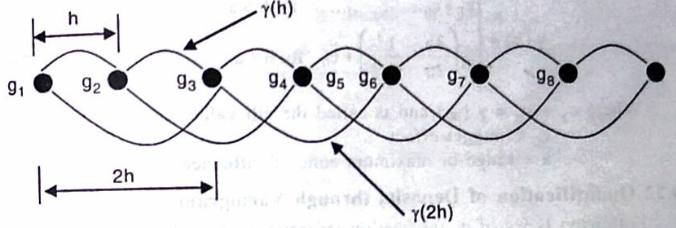


Figure 10.7 : Bore holes with Grade value

In the above figure,

h represents the distance between consecutive bore hole and is called the lag distance

g_i represents the grade values of the respectable holes

$\gamma(h)$ represents the geostatistical variance or the semi-variance

Geostatistical variance can be determined as

Geostatistical Variance for the lag distance 'h'= γ (h) = (Statistical Variance)/ 2

$$\frac{1}{2}s^2(h) = \frac{1}{2}\left[\frac{1}{n(h)}\sum_{n}(g_i - g_j)^2\right]$$

Geostatistical Variance for the lag distance '2h' = γ (2h) = (Statistical Variance)/2

$$\frac{1}{2}S^2(2h) = \frac{1}{2}\left[\frac{1}{n(2h)}\sum_{n-1}(g_i - g_j)^2\right]$$

10.21 **Trend of the Curve of Geostatistical Variance vs. Lag Distance :** There are various types of model like the linear, and the Spherical Model. But the

Spherical model (Figure 10.8) is one in which the variogram reaches a finite value as h increases indefinitely. This finite value, referred to as the sill of the spherical variogram, is the overall variance of the deposit.

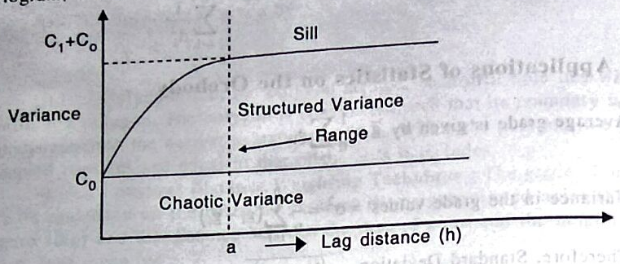


Figure 10.8 : Spherical Model

The spherical scheme is defined by the formula

$$\gamma(h) = \begin{cases} c_1 + c_0 \cdots\cdots & \cdots\cdots \text{ for } h > a \\ c_1\left(\dfrac{3h}{2a} - \dfrac{h^3}{2a^3}\right) + c_0 & \text{for } h \le a \end{cases}$$

Where $c_1 + c_0 = \gamma(\infty)$ and is called the sill value.
c_0 = nugget effect
a = range or maximum zone of influence

10.22 Quantification of Deposits through Variogram :

Following types of quantification information provided by variograms :

- A measure of continuity of mineralization.
- A measure of the area of influence of a sample.
- A measure of mineral trend of the deposita.

EXERCISE - 10

1. What is the standard deviation of the following series?
83 87 93 109 124 126 126 101 102 108
a. 14.96
b. 15.96
c. 13.96
d. 19.96

2. What is the mean of the following series?
83 87 93 109 124 126 126 101 102 108
a. 105.9
b. 150.9
c. 103.9
d. 190.9

3. What is the coefficient of variation of the following series?
83 87 93 109 124 126 126 101 102 108
a. 14.1
b. 15.9
c. 13.9
d. 19.9

4. The measures of dispersion are
a. range, variance and standard deviation
b. mean, median and variance
c. mean, mode and Skewness
d. mean, range and variance

5. The measure of central tendencies are
a. mean, median, and mode
b. mean, median and variance
c. range, variance and standard deviation
d. mean, variance and standard deviation

6. Identify the correct statement for a 'normal distribution'.
a. Mean is greater than mode but less than median
b. Mean is less than mode but greater than median
c. Mean is greater than mode and median
d. Mean, median and mode are equal [GATE 2013]

7. A measure of dispersion of a sample of data set is
a. Mean
b. Median
c. Mode
d. Standard deviation [GATE 2009]

8. In a normal distribution curve, the area between one standard deviation from mean on either side in percent is
a. 50
b. 68
c. 86
d. 95 [GATE 2009]

9. In a normal distribution curve, the area between two standard deviation from mean on either side in percent is
a. 50
b. 68
c. 86
d. 95 [GATE 2009]

10. If X is a continuous random variable and f(x) defines its probability density function, then the expected value of X is
a. $\int_{-\infty}^{+\infty} f(x)\,dx$
b. $\int_{-\infty}^{+\infty} x.f(x)\,dx$
c. $\sum_{i=-\infty}^{+\infty} x_i$
d. $\sum_{i=-\infty}^{+\infty} x_i$ [GATE 2008]

11. Daily production measured for a period of 50 days in a coal mine exhibits normal distribution with mean 1200 tpd and standard deviation 100 tpd. The 95 % confidence interval of daily production (standard normal variable z at 0.025 level of significance is 1.96) in tpd is
a. 1200 ± 120.5
b. 1200 ± 96.0
c. 1200 ± 39.6
d. 1200 ± 27.7 [GATE 2008]

12. In an iron ore deposit alumina is distributed with $\mu = 3\%$ and $\sigma = 0.5\ \%$; whereas silica is distributed with $\mu = 2.5\ \%$ and $\sigma = 0.8\ \%$. The combined alumina and silica (as impurities) has μ and σ, in percentage respectively as
a. (5.5, 0.94)
b. (5.5, 1.3)
c. (0.5, 0.3)
d. (5.5, 0.62) [GATE 2008]

13. The mean and the standard deviation of the grade of iron ore in a deposit are 62% and 5% respectively. The coefficient of variation of the grade in % is
a. 24.8 b. 12.4
c. 8.0 d. 4.0 [GATE 2006]

14. The correlation coefficient between two variables y (dependent) and x is 0.5. Given that the coefficient of determination is the ratio of explained variation and total variation of the regression model between the two variables y and x, the percentage of the total variation remains unexplained by the regression equation is
a. 25 b. 50
c. 60 d. 75 [GATE 2014]

15. A random variable, X is described by a probability density function

$$f(x) = \begin{cases} 3x^2, 0 \le x \le 1 \\ 0, \text{elsewhere} \end{cases}$$

What will be the cumulative distribution function?
a. x b. x^2
c. x^3 d. 2/3 x^2

16. A random variable, X is described by a probability density function

$$f(x) = \begin{cases} 3x^2, 0 \le x \le 1 \\ 0, \text{elsewhere} \end{cases}$$

What will be the expectation E(X)?
a. 2/3 b. 3/4
c. 1/4 d. 4/5

17. A random variable, X is described by a probability density function

$$f(x) = \begin{cases} 3x^2, 0 \le x \le 1 \\ 0, \text{elsewhere} \end{cases}$$

What will be the variance of X?
a. 0.0375 b. 0.0435
c. 0.0325 d. 0.0425

18. If the variance of random variable X is 0.76, what is the variance of random variable 2X?
a. 3.04 b. 2.34
c. 4.02 d. 3.40

19. A random variable, X is described by a probability density function

$$f(x) = \begin{cases} 3x^2, 0 \le x \le 1 \\ 0, \text{elsewhere} \end{cases}$$

What will be the value of P [X ≥ 0.6]?
a. 0.784 b. 0.765
c. 0.576 d. 0.450

20. The grade of copper in a deposit is represented by the following probability density function

$$f(x) = \begin{cases} x/2 & \text{for } 0 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

The average grade of copper in the deposit is
a. 1.00 b. 1.33
c. 1.55 d. 2.00

21. Suppose the time in minutes that a shovel has to wait at a production bench in opencast mine for a dumper, is found to be a random phenomenon, a probability function specified by the distribution function.

$$\begin{array}{ll} f(t) = 0 & \text{for } t \le 0 \\ \quad = 1/2 & 0 \le t \le 1 \\ \quad = 0 & 1 \le t \le 2 \\ \quad = 1/4 & 2 \le t \le 4 \\ \quad = 0 & t \ge 4 \end{array}$$

What is the probability that the shovel will have to wait more than 3 minutes?
a. 1/4 b. 2/3
c. 4/5 d. 1/2

22. In the above question, what is the probability that the shovel will have to wait between 1 and 3 minutes?
a. 1/2 b. 1/3
c. 1/4 d. 1/5

23. An iron ore mine recorded an average of 3 accidents per month. The number of accidents is distributed according to Poisson distribution. The probability that there will be exactly 2 accidents per month is
a. 0.22 b. 0.30
c. 0.43 d. 0.67 [GATE 2012]

24. A set of measurements is found to be normally distributed with mean μ and standard deviation σ. The interval defined by (μ ± a) includes 68 % of the measurements. The value of a is
a. 0.5σ b. σ
c. 2σ d. 3σ [GATE 2006]

25. A set of measurements is found to be normally distributed with mean μ and standard deviation s. The interval defined by (μ ± a) includes 95 % of the measurements. The value of a is
a. 0.5σ b. σ
c. 2σ d. 3σ

26. The life in hours of a radio tube has the density $f(x) = 100/x^2$, when $x \ge 100$ and zero when $x < 100$. What is the probability that none of three such tubes in a given radio set will have to be replaced during the first 150 hours of operation?
a. 1/27 b. 1/21
c. 2/27 d. 8/27

27. The life in hours of a radio tube has the density $f(x) = 100/x^2$, when $x = 100$

and zero when x < 100. What is the probability that all three of the original tubes in a given radio set will have to be replaced during the first 150 hours of operation?

a. 1/27 b. 1/8

c. 2/27 d. 8/27

28. Pull from an underground tunnel blasting is normally distributed with a mean of 100 tonnes and variance 100 (tonnes)^2. The probability that the tonnage value from a blast exceeds 110 is

a. 0.60 b. 0.80

c. 0.16 d. 0.32 [GATE 2010]

29. A bombing plane carrying three bombs flies directly above a railroad track. If a bomb falls within 10 m of the track, the track will be sufficiently damaged to disrupt traffic. With a certain sight, the density of points of impact of a bomb is

$$f(x)=\begin{cases}\dfrac{(100+x)}{10{,}000}, -100<x<0\\ \dfrac{(100+x)}{10{,}000}, 0<x<100\\ 0, \text{elsewehre}\end{cases}$$

x represents the vertical deviation from the aiming point which is the track in this case. If all the bombs are used, what is the probability that the track will not be damaged?

a. $1-(81/100)^3$ b. $1-(64/81)^3$

c. $1-(64/100)^3$ d. $1-(19/20)^3$

30. If f(x) and g(x) are real functions defined in the interval a<x<b, such that

$\int_a^b f^2(x)\,dx$ and $\int_a^b g^2(x)\,dx$ are finite then

a. $\left[\int_a^b f(x)g(x)dx\right]^2 \le \int_a^b f^2(x)\,dx \int_a^b g^2(x)\,dx$

b. $\left[\int_a^b f(x)g(x)dx\right]^2 \ge \int_a^b f^2(x)\,dx \int_a^b g^2(x)\,dx$

c. $\left[\int_a^b f(x)\,g(x)\,dx\right]^2 = \int_a^b f^2(x)\,dx \int_a^b g^2(x)\,dx$

d. None of the above

31. If X and Y are random variables taking real values then

a. $[E\,(XY)]^2 \le E(X)^2\,E\,(Y)^2$

b. $[E\,(XY)]^2 \ge E(X)^2\,E\,(Y)^2$

c. $[E\,(XY)]^2 = E(X)^2\,E\,(Y)^2$

d. None of the above

32. Let X be uniformly distributed over (0, 1), E(X^3) is

a. 1/3 b. 1/8

c. 1/4 d. 1/5 [GATE 2002]

33. The random variable X has the following probability mass function

P (4) = 1/4, P (8) = 1/4, P (12) = 1/4, P (16) = 1/4

The expected value of X is

a. 1 b. 3

c. 10 d. 12

34. In a binomial distribution, the probability of success $p \to 0$ and number of trials $n \to \infty$ such that $\lambda = np$ approaches to a finite value. The variance of the distribution is

a. $np\lambda$ b. $n\lambda$

c. $P\lambda$ d. λ [GATE 2015]

35. A and B throw one dice for a prize of Rs. 11 which is to be won by a player who first throws 6. If A has the first throw, what is the expectation of A?

a. Rs. 6 b. Rs. 5

c. Rs. 8 d. Rs. 11

36. A and B throw one dice for a prize of Rs. 11 which is to be won by a player who first throws 6. If A has the first throw, what is the expectation of B?

a. Rs. 6 b. Rs. 5

c. Rs. 8 d. Rs. 11

37. From a bag containing 5 one rupee coins and 3 coins of 20 paisa each, a person is allowed to draw 2 coins indiscriminately. Find the value of his expectation.

a. Rs. 1.40 b. Rs. 2.40

c. Rs. 3.40 d. Rs. 4.40

38. An urn contains n tickets numbered from 1 to n and m tickets are drawn one by one without replacement from the urn. What is the mathematical expectation of the sum of the numbers drawn?

a. m(n + 1)/2 b. n(m + 1)2

c. mn/2 d. m/n

39. For an oil exploration drilling, chance of striking an oil reservoir is 1 out of 15. If an oil exploration company decides to explore 5 sites, the probability of striking at least one successful oil reservoir is

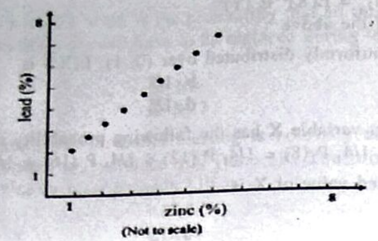
a. 0.292 b. 0.250

c. 0.034 d. 0.0024 [GATE 2011]

40. A scatter plot prepared using a set of values of lead and zinc from a lead-zinc deposit is shown in figure below. The value of correlation coefficient is

a. 1.0 b. 0.7

c. 0.5 d. 0 [GATE 2011]

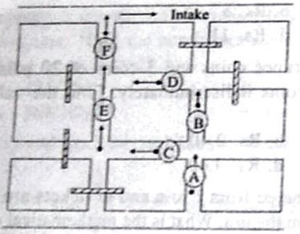


(Not to scale)

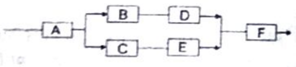
41. A company manufactured product has 2 defects per unit of product inspected. What will be the probabilities of finding a product without any defect using Poisson distribution?

a. 0.135 b. 0.143
c. 0.153 d. 0.125

42. A sudden increase of CO incidence has occurred in an underground mine section. A man at point A starts to run out to the main intake of the mine where he will be safe. Refer figure below for the mine section and the logic diagram. The probabilities that he will successfully cross the gallery sections A, B, C, D, E, and F are 0.9, 0.8, 0.7, 0.8, 0.7 and 0.9 respectively. The probability that he will successfully reach the main intake is



Section of the mine



Series-parallel logic diagram

a. 0.40 b. 0.51
c. 0.66 d. 0.77 [GATE 2011]

43. For an explosives company, the probability of producing a defective detonator is 0.02. The probability that a lot of 50 detonators produced by the company contains at most 2 defective detonators is

a. 0.621 b. 0.921
c. 0.821 d.0.721 [GATE 2014]

44. A company manufactured product has 2 defects per unit of product inspected. What will be the probabilities of finding a product with 3 defects using Poisson distribution?

a. 0.135 b. 0.180
c. 0.153 d. 0.125

45. If the probability is 0.4 that an engineering job applicant has a post graduate degree, 0.6 that he has some work experience as an engineer, and 0.3 that he has both, the probability that he would have a post graduate degree or some work experience is

a. 0.3 b. 0.1
c. 0.7 d. 0.8 [GATE 2002]

46. One fifth percent of the bolts produced by a bolts manufacturing company turn out to be defective. The bolts are supplied in packets of 10. What will be the approximate number of packets containing no defectives in a consignments of 1, 00,000, if the distribution is assumed Poisson?

a. 98020 b. 96020
c. 95020 d. 94020

47. In a mining industry, the average number of fatal accidents per month is one-half. The number of accidents per month is adequately described by a Poisson distribution. What is the probability that a months will pass without a fatal accidents?

a. 0.606 b. 0.706
c. 0.806 d. 0.906

48. A bauxite deposit has been intersected by 5 drill holes. The values of alumina (% by weight) and silica (% by weight) in these drill holes are as follows :

Drill hole number	Alumina (%)	Silica (%)
1	46	1
2	42	5
3	45	2
4	43	4
5	44	3

The relationship between alumina and silica is

a. positive linear b. exponential
c. negative linear d. random

49. In the above question the unbiased estimate of variances of alumina and silica in $(\%)^2$ respectively are

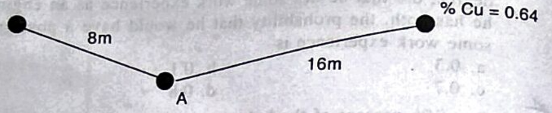
a. 2.5 2.5 b. 2.0 2.5
c. 2.5 2.0 d. 2.0 2.0

50. The grade values of 30 iron ore samples in a mine are normally distributed. The sample mean (Fe % by weight) and the standard deviation of these sample observations are 60% and 4% respectively. The 95% confidence limits for the population- mean are (given $Z_{0.975} = 1.96$)

a. 60 ± 1.43 b. 60 ± 1.79
c. 60 ± 2.01 d. 60 ± 3.50

51. The grade of copper (%) at point A using the inverse distance weighting method is

%.Cu=0.56



a. 0.47 b. 0.58
c. 0.61 d. 1.20

52. Assume the grade at A to be the average grade of copper, mill recovery to be 85 % and the smelting & refining loses to be 1 kg of copper per tonne of ore. The amount of saleable copper in kg/te of ore is

a. 2.896 b. 3.896
c. 4.896 d. 5.896

53. Five sample observations have been taken from a lead-zinc deposit. The observed values of zinc % and lead % are given in the following table

Zinc %	Lead%
6.0	4.2
5.2	3.0
6.0	6.8
2.4	2.0
4.0	3.2

The sample covariance between two variables is

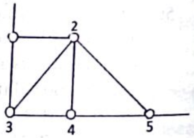
a. 0.80 b. 1.72
c. 2.14 d. 3.00

54. The thickness of a coal deposit is represented by a spherical semi-variogram model with sill of 5m^2. If the semi-variogram value at lag distance (h) is 3m^2, the correlogram value at the same lag distance is

a. 0.4 b. 2.0
c. 2.5 d. 5.0 [GATE 2011]

55. A flat bauxite deposit has thickness of 10 m with an average density of 2200 kg/m^3. The grade values and the sample coordinates are as shown in the table. To carry out reserve estimation using triangular method, the triangles are constructed as shown in the figure.

Sample No	1	2	3	4	5
Alumina %	3	4	3	4	4
x coordinate, m	0	20	0	20	50
y coordinate, m	30	30	0	0	0



The alumina content in million tonnes, in the region comprising the three triangles is

a. 0.00726 b. 0.00826
c. 0.00926 d. 0.01026 [GATE 2013]

56. Economic analysis of an iron ore deposit reveals that the net value of the ore is related to the grade mined as shown in the table.

Grade (% Fe)	Net value of ore (Rs/tonne)
64.5	3200
60.2	1800

Assuming linear relationship between the net value and grade, the break-even cut-off grade in % Fe is

a. 52.2 b. 54.7
c. 58.0 d. 62.2 [GATE 2013]

57. In the above problem, assuming that the grade follows normal distribution with mean 62.7%, and standard deviation 10.0% (A portion of the standard normal distribution table is given below),

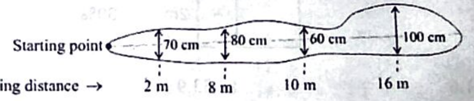
z	0.00	0.01	0.02	0.03	0.04
0.6	0.72575	0.72907	0.73237	0.73565	0.73891
0.7	0.75803	0.76115	0.76424	0.76730	0.77035
0.8	0.78814	0.79103	0.79389	0.79673	0.79954
0.9	0.81594	0.81859	0.82121	0.82381	0.82639
1.0	0.84134	0.84375	0.84613	0.84849	0.85083

The percentage of waste in the deposit based on the break-even cut-off grade is

a. 78.8 b. 71.2
c. 28.8 d. 21.2 [GATE 2013]

58. The chip sampling data, spaced irregularly for a gold vein deposit, are shown in figure. The sample points have equal influence on both the sides.

Distance of sample from starting point (m)	2	8	10	16
Width (cm)	70	80	60	100
Assay (g/te)	5	7	6	4



The mean assay value in g/te is

a. 6.52 b. 5.50
c. 5.19 d. 4.50 [GATE 2012]

59. The injury rates of mine workers in an underground coal mine based on age group are given below :

Age group of mine workers	Age-specific injury rate (per 1000 persons)	Age-specific population in the mine
18-32	1.8	1000
33-46	2.5	500
47-60	4.5	300

The injury rate per 1000 persons employed in the mine for the total population is

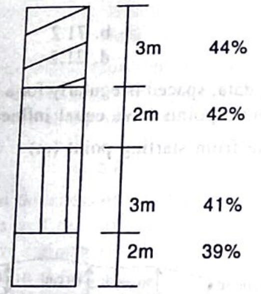
a. 0.24 b. 2.44
c. 8.80 d. 24.40 [GATE 2012]

60. Block economic values in a 2D block model are shown below. Then based on the assumption of 1:1 slope angle, the blocks (identified by row and column numbers) that constitute the ultimate pit are

	1	2	3	4
1	−1	−1	1	−1
2	−1	1	3	−1
3	−1	−1	−1	−1

a. (1,1), (1,2), (1,3), (2,2) b.(1,2), (1,3), (1,4), (2,3)
c. (1,3), (2,4) d.(1,3), (1,4), (2,4) [GATE 2008]

61. An assay value of alumina in a borehole from a bauxite deposit is as shown below. If the cut-off grade is 40 %, the composite value of ore in the borehole in percent is



a. 31.6 b. 33.9
c. 41.7 d. 42.2 [GATE 2008]

62. An iron ore deposit has a mean grade of 63 % Fe. During the course of mining, 30 % fines by weight are generated at a grade of 72 % Fe which are rejected. The effective mean grade of the deposit in Fe percentage is

a. 59.1 b. 53.1
c. 50.4 d. 41.4 [GATE 2009]

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63. A mine having a reserve of 320 Mt produces 4 Mt of ore at the end of 1st year. If the mine increases production by 10 % every year, the percentage of the reserve that still remains at the end of 21st year is

a. 50 b. 35
c. 25 d. 20 [GATE 2009]

64. Cause wise data of injuries in an underground coal mine for a five-year period is given below :

Cause of injury	Number of injuries
Fall of roof	27
Fall of person	22
Rope haulage	17
Explosives	5
Other causes	4

The cumulative probability of injury due to fall of roof and fall of person is

a. 0.65 b. 0.50
c. 0.36 d. 0.29 [GATE 2009]

65. The following data represent the number of workers suffering from pneumoconiosis in 10 coal mines.

Mine	I	II	III	IV	V	VI	VII	VIII	IX	X
Number	10	16	14	15	14	12	17	13	15	12

The number of mines falling above the 50th percentile in terms of the number of workers suffering from pneumoconiosis is

a. 2 b. 3
c. 4 d. 5 [GATE 2009]

66. The covariance of copper grade for a certain lag distance in an ore body is 6.0 (%)2. If the sill is 10 (%)2, the semivariogram for the same lag distance in (%)2 is

a. 4.0 b. 16.0
c. 2.0 d. 64.0 [GATE 2015]

Answer

1. a 2. a 3. d 4. a 5. a 6. d 7. d
8. b 9. d 10. b 11. d 12. a 13. c 14. d
15. c 16. b 17. a 18. a 19. a 20. b 21. a
22. c 23. a 24. b 25. c 26. a 27. d 28. c
29. a 30. b 31. a 32. c 33. c 34. d 35. a
36. b 37. a 38. a 39. a 40. a 41. a 42. c
43. b 44. b 45. c 46. a 47. a 48. c 49. d
50. a 51. b 52. b 53. b 54. a 55. a 56. b
57. d 58. c 59. b 60. b 61. c 62. a 63. d
64. a 65. c 66. a

Solutions of Exercise 10

1. Given data :

83 87 93 109 124 126 126 101 102 108

The standard deviation of the above data set is given by

$$\text{Group Standard Deviation } (\sigma) = \sqrt{\left\{\frac{1}{N}\sum_i (x_i - \bar{x})^2\right\}}$$

Where, x_i is the ith data
N is number of data
$\bar{x}$ is mean of the data

Therefore

$$\sigma = \sqrt{\left[\frac{1}{10}\begin{Bmatrix}(83-105.9)^2+(87-105.9)^2+(93-105.9)^2+(109-105.9)^2+\\(124-105)^2(126-105.9)^2+(126-105.9)^2+(101-105.9)^2+\\(102-105.9)^2+(108-105.9)^2\end{Bmatrix}\right]}$$

$$\Rightarrow \sigma = 14.96$$

2. Given data :

83 87 93 109 124 126 126 101 102 108

The mean of the above data set is given by

$$\text{Mean } (\bar{x}) = \frac{\sum_i^N x_i}{N} = \frac{83+87+93+109+124+126+126+101+102+108}{10} = 105.9$$

3. Given data :

83 87 93 109 124 126 126 101 102 108

The coefficient of variation of the above data set is given by

$$\text{Coefficient of Variation} = \frac{\text{Standard Deviation}(\sigma)}{\text{Mean (M)}} * 100\% = \frac{14.96}{105.9} * 100$$

$$= 14.12$$

11. Given data :

Mean production (μ) = 1200 tpd

Standard deviation (σ) = 100 tpd

Number days/sample (n) = 50

Standard normal variable z at 0.025 level of significance = $z_{0.025}$ = 1.96

Therefore, daily production at 95% confidence level is given by

$$Q = \mu \pm 1.96\frac{\sigma}{\sqrt{n}} = 1200 \pm 1.96\frac{100}{\sqrt{50}} = 1200 \pm 27.7$$

12. Given data :

Alumina is distributed in iron ore deposit as $(\mu_a \pm \sigma_a) = 3\% \pm 0.5\ \%$
Silica is distributed in iron ore deposit as $(\mu_s \pm \sigma_s) = 2.5\% \pm 0.8\ \%$
Let the combined mean and standard deviation of impurities are μ and s respectively.

We have,

Comibined mean, $\mu = \mu_a + \mu_s = 3 + 2.5 = 5.5$

= combined standard deviation, σ

$$= \sqrt{\text{Variance of alumina + Variance of silica + Covariance of Alumina and Silica}}$$

$$\Rightarrow \sigma = \sqrt{a_a^2 + \sigma_s^2 + 0} = \sqrt{0.5^2 + 0.8^2} = \sqrt{0.89} = 0.943$$

13. Given data :

Mean grade percentage (μ) = 62%

Standard deviation (σ) = 5%

The coefficient of variation of the grade can be determined as

$$\text{Coefficient of Variation} = \frac{\text{Standard Deviation}(\sigma)}{\text{Mean}(\mu)} * 100\% = \frac{5}{62} * 100$$

$$= 8.06\%$$

14. The correlation coefficien (r) between y (dependent) and x (independent variables is 0.5

The coefficient of determination can be determined as

Coefficient of determination (r^2) = 0.5^2 = 0.25 = 25%

The coefficient of determination represents the percent of data that is the closest to the line of best fit. Therefore, the value of coefficient of determination, means that 25 % of the total variation in y can be explained by the linear relationship between x and y. The other 75 % of the total variation remains unexplained.

15. The given probability density function is

$$f(x) = \begin{cases} 3x^2, 0 \le x \le 1 \\ 0, \text{elsewhere} \end{cases}$$

The cumulative probability density can be determined as

$$F(x) = P(-\infty \le X \le \infty) = \int_{-\infty}^{+\infty} f(x)dx$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{+\infty} f(x)\,dx$$

$$\Rightarrow F(x) = \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} 3x^2 dx + \int_{1}^{+\infty} 0\,dx = 0 + x^3 + 0 = x^3$$

16. Given data :

$$f(x)=\begin{cases}3x^2, 0\le x\le 1\\ 0, \text{elsewhere}\end{cases}$$

The expectation, E(x), can be determined as

$$E(X)=\int_{-\infty}^{+\infty} xf(x)dx=\int_{-\infty}^{0} xf(x)\,dx+\int_0^1 xf(x)\,dx+\int_1^{+\infty} x\,f(x)dx$$

$$\Rightarrow E(X)=\int_{-\infty}^{+\infty} x*0\,dx+\int_0^1 x*3x^2\,dx+\int_1^{+\infty} x*0\,dx$$

$$\Rightarrow E(X)=0+\int_0^1 x*3x^2dx+0=\int_0^1 3x^3\,dx=\frac{3}{4}\left[x^4\right]_0^1$$

$$\Rightarrow E(X)=\frac{3}{4}\left(1^4-0\right)=\frac{3}{4}$$

17. The given probability density function is

$$f(x)=\begin{cases}3x^2, 0\le x\le 1\\ 0, \text{elsewhere}\end{cases}$$

The mean of a random variable x, denoted as μ, is equal to the expected value i.e.

μ = E(X) = 3/4 (determined in above solution)

The variance of X can be determined as

$$\text{Variance}\left(\sigma^2\right)=\int_{-\infty}^{+\infty}(x-\mu)^2\,f(x)\,dx$$

$$\Rightarrow \sigma^2=\int_{-\infty}^{0}(x-\mu)^2 f(x)\,dx+\int_0^1(x-\mu)^2 f(x)dx+\int_1^{+\infty}(x-\mu)^2\,f(x)\,dx$$

$$\Rightarrow \sigma^2=\int_{-\infty}^{0}(x-\mu)^2*0\,dx+\int_0^1(x-\mu)^2 3x^2dx+\int_1^{+\infty}(x-\mu)^2*0\,dx$$

$$\Rightarrow \sigma^2=0+\int_0^1(x-3/4)^2*3x^2\,dx+0\int_0^1\left(x^2-\frac{3}{2}x+\frac{9}{10}\right)*3x^2dx$$

$$\Rightarrow \sigma^2=\int_0^1\left(3x^4-\frac{9}{2}x^3+\frac{27}{16}x^2\right)dx=\left[\frac{3}{5}x^5-\frac{9}{8}x^4+\frac{27}{16}x^3\right]_0^1$$

$$=\frac{3}{5}*1-\frac{9}{8}*1+\frac{9}{16}*1$$

$$\Rightarrow \sigma^2=0.0375$$

18. Variance of a random variable, X is 0.76

Therefore, the variance of random variable, cX is given by

Var (cX) = c^2 Var (X)

$$\Rightarrow \text{Var}(2X)=2^2\,\text{Var}(X)=4*0.76=3.04$$

19. The given probability density function of a random variable x is

$$f(x)=\begin{cases}3x^2, 0\le x\le 1\\ 0, \text{elsewhere}\end{cases}$$

The probability that the value is greater and equal to 0.6 is given by

$$P(X\ge 0.6)=\int_{0.6}^{+\infty} f(x)\,dx=\int_{0.6}^{1} f(x)\,dx+\int_1^{+\infty} f(x)\,dx=\int_{0.6}^{1} 3x^2dx+0$$

$$\Rightarrow P(X\ge 0.6)=\left[x^3\right]^1{}_{0.6} \qquad =1-0.6^3=1-0.216=0.784$$

20. The grade of copper is represented by following distribtion function :

$$f(x)=\begin{cases}\dfrac{x}{2}, \text{ for } 0<x<2\\ 0, \text{ otherwise}\end{cases}$$

The average grade of copper can be determined as :

$$\text{Mean grade}(\mu)=E(X)=\int_{-\infty}^{+\infty} xf(x)\,dx$$

$$\Rightarrow \mu=\int_{-\infty}^{0} xf(x)\,dx+\int_0^2 xf(x)\,dx+\int_2^{+\infty} xf(x)\,dx$$

$$\Rightarrow \mu=\int_{-\infty}^{0} x*0\,dx+\int_0^2 x*\frac{x}{2}\,dx+\int_2^{+\infty} x*0\,dx=0+\int_0^2\frac{1}{2}x^2dx+0$$

$$\Rightarrow \mu=\left[\frac{x^3}{6}\right]_0^2=\left[\frac{8}{6}-\frac{0}{6}\right]=1.33$$

21. The probability density function of waiting time is given as :

$$f(t)=\begin{cases}0, & \text{for } t\le 0\\ \dfrac{1}{2}, & 0\le t\le 1\\ 0, & 1\le t\le 2\\ \dfrac{1}{4}, & 2\le t\le 4\\ 0, & t\le 4\end{cases}$$

The probability that the person will have to wait more than 3 minutes is

$$P(t>3)=1-P(t\le 3)=1-\int_{-\infty}^{3} f(t)\,dt$$

$$\Rightarrow p(t>3)=1-\left[\int_{-\infty}^{0} f(t)\,dt+\int_0^1 f(t)\,dt+\int_1^2 f(t)dt+\int_2^3 f(t)\,dt\right]$$

$$\Rightarrow p(t>3)=1-\left[\int_{-\infty}^{0} 0\,dt+\int_0^1\frac{1}{2}dt+\int_1^2 0\,dt+\int_2^3\frac{1}{4}\,dt\right]$$

$$\Rightarrow p(t>3)=1-0-\left[\frac{x}{2}\right]_0^1-0-\left[\frac{x}{4}\right]_2^3=1-\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$$

22. The probability that the person will have to wait between 1 and 3 minutes is

$$P(1 \le t \le 3) = \int_1^3 f(t)\,dt = \int_1^2 f(t)\,dt + \int_2^3 f(t)\,dt$$

$$\Rightarrow P(1 \le t \le 3) = \int_1^2 0\,dt + \int_2^3 \frac{1}{4}\,dt$$

$$\Rightarrow P(1 \le t \le 3) = 0 + \left[\frac{t}{4}\right]_2^3 = \frac{1}{4}$$

23. Average accident rate (λ) = 3/month

The number of accidents is distributed according to Poisson distribution. The probability function of Poisson distribution is

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

Where, P (x) = probability of x number of occurrence

λ = average rate

x = number of occurences

Therefore, the probability that there will be exactly 2 accidents per month is given by

$$P(x=2) = \frac{2.718^{-3} * 3^2}{2!} = \frac{0.05*9}{2} = 0.224$$

26. The given probaility density function of life of a radio tube, x is

$$f(x) = \begin{cases} \dfrac{100}{x^2} & \text{fox } x > 100 \\ 0, & \text{fox } x < 100 \end{cases}$$

The probability that one of such tube in a given radio set will not have to be replaced during first 150 hrs of operation is given by

$$P(100 \le x \le 150) = \int_{100}^{150} f(x)dx$$

$$\Rightarrow P(100 \le x \le 150) = \int_{100}^{150} \frac{100}{x^2}dx = \left[-\frac{100}{x}\right]_{100}^{150} = -\frac{100}{150} + \frac{100}{100} = \frac{1}{3}$$

Therefore, the probability that none of three tube in a given radio set will not have to be replaced during first 150 hrs of operation is = $(1/3)^3$ = 1/27

27. *In the above question, the probability that one tube in a given radio set will have to be replaced during first 150 hrs of operation is given by*

$$P(x < 150) = 1 - P(100 \le x \le 150) = 1 - \frac{1}{3} = \frac{2}{3}$$

Hence the probability that all three tubes failing during first 150 hours is $(2/3)^3$ = 8/27

28. Given data :

Mean value of extraction (μ) = 100 te

Variance (Var) = 100 te^2

Therefore, Standard deviation (σ) = $\sqrt{Var} = 10$ te

It is given that the extraction from tunnel blasting is normally distributed.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}(x-\mu)^2/\sigma^2} = \frac{1}{\sqrt{2*3.14*10}} e^{-\frac{1}{2}(x-100)^2/100} = 0.126e^{-\frac{1}{200}(x-100)^2}$$

We can determine the probability from the standard normal distribution table rather integrating the complex function.

The probability that tonnage value from a blast exceeds 110 is given by

$$P(X > 110) = P(X - \mu > 110 - \mu) = P\left(\frac{x-\mu}{\sigma} > \frac{110-\mu}{\sigma}\right) = P\left(Z > \frac{110-100}{10}\right)$$

$\Rightarrow$ P (z > 1) = 1 − 0.8413 = 0.1587 ≅ 0.16

Note : The value of Z taken from normal distribution table corresponding to z = 0 (in row) and 1 (in column).

29. The density of points of impact of a bomb is given as:

$$f(x) = \begin{cases} \dfrac{100+x}{10000}, & \text{for } -100 < x < 0 \\ \dfrac{100-x}{10000}, & 0 < x < 100 \\ 0, & \text{elsewhere} \end{cases}$$

The probability that the bomb will fall within10 m of rail track is given by

$$P(-10 \le x \le 10) = \int_{-10}^{+10} f(x)dx = \int_{-10}^{0} f(x)\,dx + \int_0^{10} f(x)\,dx$$

$$\Rightarrow P(-10 \le x \le 10) = \int_{-10}^{0} \frac{100+x}{10000}dx + \int_0^{10} \frac{100-x}{10000}dx$$

$$\Rightarrow P(-10 \le x \le 10) = \left[\frac{200x + x^2}{20000}\right]_{-10}^{0} + \left[\frac{200x - x^2}{20000}\right]_0^{10}$$

$$\Rightarrow P(-10 \le x \le 10) = \left[0 - \frac{-2000+100}{20000}\right] + \left[\frac{2000-100}{20000} - 0\right] = \frac{3800}{20000} = 19/100$$

Therefore, the probability that the track will not damage by one bomb =1-19/100 = 81/100

Hence, the probability that none of the bombs will damage = $(81/100)^3$

The probability that the track will damage = 1 - $(81/100)^3$

32. Given that x is uniformly distributed over (0, 1)

Therefore, f(x) = x

We have,

$$E(X^3)=\int_{-\infty}^{+\infty}x^3f(x)dx=\int_{-\infty}^{0}x^3f(x)\,dx+\int_{0}^{1}x^3f(x)dx+\int_{1}^{+\infty}x^3f(x)dx$$

$$\Rightarrow E(x^3)=\int_{-\infty}^{0}x^3*0dx+\int_{0}^{1}x^3 1dx+\int_{1}^{+\infty}x^3*0\,dx$$

$$\Rightarrow E(x^3)=0+\left[\frac{x^4}{4}\right]_0^1+0=1/4$$

33. The probability mass functions of a random variable x are given as :

P(4) = ¼, P(8) = ¼, P(12) = ¼, P(16) = ¼

Expected value of x is given as

$$E(x)=x_1P(x_1)+x_2P(x_2)+x_3P(x_3)+x_4P(x_4)$$

$$\Rightarrow E(x)=4*\frac{1}{4}+8*\frac{1}{4}+12*\frac{1}{4}+16*\frac{1}{4}$$

$$\Rightarrow E(x)=1+2+3+4=10$$

35. Let p is the probability of A's and B's throwing 6 and q is the probability of A's and B's not throwing 6.

Therefore, $p = 1/6$ and $q = 5/6$

The probabilities of A's success in 1st, 2nd ...chances are

$p, q^2p, q^4p, \ldots\ldots$

Expectation of A's wining prize is given by

$E(A) = 11*p + 11*q^2p + 11*q^4p \ldots\ldots\ldots\ldots\infty$

$$\Rightarrow E(A)=11p\left(1+q^2+q^4\ldots\infty\right)$$

$$\Rightarrow E(A)=11p*\frac{1}{1-q^2}=11*\frac{1}{6}*\frac{1}{(1-25/36)}=\frac{66}{11}=\text{Rs. }6$$

36. In the above problem, the probabilities of B's success in 1st, 2nd ... chances are

$qp, q^3p, q^5p, \ldots$

Expectation of B's wining prize is given by

$E(B) = 11*qp + 11*q^3p + 11*q^5p \ldots\ldots\infty$

$\Rightarrow E(B) = 11pq\ (1+ q^2 + q^4 \ldots\ldots\infty)$

$$\Rightarrow E(B)=11pq*\frac{1}{1-q^2}=11*\frac{1}{6}*\frac{5}{6}*\frac{1}{\left(1-\frac{25}{36}\right)}=\frac{11*5*36}{36*11}=\text{Rs. }5$$

37. *Given data :*

Number of 1 rupee coins is 5

Number of 20 paisa coins is 3

Therefore, the number of ways 2 coins can be drawn = $8C_2$

The probability that the 2 coins drawn are 1 rupee = $5C_2/8C_2 = 10/28$

The probability that the 2 coins drawn are 20 paisa = $3C_2/8C_2 = 3/28$

The probability that coins are 1-1 rupee and 1-20 paisa $=5C_1*3C_1/8C_2 = 15/28$

Hence, the value of his expectation = 2*10/28 + 0.4*3/28 + 1.2*15/28

= 39.2/28 = Rs. 1.40

38. Let the variable associated with ith (i = 1 to n) ticket be x

Now expectation of x_i is given by

$$E(x_i)=\sum_{i=1\text{ to }n}p_ix_i=\frac{1}{n}*\sum x_i=\frac{n(n+1)}{2n}=\frac{n+1}{2}$$

[where $p_1 = p_2\cdots = p_n = 1/n$]

Therefore, expected value of the sum of m number of drawn is given by

$$E(s)=\sum_{m}E(x_i)=mE(x_i)=m\frac{(n+1)}{2}$$

39. Given data :

Probability of striking an oil reservoir during exploration = P(x) = 1/15

Therfore, the probability of not striking an oil reservoir during exploration

= P (x') = 1 − 1/15=14/15

Probability of striking at least one successful oil reservoir in 5 attempts is given by

P (x ≥ 1) = 1 − P (x')*P(x')*P(x')*P(x')*P(x')

[All the five sites are independent to each other]

$P(x \geq 1) = 1 - (14/15)^5 = 0.292$

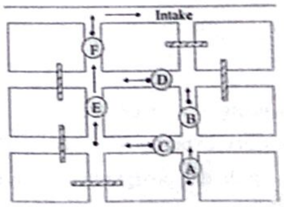
40. Since all the points lie in a straight line, the correlation coefficient is equal to 1.

41. The probability of finding a product without any defect using Poisson distribution is given by

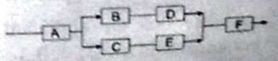
$P(0) = e^{-m} = 2.718^{-2} = 0.135$

[where, m = mean number of defect per unit of product = 2]

42. Given configuration :



Section of the mine



Series-parallel logic diagram

Given data :

Probability of successfully crossing the galley A = P(A) = 0.9

Probability of successfully crossing the galley B = P(B) = 0.8

Probability of successfully crossing the galley C = P(C) = 0.7

Probability of successfully crossing the galley D = P(D) = 0.8

Probability of successfully crossing the galley E = P(E) = 0.7

Probability of successfully crossing the galley F = P(F) = 0.9

The probability that the man will reach the main intake from point A is given by

$$P = P(A) * P\{(B \cap D) \cup (C \cap E)\} * P(F)$$

$$\Rightarrow P = P(A) * [P(B \cap D) + P(C \cap D) - P(B \cap D) * P(C \cap D)] * P(F)$$

[Since all the events are independent to each other, we have $P(X \cap Y)$ $P(X)\ P(Y)$]

$$\Rightarrow P = P(A) * [P(B)P(D) + P(C)P(E) - P(B)P(D) * P(C)P(E)] * P(F)$$

$$\Rightarrow P = 0.9 * [0.8 * 0.8 + 0.7 * 0.7 - 0.8 * 0.8 * 0.7 * 0.7] * 0.9$$

$$\Rightarrow P = 0.9 * [0.64 + 0.49 - 0.3136] * 0.9 = 0.81 * 0.8164 = 0.661284$$

43. The probability of producing a defective detonator is

$$P(d) = 0.02$$

Therefore, the probability of producing a non-defective detonator is

$$P'(d) = 1 - 0.02 = 0.98$$

Probability that a lot of 50 detonators produced by the company contain at most 2 defective detonators is given by

$$P(\leq 2) = P(0) + P(1) + P(2)$$

$$\Rightarrow P(\leq 2) = (0.98)^{50} + 50C_1 * 0.02 * (0.98)^{49} + 50C_2\ (0.02)^2\ (0.98)|^{48}$$

$$\Rightarrow P(\leq 2) = 0.364 + 0.372 + 0.185 = 0.921$$

44. The average defective rate of product manufactured is given as

$$m = 2$$

Therefore, the probability of finding a product with 3 defects using Poisson distribution is given by

$$P(3\ \text{defective iems}) = \frac{m^3}{3!}e^{-m} = \frac{2^3}{6} * 2.718^{-2} = \frac{8}{6} * 0.135 = 0.18$$

45. Given data :

Probability that the applicant has postgraduate degree = P(A) = 0.4

Probability that the applicant has work experience = P(B) = 0.6

Probability that the applicant has both the postgraduate degree and work experience = $P(A \cap B)$ = 0.3

Therefore, the Probability that the applicant has either postgraduate degree or work experience = $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.6 - 0.3 = 0.7$

46. Given data :

Probability of bolts turned out to be defective = p = 1/5 % = 1/500

Probability that a bolts in a pack of 10 turns out to be defective = m= np = 10*1/500 = 1/50 = 0.02

Therefore, the number of packets containing r (r= 0) defective bolts in a consignment of N (=100,000) for Poisson distribution is given by

$$X = Ne^{-m}\frac{m^r}{r!} = 100000 * (2.718)^{-0.02} * 1 = 98020$$

47. Given data :

Average number of fatal accidents in a mine = m= ½ = 0.5

It is given that number of accidents per month described by Poisson distribution.

Therefore, the probability that a months will pass without a fatal accidents is

$$P = e^{-m}\frac{m^r}{r!} = (2.718)^{-0.5}\left(\frac{0.5^0}{0!}\right) = 0.606 * \frac{1}{1} = 0.606$$

48. Given data :

Drill hole number	Alumina (%)	Silica (%)
1	46	1
2	42	5
3	45	2
4	43	4
5	44	3

The above data set in the table clearly indicate that there is an inverse relationship between percentage of alumina and percentage of silica. That is, on linear increasing of one parameter, the other is linearly decreases. Therefore, the relationship is negative linear.

49. In the above data, the unbiased estimate of variances of alumina and silica can be determined as :

Variance of Alumina (V_{al}) = [Standard Deviation (σ)]^2

$$= \left[\sqrt{\left\{\frac{1}{N}\sum_i (x_i - \bar{x}_{al})^2\right\}}\right]^2$$

$$\Rightarrow V_{al} = \frac{1}{5}\left[(46-44)^2 + (45-44)^2 + (44-44)^2 + (43-44)^2 + (43-44)^2\right] = 2$$

Similarly,

Variance of Silica (V_{si}) = [Standard Deviation (σ)]^2

$$= \left[\sqrt{\left\{\frac{1}{N}\sum_i (x_i - \bar{x}_{si})^2\right\}}\right]^2$$

$$\Rightarrow V_{si} = \frac{1}{5}\left[(1-3)^2 + (5-3)^2 + (2-3)^2 + (4-3)^2 + (3-3)^2\right] = 2$$

Where,

$\bar{x}_{al}$ and $\bar{x}_{si}$ are the mean percentage of weight of alumina and silica respectively.

N is number of samples or drill holes

x_i are the percentage values of ith sample.

50. Given data :

Number of iron ore sample (N) = 30

Mean grade (μ) = 60 %

Standard deviation (s) = 4 %

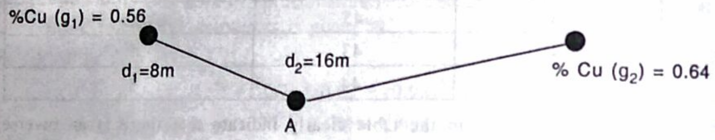
$Z_{0.975}$ = 1.96

It is given that the grade values are normally distributed.

The 95 % confidence limits for the population mean can be determined as

$$G = \mu \pm Z_{0.975}\frac{\sigma}{\sqrt{n}} = 60 \pm 1.96\frac{4}{\sqrt{30}} = 60 \pm 1.43$$

51. Given that,



Let the grade of copper percentage at A is G

Applying inverse distance weighting method, G can be determined as

$$\text{Grade percentage at A (G)} = \frac{g_1/d_1^2 + g_2/d_2^2}{1/d_1^2 + 1/d_2^2} = \frac{0.56/8^2 + 0.64/16^2}{1/8^2 + 1/16^2} = \frac{0.56+0.16}{1+0.25}$$

$$= 0.576\%$$

52. Given that, average grade of copper A is grade of copper at A.

Mill recovery (η) = 85 % = 0.85

Smelting and refining losses = 1 kg of copper per tonne of ore

Therefore, the amount of saleable copper in kg/te of ore is

= $G^*\eta^*1000 - 1$ = (0.576/100)*0.85*1000-1 =4.896-1 = 3.896 kg/te of ore

53. Given data of sample observations :

Zinc %	Lead%
6.0	4.2
5.2	3.0
6.0	6.8
2.4	2.0
4.0	3.2

Mean value of Zn percentage ($\bar{x}_{zn}$) = (6 + 5.2 + 6 + 2.4 + 4)/5 = 4.72

Mean value of Pb perentage ($\bar{y}_{pb}$) = (4.2 + 3 + 6.8 + 2 + 3.2)/5 = 3.84

Sample covariance between two variables is given by

$$\mathrm{Cov}(Zn, Pb) = \sum_{i=1}^{N}\frac{(x_i - \bar{x}_{zn})(y_i - \bar{y}_{pb})}{N}$$

Where xi and yi are the value of Zn percentage and Pb percentage respectively of ith sample.

N is the number of sample

$\Rightarrow \mathrm{cov}(Zn, Pb)$

$$\frac{(6-4.72)(4.2-3.84)+(5.2-4.72)(3-3.84)+(6-4.72)(6.8-3.84)+(2.4-4.72)(2-3.84)+(4-4.72)(3.2-3.84)}{5}$$

$$\Rightarrow \mathrm{cov}(Zn, Pb) = \frac{8.576}{5} = 1.715 \cong 1.72$$

54. Given data :

Sill value of semi-variogram model = C_0 = 5 m^2 (1)

Semi-variogram value at lag distance h = γ (h) = C_0–C_h =3 m^2 (2)

From above two eqns. we get,

C_h = 2

Correlogram value at lag distance h is given by

$$\text{Correlogram} \quad = \rho(h) = \frac{c_h}{c_0}$$

Where, C_h = covariance as a function of lag distance h

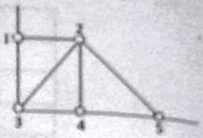
C_0 = variance at zero distance

Therefore,

$$\rho(h) = \frac{2}{5} = 0.4$$

55. Given data :

Sample No	1	2	3	4	5
Alumina %	3	4	3	4	4
x coordinate, m	0	20	0	20	50
y coordinate, m	30	30	0	0	0



Thickness of deposit (t) = 10m

Average density (γ) = 2200 kg/m^3

Average grade of Δ 123 for uniform tickness is given by

$$g_{123} = \frac{g_1+g_2+g_3}{3} = \frac{3+4+3}{3} = \frac{10}{3}\%$$

Similarly, for Δ 234

$$g_{234} \frac{g_2+g_3+g_4}{3} = \frac{4+3+4}{3} = \frac{11}{3}\%$$

And, $g_{245} = \frac{g_2+g_4+g_5}{3} = \frac{4+4+4}{3} = \frac{12}{3}\%$

Alumina content in reserve (Q)

$= g_{123}$ *Area of Δ123 + g_{234} *Area of Δ234 + g_{245} *Area of Δ245 *g*γ

$$\Rightarrow Q = \left[\frac{10}{300} * \frac{1}{2}\left\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right\} + \frac{11}{300} * \frac{1}{2}\left\{x_2(y_3-y_4)+\right.\right.$$

$$\left.\left.x_3(y_4-y_2)+x_4(y_2-y_3)\right\} + \frac{4}{100} * \frac{1}{2}\left\{x_2(y_4-y_5)+x_4(y_5-y_,)+x_5(y_2-y_4)\right\}\right] *10*$$

2200

$$\Rightarrow Q = \left[\frac{5}{300} * \left\{0(30-0)+20(0-30)+0(30-30)\right\} + \frac{5.5}{300} * \left\{20(0-0)+0(0-30)+\right.\right.$$

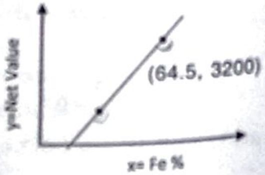
$$\left.\left.20(30-0)\right\} + \frac{2}{100} * \left\{20(0-0)+20(0-30)+50(30-0)\right\}\right] * 22000$$

$\Rightarrow$ Q = (10+11+12)*22000 = 726000kg = 7260 te = 0.00726 Mt

56. Given data :

Grade (% Fe)	Net value of ore (Rs/tonne)
64.5	3200
60.2	1800

It is given that the relationship between grade percentage and net value of ore is linear. The value can be plotted as :



The equation of the line can be written as

$$Y-3200 = \frac{3200-1800}{64.5-60.2}(x-64.5)$$

At break-even cut-off grade point (X_{ceg}), the net value of ore is equal to zero.

That is, y = 0

We have,

$$0-3200 = \frac{3200-1800}{64.5-60.2}(x_{ceg}-64.5)$$

$$\Rightarrow -3200 = \frac{1400}{4.3}(x_{ceg}-64.5)$$

$$\Rightarrow (x_{ceg}-64.5) = -9.83$$

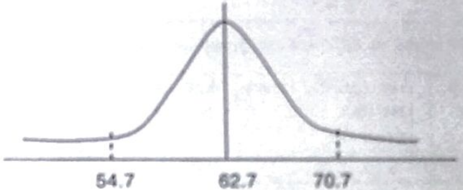
$$\Rightarrow x_{ceg} = 54.67\%$$

57. In the above problem, it is given that the grade follows normal distribution with mean and standard deviations as

Mean (μ) = 62.7 %

Standard deviation (σ) = 10 %

Cut off grade (g) = 54.67 % ≈ 54.7 %



The probability that grade value less than the cut off grade (g) is given by

$$P(x<54.7) = P(X-\mu<54.7-\mu) = P\left(\frac{x-\mu}{\sigma}<\frac{54.7-\mu}{\sigma}\right) = P\left(Z<\frac{54.7-62.7}{10}\right)$$

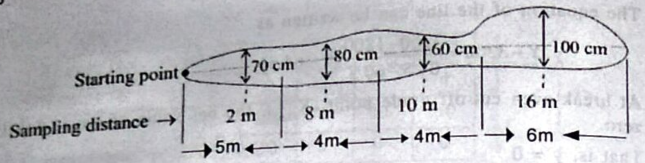
$$\Rightarrow P(Z<-0.8) = 0.2119 \approx 0.212$$

Note: The value of Z taken from normal distribution table corresponding to z = 0 (in row) and -0.8 (in column).

Therefore, the percentage of waste in the deposit based on the cut-off-grade is 21.2 %

58. Given data :

Distance of sample from starting point (m)	2	8	10	16
Width (cm)	70 (t_1)	80 (t_2)	60 (t_3)	100 (t_4)
Assay (g/te)	5 (g_1)	7 (g2)	6 (g_3)	4 (g_4)



The mean assay value can be determined as :

$$\text{Mean assay (g)} = \frac{A_1 * t_1 * g_1 + A_2 * t_2 * g_2 + A_3 * t_3 * g_3 + A_4 * t_4 * g_4}{A_1 * t_1 * A_2 * t_2 * A_3 * t_3 * A_4 * t_4}$$

A_1, A_2, A_3 and A_4 are the distance of influence of four sampling locations respectively.

$$\text{Therefore, Mean assay (g)} = \frac{5*70*5+4*80*7+4*60*6+6*100*4}{5*70+4*80+4*60+6*100} = 5.185$$

59. Given data of injury rates of mine workers are given as :

Age group of mine workers	Age-specific injury rate (per 1000 persons)	Age-specific population in the mine
18-32	1.8	1000
33-46	2.5	500
47-60	4.5	300

Number of injury in different age group can be calculated as :

Age group of mine workers	Age-specific injury rate (per 1000 persons)	Age-specific population in the mine	Number of injuries in different age groups
18-32	1.8	1000	1.8
33-46	2.5	500	(2.5/1000)*500=1.25
47-60	4.5	300	(4.5/1000)*300=1.35

Therefore, the injury rate per 1000 persons employed in the mine is given by

Injury rate = (Total number of injuries/ Total number of mine workers)*1000

= [(1.8+1.25+1.35)/(1000+500+300)]*1000 = (4.4/1800)*1000 = 2.44

60. The economic values of 2D blocks are given as :

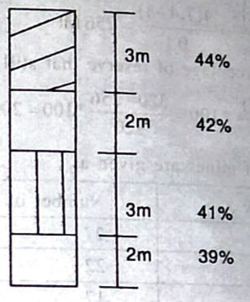
	1	2	3	4
1	–1	–1	1	–1
2	–1	1	3	–1
3	–1	–1	–1	–1

For Slope angle is 1:1, the block constitutes the ultimate pit for most economical extraction is

(1, 2), (1, 3), (1, 4), ((2, 3)

Economic value of extracted blocks = -1 + 1 -1 + 3 = 2

61. Given data :



Let the area of cross-section of the bore hole is A.

Composite value of ore in the borehole is given by

$$\text{Composite value of ore (G)} \frac{A * t_1 * g_1 + A * t_2 * g_2 + A * t_3 * g_3 + A * t_3 * g_3}{A * t_1 + A * t_2 * A * t_3}$$

$$\Rightarrow \text{Composite value of ore (G)} = \frac{3*44+2*42+3*41+2*39}{3+2+3+2} = \frac{132+84+123+78}{10}$$

$$\Rightarrow G = \frac{417}{10} = 41.7$$

62. Given data :

Mean grade of iron ore deposit (μ_1) = 63 %

During the course of mining, 30 % ores are rejected due to fineness. The average grade of the rejected ore is equal to

μ_2 = 72 %

Let the effecteve mean grade of the deposit used in plants is μ and the weight of in-situ ore is W

The effecteve mean grade can be determined as

$$\mu_1 = \frac{0.7W * \mu + 0.3W * \mu_2}{W}$$

$$\Rightarrow 63 = 0.7\mu + 0.3 * 72$$

$$\Rightarrow \mu = \frac{63 - 21.6}{0.7} = \frac{41.4}{0.7} = 59.14\%$$

63. Given data :

Total reserve (Q) = 320 Mt

Production in 1st year (q) = 4 Mt

It is given that production increases 10% every year.

Therefore, production in 21 years

$Q_1 = q + 1.1q + 1.1^2q + 1.1^3q.... 1.12^{20}q$

$$\Rightarrow Q_1 = \frac{4(1.1^{21}-1)}{1.1-1} = \frac{4(7.4-1)}{0.1} = 256 \text{ Mt}$$

Therefore, the percentage of reserve that still remains after 21 years

$$= \frac{Q-Q_1}{Q} * 100 = \frac{320-256}{320} * 100 = 20\%$$

64. The injury data of mines are given as :

Cause of injury	Number of injuries
Fall of roof	27
Fall of person	22
Rope haulage	17
Explosives	5
Other causes	4

The cumulative probability of injury due to fall of roof and fall of person is given by

$$P = \frac{N_1+N_2}{(N_1+N_2+N_3+N_4+N_5)} = \frac{27+22}{27+22+17+5+4} = \frac{49}{75} = 0.65$$

65. The number of workers suffering from pneumoconiosis in 10 coal mines is given as

Mine	I	II	III	IV	V	VI	VII	VIII	IX	X
Number	10	16	14	15	14	12	17	13	15	12

To calculate percentiles, sort the data (x_i) in increasing order.

Sorted data :

Mine	I	VI	X	VIII	III	V	IV	IX	II	VII
Number (x_i)	10	16	14	15	14	12	17	13	15	12
I	1	2	3	4	5	6	7	8	9	10

P_I percentile of data set can be determined as

$$P_I = 100 * \frac{I-0.5}{n}$$

Where, P_i is percentile

I = the sequence number

n = number of observation

$$\Rightarrow 50 = 100 * \frac{I-0.5}{10}$$

$\Rightarrow I = 5 + 0.5 = 5.5 = f\ (=0.5) + k\ (=5)$

Therefore, 50% percentile value $(x_{5.5}) = (1-f)*x_k + f*x_{k+1}$

$= 0.5*14 + 0.5*14 = 14$

Thus, the number of mines falling above the 50th percentile in terms of the number of workers suffering from pneumoconiosis is 4.

66. Given data :

Sill = 10 $(\%)^2$

Covariance at a particular lag distance = c(h) = 6$(\%)^2$

The semi-variance at the same lag distance can be determined as

γ (h) = Sill – c(h) = 10 – 6 = 4 $(\%)^2$

■